Research on Inventory and Transportation Integrated Optimization for Train Fuel Supply under VMI Model

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Abstract. The efficient and economic organization of the railway fuel supply and good diesel locomotive fuel supply management is very significant to ensure the railway transport safety, punctuality, good service, transportation and production costs reduced, and it also improves the economic efficiency. This study comes from the reality of China's rail fuel supply business processes. It established the rail fuel inventory—transportation integrated optimization model whose goal is the total cost of the minimum and uses iterative genetic algorithm to solve it. Then we get the final optimal order quantity, total cost and transport path. The result provides a reference for the optimization of train fuel supply chain.

Introduction

Inventory and transportation are the links that cost most of railway fuel supply system. While the characteristics of the large demand, long route and strict demand of the inventory of the railway fuel supply make it hard to manage the inventory and transportation. It is important to make a strategic of inventory and transportation of railway fuel supply to decrease the cost of the whole supply chain and achieve maximum system efficiency.

At present, the research on the integrated optimization of inventory and transportation both domestic and abroad has been studied. Chien W (1989) \cite{1} studied the problem at day cycle. A mixed integer programming model is established to compare the stock and transportation of several days after another. Then it solve the daily maximum profit using large relaxation method. Viswanat han and Mathur (1997) \cite{2} studied a variety of products, a single warehouse and multiple retailers distribution system, using the heuristic algorithm to solve the minimum total cost of inventory and transportation system, and determine the specific distribution and distribution route. Armand Baboli (2006) \cite{3} studied the problem of the lowest total cost of inventory and transportation composed of a single supplier and a single retailer. The conclusion of integrated optimization is better than not integrated optimization was drawn.

The model of Inventory and Transportation Integrated Optimization

Problem Description

Railway fuel supply system is a two level supply system composed of one refinery, China Railway Materials Company Limited, several fuel stations of railway administration. China
Railway Materials Company Limited takes VMI inventory management model to find out the situation of fuel usage and inventory levels and make the strategic of transportation and inventory. The paper studies the travelling salesman problem under VMI model based on railway transportation, and seeks the optimal order quantity and transport routes to minimize the cost of the whole system.

Symbolic Interpretation of the Model

In order to take the third-party transportation companies into inventory management analysis, the costs of transport are separated from ordering costs in this paper. The total cost of the fuel supply system contains the purchase costs, ordering costs, inventory carrying costs, and transportation costs. To simplify the model, the rail fuel supply chain was constrained.

### Symbolic Interpretation

1. **Cost of Purchase**: The cost of purchase can be expressed as:
   \[ C_i = \sum_{j} c_{ij} q_{ij} \]
   \[ C_i = \sum_{j} c_{ij} q_{ij} \]
   Wherein,
   - \( n \): The number of fuel station;
   - \( p \): The number of types of fuel;
   - \( i \): The label of fuel station; \( i = 1, 2, \ldots, n \) (the label of refinery is 0);
   - \( j \): The label of types of fuel; \( j = 1, 2, \ldots, p \);
   - \( D_i \): The demand of fuel station \( i \) for fuel \( j \) during a planned period (tank);
   - \( \alpha \): The unit purchase price of fuel \( j \).

2. **Ordering Costs**: The ordering costs can be expressed as:
   \[ O = \beta Q \]
   \[ O = \beta Q \]
   Wherein,
   - \( \beta \): The cost-per-order (yuan);
   - \( D \): The total demand of fuel station \( i \) for fuel \( j \) during a planned period (tank);
   - \( Q \): The cost-per-order of the whole system;

3. **Inventory Holding Costs**: The inventory holding costs can be expressed as:
   \[ H = \gamma \]
   \[ H = \gamma \]
   Wherein,
   - \( \gamma \): The unit inventory holding costs of fuel \( j \) during one planned period (yuan).

4. **Transportation Costs**: The transportation costs can be expressed as:
   \[ T = \sum_{k} c_{ik} x_{ik} + \sum_{k} c_{kj} y_{kj} \]
   Wherein,
   - \( c_{ik} \): The cost of transportation from fuel station \( i \) to transport route \( k \);
   - \( c_{kj} \): The cost of transportation from transport route \( k \) to fuel station \( j \);
   - \( x_{ik} \): The amount of fuel transported from fuel station \( i \) to transport route \( k \);
   - \( y_{kj} \): The amount of fuel transported from transport route \( k \) to fuel station \( j \).

