Optimization of 3PL Transportation Schedule Based on LR and BB Algorithm in Supply Chain Management

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Abstract. Aiming at the characteristics of diversity of the 3PL transportation schedule, the algorithm of LR and BB was applied to the study of the Optimization of 3PL transportation schedule in Supply Chain Management. Firstly, the algorithm of LR and BB was applied in 3PL transportation schedule, the constraints of 3PL transportation schedule was identified. Secondly, the model of transportation scheduling based on LR and BB algorithm was put forward and measured. Finally, the numerical examples show that the transport efficiency can be promoted significantly by the study of the 3PL transportation schedule based on LR and BB algorithm, improving the transportation schedule problem.

Introduction

At present, many domestic and foreign literature dedicated to research all aspects of 3PL transportation schedule. In reference [1], the problem of transportation with time window constraints was studied by Jean in 2004, and put forward a solution of parallel genetic algorithm. In reference [2], the MTO production and 3PL transportation schedule problem was raised by Dennis in 2007. The research of 3PL started late in China. In reference [3], the problem of 3PL transportation schedule based on JIT was studied by Kunpeng Li in 2008. In reference [4], the vehicle loading problem of multi products and multi vehicles was studied by Wu Ying in 2008, and the BB algorithm was applied to solve the problem. Many achievements have been achieved about 3PL transportation problem in building model and algorithm application etc, but this problem involves many factors, the study for the multi-objective problem is not enough currently. This paper analyzes the constrains of 3PL transportation schedule, the LR and BB algorithm was applied to measure and to minimize the transportation cost and the earliness/tardiness penalties, such can provide a new method with Stability, efficiency and adaptability to solve the problem of combined transportation for manufacturers and transporters.

The Basic Model of 3PL Transportation Problem

Problem Description

The 3PL service providers to provide fixed transportation service plan, this transporter includes F transport vehicles, the transport fleet have Des\_B starting points and Des\_E end points, the maximum loading capacity Hj, unit transportation cost Cj, transportation destination Desj, departure time is Depj and the stop time is Arrj. Each order i has the size Qi, destination
Desi and the delivery Datei. If the arrival time in advance of delivery, it will produce the earliness penalty cost \( E_{ij} \), if the arrival time after delivery, it will produce the tardiness penalty cost \( L_{ij} \), the suppliers/manufacturers has a limit of maximum capacity, the maximum supply capacity in unit time is \( R_m \).

The Mathematical Model of Problem

If the penalty cost coefficient is \( \alpha_i \) of order \( i \) in advance of arrival, the penalty cost coefficient is \( \beta_i \) of order \( i \) after arrival, the order \( i \) was transported by vehicle \( j \), the unit penalties cost of earliness/tardiness is:

\[
E_{ij} = \max\left(0, Date_i - Arr_j\right) \times \alpha_i
\]

\[
L_{ij} = \max\left(0, Arr_j - Date_i\right) \times \beta_i
\]

The selection of decision variables:

\[
X_{ij} = \begin{cases} 
1 & \text{if order } i \text{ was distributed by vehicle } j \\
0 & \text{if order } i \text{ was not distributed by vehicle } j
\end{cases}
\]

The mathematical model of transportation problem is as follows:

\[
(IP) \quad Z = \min \sum_i \sum_j C_{ij} X_{ij} + \sum_i \sum_j X_{ij} Q_i \left(E_{ij} + L_{ij}\right)
\]

Subject to:

\[
\sum_j X_{ij} = 1 \quad \text{for all } i
\]

\[
\theta X_{ij} \Delta_{ij} < 1 \quad \text{for all } i, j
\]

\[
\sum_i X_{ij} Q_i \leq H_j \quad \text{for all } j
\]

\[
\sum_{j=1}^{J} \sum_{i} X_{ij} Q_i \leq D_j \sum_{m=1}^{M} R_m \quad \text{for all } j
\]

\[
X_{ij} \in \{0,1\} \quad \text{for all } i, j
\]

The above model, the formula (3) shows that minimizing the total distribution cost, the formula (4) limits that a order only transported by a vehicle, the formula (5) makes sure the destination of order \( i \) and vehicle \( j \) is same, the formula (6) limits that the number of vehicle loading orders cannot exceed the maximum capacity of vehicle loading, the formula (7) ensure the number of loaded products when the vehicle started.
The Design of Lagrangian Relaxation Method