In summary, the total cost of the fuel supply system contains the purchase costs, ordering costs, inventory carrying costs, and transportation costs. The model seeks the optimal order quantity and transport routes to minimize the cost of the whole system.
Wherein,

- $C^T(Q)$: The cost of transportation of through train;
- $C^S(Q)$: The cost of single tank car transportation;
- $c_f$: The fixed cost of single tank car transportation;
- $c_V$: The changeable cost of the route of tank car;
- $c_E$: The transportation cost of each empty tank running a unit distance;
- $d_{ij}$: The distance between point $i$ and point $j$, $i, j = \{1, 2, \cdots, n\}$ (kilometers);
- $M$: The total number of trains available;
- $x$: The total number of through trains, $x = \sum_{i=1}^{n} x_i$;
- $n_k$: The number of fuel station delivered by the train $k$, $0 \leq n_k \leq n$, and
  \[\text{sign}(n_k) = \begin{cases} 
1, & n_k \geq 1 \\
0, & n_k < 1
\end{cases}\]
- $r_{ki}$: The order of fuel station $i$ in the transportation path of the train $k$, let $r_{k0} = 0$ means refinery, $R_k$ expresses the transportation route of the train $k$, $R_k = \{ r_{ki} \mid r_{ki} \in \{1, 2, \cdots, n\}, i = 1, 2, \cdots, n_k \}$.

**Establish the Model**

In summary, the model of inventory and transportation integrated optimization can be expressed as:

\[
min C(Q) = min(C_f(Q) + C_S(Q) + C_V(Q) + C_E(Q))
\]

\[
= \min \left\{ \sum_{j=1}^{p} \sum_{i=1}^{n} D_{ij} x_j + \beta D / Q + \frac{1}{2} Q \sum_{j=1}^{p} \sum_{i=1}^{n} D_{ij} y_j + \sum_{i=1}^{n} x_i (c_f + c_v d_{i0} W + c_E d_{i0} W) \right\}
\]

\[
+ \sum_{k=1}^{M-x} \text{sign}(n_k) \left[ c_f + \sum_{i=1}^{n_k} c_v d_{i} y_i \sum_{m=0}^{n_k} y_{i} \sum_{i=1}^{n_k} C_E d_{i} (i-1) y_{i-1} \sum_{m=0}^{i-1} y_{i-m} \right]
\]  \hspace{1cm} (7)

s.t.

\[
Q_i = x_i W + y_i
\]  \hspace{1cm} (8)

\[
x = \sum_{i=1}^{n} x_i
\]  \hspace{1cm} (9)

\[
\sum_{i=1}^{n_k} y_i \leq W
\]  \hspace{1cm} (10)

\[
0 \leq n_k \leq n
\]  \hspace{1cm} (11)

\[
\sum_{k=1}^{M-x} n_k = n
\]  \hspace{1cm} (12)

\[
R_{k1} \cap R_{k2} = \emptyset, \quad \forall k_1 \neq k_2
\]  \hspace{1cm} (13)
Wherein,  
\begin{align*}
&\text{C}(Q) \text{: Total inventory and transportation costs (yuan);} \\
&d \text{: The longest distance of a train running one time (kilometers)}
\end{align*}

In the above model, the Eq. 8 calculates the number of tank cars according to the number of ordering; the Eq. 9 represents the sum of the number of through trains of each fuel stations is the total number of through trains; the Eq. 10 represents the numbers of tank cars cannot exceed the total number of the tank cars of the through train; the Eq. 11 represents that the number of the fuel station delivered by the train \( k \) cannot exceed the total number of the fuel station; the Eq. 12 guarantee each fuel station can get delivery services; the Eq. 13 represents each fuel station can only be served by one train; the Eq. 14 guarantee each train can only run a distance within its capabilities; \( \text{sign}(n_k) = 1 \) of the Eq. 15 represents the number of the fuel station served by the train \( k \) cannot be 0; \( \text{sign}(n_k) = 0 \) means the train \( k \) does not participate in the transportation.

Solve the Model

The Solution of the Model Based on IM-GA

The Iterative Method embedded in Genetic Algorithm, the IM-GA method has been used to solve the model. The genetic algorithm program is nested within an iterative procedure. The paper chooses Microsoft Visual Studio 2010 as a development tool, and solve the rail fuel inventory - transportation integrated optimization model through C# programming language. Use the EOQ method for solving the initial optimal order quantity, then plug this order into the cost of transportation part making use of genetic algorithms to calculate the transportation costs, at last, plug the transportation costs back into the optimization model in order to get the optimal order quantity. When the iteration reaches a certain number of iterations, it is terminated, a single distribution volume output and minimum total cost of the corresponding order quantity, order number, transport routes, as well as the different oil oiling point are output. Then end the calculation.

The Iterative Process of IM-GA

Based on the above ideas, solving process model can be expressed as the following step by Eq.:

\begin{align*}
\text{Step0:} & \\
&k = 0, \beta = \beta^{'}, Q^0 \text{, calculate } C_4(Q^0) \text{ of the MA,} \\
&\text{return to the main to calculate } C(Q^0) \text{, and let } \text{min} C(Q) = C(Q^0), \text{ go Step1;} \\
\text{Step1:} & \\
&k = k + 1, \beta = \beta + C_4(Q^{k-1}), Q^k \text{, calculate } C(Q^k), C_4(Q^k) \text{ and } C(Q^k)
\end{align*}
Step 2: If \( C(Q^k) < C(Q^{k-1}) \), then return Step 1; or let \( \min C(Q) = C(Q^{k-1}) \), go Step 3;

Step 3: \( k = k + 1 \), let \( Q^i = \frac{Q^{k-1} + Q^{k-2}}{2} \), calculate \( C_i(Q^k) \) and \( C(Q^k) \), if \( C(Q^k) \geq \min C(Q) \), then go Step 4, or stop the calculation, take \( Q^k \) as the best order quantity and \( C(Q^k) \) as the minimum cost;

Step 4: \( k = k + 1 \), let \( Q^k = \frac{Q^{k-1} + Q^d}{2} \), \( Q^d \) is the value of \( Q \) in \( \min \{ C(Q^{k-2}), C(Q^{k-3}) \} \), calculate \( C_i(Q^k) \) and \( C(Q^k) \);

Step 5: If \( C(Q^k) \geq \min C(Q) \), then go Step 4 (terminated when the iterations reach a certain number), or end the iterations, take \( Q^k \) as the best order quantity and \( C(Q^k) \) as the minimum cost.

The Transport Path Selection for IM-GA

1. Determine new coding scheme solutions
The paper codes the route of trains in the way of serial number coding. Use 0, 1, 2, \( n + M - x - 1 \) to express the refinery and all the fuel station. Thus the permutations and combinations of the subsets with the number of \( n + M - x \) means one transportation plan.

2. Generate initial species randomly
Generate a combination with 0, 1, 2, \( n + M - x - 1 \), then check the transportation route between two neighbor refineries (including actual and virtual refinery) and find out if the route meet the constraint condition of Eq. 10 and Eq. 14. If the route meet the constraint condition, then the sequence can be used as the initial population; otherwise repeat to generating random sequence, and check whether the constraint condition is satisfied until the sequence has been found to meet the constraints.

3. Fitness evaluate
The method of coding and generating initial species which is taken by this paper can meet constrains Eq. 10 ~ Eq. 14. Thus we only need to solve the minimum value of the objective function when fitness evaluation. Therefore, take the reciprocal value of the objective function for evaluating fitness, which means the fitness function \( F_i \) of the individual \( i \) is:

\[
F_i = \frac{1}{C_i^3(Q)} \quad (16)
\]

4. Select
Arrange the \( N \) number of individuals of generation \( i \) as an order of the size of fitness from large to small, copy an individual ranked 1 directly into the next generation, and rank it as number 1, generate the \( N - 1 \) number of remaining individuals with wheel selection method.

5. Take crossover operation
Directly copy the first individual into the next generation and cross the remains with Genetic Operators. Generate two integer between \([1, n + M - x - 1]\). Exchange these two gens according to a certain probability to get a new individual and inherit it into next generation. Then calculate the fitness of the new species and check if they meet the conditions of termina-
tion. If it meet the condition, end the calculation. Otherwise generate new species until we find out new species meet the conditions.

Example Analysis

The fundamental Information of the Example

The distance between the refineries and the fuel stations of Shanghai Railway Administration and the demand of the three kinds of fuel (0#, 0#(m) and -10#) and the fee of purchasing and storing the fuel are shown as the Table 1, 2 and 3.

Table 1. Distance between Refinery and Requirement Points [Units: km].