The vehicle capacity constraints (6) and the supply capacity constraints (7) of the original problem are relaxed by using the Lagrange multiplier \( A_j \) and \( B_j \), \( A_j, B_j \geq 0, j=1,2...,J \). The relaxation problem is:

\[
LR_{A,B} = \min \sum_i C_i X_{ij} Q_i + \sum_j E_{ij} + L_{ij} + \sum_j A_j \left( \sum_i X_{ij} Q_i - H_j \right) + \sum_j B_j \left( \sum_i X_{ij} Q_i - D_j \sum_m R_m \right)
\] (9)

Subject to: The formula (4), (5) and (8), \( A_j, B_j \geq 0 \). If (LR) has not the feasible solution, so is original problem (IP). If the optimal solution of (LR) is existential, the optimal solution is a lower bound of (IP). \( \forall A, B \geq 0 \Rightarrow LR_A, B \leq Z \). The (LR) is a lower bound of (IP), so the following dual problem (PD) need to be solved, and find the closest lower bound of the optimal solution.

\[
(PD) Z_{LD} = \max LR_{A,B}
\] (10)

Solving the problem is to find the smallest difference between ZIP and ZLD, to get better calculation by getting a better \( X_{ij}, A_j \) and \( B_j \). The optimal Lagrange multiplier is obtained by applying the subgradient algorithm (PD). According to the principle of subgradient algorithm and give the initialization of \( A_0 j=0 \) and \( B_0 j=0 \) of Lagrange multiplier, by using the following formula to iterate:

\[
A_{j+1}^n = \max \left\{ 0, A_j^n + t_n \left( \sum_i X_{ij} Q_i - H_j \right) \right\}
\] (11)

\[
B_{j+1}^n = \max \left\{ 0, B_j^n + t_n \left( \sum_j X_{ij} Q_i - D_j \sum_m R_m \right) \right\}
\] (12)

\[
t_n = \frac{\sigma_n (Z - LR)}{\sum_j \left( \sum_i X_{ij} Q_i - H_j \right)^2 + \sum_j \left( \sum_i X_{ij} Q_i - D_j \sum_m R_m \right)^2}
\] (13)

The \( t_n \) is the step of stage \( n \), its calculation value is positive.

The Design of Branch and Bound Algorithm

The transportation problem in this paper needs to get the allocation scheme of each vehicle order. Assuming that \( \phi_j \) is the set of orders which vehicle \( j \) can be fully loaded, and \( \psi_j \subset \phi_j \) said vehicle \( j \) is not loaded fully. Node selection: If the current node is not lost, then the next step will be to consider the sub node of this node, if the current node has been lost, to find back along the branch from the node to the root node. Branching strategy: When a node generates a new branch, increasing an order of partial orders set in vehicle \( j \), so as to obtain a new branch. If the current node is corresponding to a full set, vehicle \( j \) has finished loading and produced new partial order set \( \psi_{j+1} \) and get a new branch in vehicle \( j+1 \), so it ensure the traversal of all branches. Pruning principle: According to the upper and lower bounds on the above IP\( j \). If the new branch is bigger than upper bound obtained, cut the branch. Another situation need to cut the branch is: The new branch of non feasible solution; The problem has been a feasible solution to meet the requirements. According to convergence: If the lower bound of a node is the optimal upper bound, the algorithm stops and the feasible solution obtained is the optimal
solution of original problem. If there are no such nodes that should be traversal of all branches and select the minimum calculation of the upper bound of the leaf nodes, where the branch is the optimal solution.

Case Analysis

The required transport building materials as the experimental object this paper, and the cement transport as an example to analyze this kind of transportation problem. This section proposes two transport models and to study them separately, one is the transport model of vehicle resource constrains and another is the transport model of subsection.

The Transport Model of The Vehicle Resource Constrains and The Subsection

According to the mentioned standard transport model above, using the Lagrange multiplier \( A_j \) and \( B_j \) to relax the vehicle capacity constraints and the supply capacity of supplier of original problem respectively, \( A_j, B_j \geq 0, j=1,2,...,J \). The relaxation problem expressed as follows:

\[
LR_{A,B} = \text{Min} \sum_{t \omega} \sum_{j} C_j X_{t \omega j} Q_{t \omega} + \sum_{t \omega} \sum_{j} X_{t \omega j} Q_{t \omega} L_{t \omega j} + \rho \sum_{t \omega} \sum_{j} (1 - X_{t \omega j}) Q_{t \omega} + \\
\sum_{j} \left( \sum_{t \omega} X_{t \omega j} Q_{t \omega} - H_j \right) + \sum_{j} \left( \sum_{t \omega} X_{t \omega j} Q_{t \omega} - D_j \sum_{m=1}^{M} R_m \right)
\]

\( (14) \)

According to the principle of Lagrange relaxation method to relax the vehicle capacity constraints and the supply capacity constraints, the relaxation problems expressed as follows:

\[
LR_{A,B} = \text{Min} \sum_{t \omega} \sum_{j} C_j X_{t \omega j} Q_{t \omega} + \sum_{t \omega} \sum_{j} X_{t \omega j} Q_{t \omega} L_{t \omega j} + \\
\sum_{j} \left( \sum_{t \omega} X_{t \omega j} Q_{t \omega} - H_j \right) + \sum_{j} \left( \sum_{t \omega} X_{t \omega j} Q_{t \omega} - D_j \sum_{m=1}^{M} R_m \right)
\]

\( (15) \)

Analysis and Verification

To cement transportation as an example to analyze two models above, and setting the parameters of cement transportation, as shown in table 1:

<table>
<thead>
<tr>
<th>parameters</th>
<th>instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantity and size of order (per day)</td>
<td>quantity: 10, size: obey the uniform distribution [15,30], unit: tons</td>
</tr>
<tr>
<td>quantity and capacity of vehicles (per day)</td>
<td>quantity: 6, capacity: obey the uniform distribution [15,30], unit: tons</td>
</tr>
<tr>
<td>unit transport cost and destination of vehicles</td>
<td>unit transport cost: obey the uniform distribution [350,400], unit: ¥ per ton, destination: {1, 2}, if it is a direct transportation, the destination is 1</td>
</tr>
</tbody>
</table>
departure time and arrival time of vehicles
delivery date and destination of orders
deliverability of suppliers
Penalties

departure: \{1,2,3\}, if it is a direct transportation, the departure time is 1, arrival time: \{3,4,5,6\}, the probability of distribution of 25\% to each order
delivery date: \{3,4\}, the probability of distribution of 25\% to each order, the destination is 1
supplier 1: 100 tons, productivity: 12 tons per hour, supplier 2: 120 tons, productivity: 12 tons per hour

The Comparison between Vehicle Resource Constraints and Supply Capacity Constraints. To cement transport as an example, analyzing the influence of vehicle resource shortage for target cost. When the transport is peak, the demand reached 7000 tons per month, according to the capacity of 25 tons per car to calculate, it need about 10 vehicles every day. In fact, the vehicles are not enough, it has only six vehicles daily. From that, increasing the number of vehicles can reduce much marginal cost. As shown in table 2, increasing a vehicle with the total cost will reduced substantially.

Table 2. Sensitive Analysis of Quantity and Cost of Vehicles.

<table>
<thead>
<tr>
<th>quantity of vehicles</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>total cost(thousand)</td>
<td>1600</td>
<td>1400</td>
<td>950</td>
<td>700</td>
<td>600</td>
</tr>
</tbody>
</table>

Similarly, when the transport is peak, the supply capability of two supplier only reaches about 170 tons per day. Thus, reducing the cost of target by improve the supply capability. As shown in table 3, with the increase of the supply capacity, the total cost started greatly reduce and there is a slight rise to stable state later.

Table 3. Sensitive Analysis of Deliverability and Cost.

<table>
<thead>
<tr>
<th>deliverability(ton per day)</th>
<th>170</th>
<th>180</th>
<th>190</th>
<th>200</th>
<th>210</th>
<th>220</th>
<th>230</th>
<th>240</th>
<th>250</th>
<th>260</th>
<th>270</th>
<th>280</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost(ten thousand)</td>
<td>155</td>
<td>140</td>
<td>130</td>
<td>110</td>
<td>115</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
<td>123</td>
</tr>
</tbody>
</table>

Due to the funds is limited and there were 700 thousand which only get a maximum of three new cars. To get the optimal funds allocation scheme by combining with increasing vehicles and improving supply capacity, while five calculation examples take from every combination to obtain the average value and all the combination of target cost, as shown in table 4 (PR: tons per day; VN: vehicles; cost: thousand).
Table 4. The Influence of Increasing the Number of Vehicles and Improving the Deliverability to Cost.

<table>
<thead>
<tr>
<th>PR VN</th>
<th>180</th>
<th>190</th>
<th>200</th>
<th>210</th>
<th>220</th>
<th>230</th>
<th>240</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1472.8</td>
<td>1323.5</td>
<td>1344.2</td>
<td>1317.4</td>
<td>1356.6</td>
<td>1389.2</td>
<td>1422.3</td>
<td>1446.3</td>
</tr>
<tr>
<td>8</td>
<td>1254.7</td>
<td>1203.8</td>
<td>1175.8</td>
<td>1129.1</td>
<td>1123.8</td>
<td>1175.7</td>
<td>1183.6</td>
<td>1215.5</td>
</tr>
<tr>
<td>9</td>
<td>1149.6</td>
<td>1085.6</td>
<td>1019.7</td>
<td>1007.3</td>
<td>1014.9</td>
<td>1065.3</td>
<td>1092.8</td>
<td>1119.3</td>
</tr>
<tr>
<td>10</td>
<td>1102.9</td>
<td>1057.1</td>
<td>976.9</td>
<td>970.1</td>
<td>972.6</td>
<td>1018.4</td>
<td>1046.7</td>
<td>1064.2</td>
</tr>
</tbody>
</table>

Table 4 shows that increasing 3 vehicles and maintaining the supply capacity of 210 tons per day can make the total cost lowest. After this scheme was worked that reduces the daily cost (including penalty cost) of about 450 thousand.

**The Comparison between Direct Transportation and Transportation of Subsection.**

Due to the presence of the large uncertainties in the transportation time, so the staging was used that can make the transportation normative, and reduce unfavorable factors of the transportation of next day and the fatigue driving. To show that which scheme enable benefit become better, this paper chooses the transportation data of 14 days by calculation and comparative experiments, as shown in table 5 (cost: thousand).

<table>
<thead>
<tr>
<th>days</th>
<th>cost of DT</th>
<th>cost of TS</th>
<th>days</th>
<th>cost of DT</th>
<th>cost of TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1080</td>
<td>560</td>
<td>8</td>
<td>1480</td>
<td>1130</td>
</tr>
<tr>
<td>2</td>
<td>1420</td>
<td>1180</td>
<td>9</td>
<td>1230</td>
<td>820</td>
</tr>
<tr>
<td>3</td>
<td>930</td>
<td>1320</td>
<td>10</td>
<td>1150</td>
<td>760</td>
</tr>
<tr>
<td>4</td>
<td>1370</td>
<td>970</td>
<td>11</td>
<td>1260</td>
<td>1130</td>
</tr>
<tr>
<td>5</td>
<td>1180</td>
<td>1060</td>
<td>12</td>
<td>1390</td>
<td>990</td>
</tr>
<tr>
<td>6</td>
<td>1450</td>
<td>1290</td>
<td>13</td>
<td>1040</td>
<td>870</td>
</tr>
<tr>
<td>7</td>
<td>1230</td>
<td>950</td>
<td>14</td>
<td>1280</td>
<td>1120</td>
</tr>
</tbody>
</table>

Table 5 shows that the average target cost for transportation of subsection (including penalty cost and transportation cost) less than the one of direct transportation. By transportation of subsection can improve the transportation efficiency and economic benefits effectively.

**Summary**

Analyzing the research direction of 3PL transportation schedule problem and optimizing the mode of transportation schedule with the background of supply chain management in this paper. The vehicle resource constraints and 3PL transport model based on transportation of subsection were built, the verification and analysis shows that LR and BB hybrid intelligent algorithm can solve the transportation problem of high-order optimal solution effectively. The numerical
examples show that the vehicles resource constraints problem can be solved by increasing the number of vehicles and improving the supply capacity, and the transportation cost of transportation of subsection is lower than the cost of direct transportation.

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References