<table>
<thead>
<tr>
<th>Distance</th>
<th>Refinery 0</th>
<th>Fuel Station1</th>
<th>Fuel Station2</th>
<th>Fuel Station3</th>
<th>Fuel Station4</th>
<th>Fuel Station5</th>
<th>Fuel Station6</th>
<th>Fuel Station7</th>
<th>Fuel Station8</th>
<th>Fuel Station9</th>
<th>Fuel Station10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refinery 0</td>
<td>0</td>
<td>50</td>
<td>60</td>
<td>80</td>
<td>90</td>
<td>60</td>
<td>70</td>
<td>150</td>
<td>80</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Fuel Station1</td>
<td>50</td>
<td>0</td>
<td>110</td>
<td>100</td>
<td>40</td>
<td>50</td>
<td>120</td>
<td>40</td>
<td>210</td>
<td>140</td>
<td>100</td>
</tr>
<tr>
<td>Fuel Station2</td>
<td>60</td>
<td>110</td>
<td>0</td>
<td>50</td>
<td>150</td>
<td>120</td>
<td>40</td>
<td>210</td>
<td>140</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Fuel Station3</td>
<td>80</td>
<td>100</td>
<td>50</td>
<td>0</td>
<td>140</td>
<td>140</td>
<td>70</td>
<td>120</td>
<td>160</td>
<td>130</td>
<td>120</td>
</tr>
<tr>
<td>Fuel Station4</td>
<td>90</td>
<td>40</td>
<td>150</td>
<td>140</td>
<td>0</td>
<td>30</td>
<td>160</td>
<td>120</td>
<td>140</td>
<td>140</td>
<td>120</td>
</tr>
<tr>
<td>Fuel Station5</td>
<td>60</td>
<td>50</td>
<td>120</td>
<td>140</td>
<td>30</td>
<td>0</td>
<td>130</td>
<td>90</td>
<td>110</td>
<td>110</td>
<td>90</td>
</tr>
<tr>
<td>Fuel Station6</td>
<td>70</td>
<td>120</td>
<td>40</td>
<td>70</td>
<td>160</td>
<td>130</td>
<td>0</td>
<td>220</td>
<td>150</td>
<td>60</td>
<td>110</td>
</tr>
<tr>
<td>Fuel Station7</td>
<td>150</td>
<td>140</td>
<td>210</td>
<td>120</td>
<td>120</td>
<td>90</td>
<td>220</td>
<td>0</td>
<td>100</td>
<td>200</td>
<td>160</td>
</tr>
<tr>
<td>Fuel Station8</td>
<td>80</td>
<td>130</td>
<td>140</td>
<td>160</td>
<td>140</td>
<td>110</td>
<td>150</td>
<td>100</td>
<td>0</td>
<td>110</td>
<td>100</td>
</tr>
<tr>
<td>Fuel Station9</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>130</td>
<td>140</td>
<td>110</td>
<td>60</td>
<td>200</td>
<td>110</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>Fuel Station10</td>
<td>40</td>
<td>90</td>
<td>100</td>
<td>120</td>
<td>120</td>
<td>90</td>
<td>110</td>
<td>160</td>
<td>100</td>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Demand of Requirement Points for a Quarter [Units: tank].

<table>
<thead>
<tr>
<th>Demand</th>
<th>Fuel 1</th>
<th>Fuel 2</th>
<th>Fuel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Station 1</td>
<td>18</td>
<td>41</td>
<td>26</td>
</tr>
<tr>
<td>Fuel Station 2</td>
<td>82</td>
<td>35</td>
<td>165</td>
</tr>
<tr>
<td>Fuel Station 3</td>
<td>56</td>
<td>29</td>
<td>117</td>
</tr>
<tr>
<td>Fuel Station 4</td>
<td>48</td>
<td>70</td>
<td>132</td>
</tr>
<tr>
<td>Fuel Station 5</td>
<td>70</td>
<td>52</td>
<td>28</td>
</tr>
<tr>
<td>Fuel Station 6</td>
<td>91</td>
<td>75</td>
<td>176</td>
</tr>
<tr>
<td>Fuel Station 7</td>
<td>29</td>
<td>27</td>
<td>134</td>
</tr>
<tr>
<td>Fuel Station 8</td>
<td>13</td>
<td>85</td>
<td>56</td>
</tr>
<tr>
<td>Fuel Station 9</td>
<td>28</td>
<td>13</td>
<td>133</td>
</tr>
</tbody>
</table>
Table 3. Purchase Price and Storage Fee of the Fuel.

<table>
<thead>
<tr>
<th></th>
<th>Fuel 1</th>
<th>Fuel 2</th>
<th>Fuel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit price [yuan]</td>
<td>8785</td>
<td>9330</td>
<td>9751</td>
</tr>
</tbody>
</table>

The Results of the Calculation

The study program has run 15 times and record the results of each run, and take the minimum total cost of the relevant data among the 15 times for the optimal solution. The approximate optimal solution are as follows:

1. Order Quantity: 212;
2. Order Times: 10;
3. Transportation Route: 0-2-3-14-1-4-12-10-8-11-6-13-5-7-15-9-0 (0 represents refinery, 1-10 represents fuel station, 11-15 represents virtual refinery);
4. Total Costs: 22296731.23 yuan;
5. The distribution amount of requirement points for each time are shown as table 4.

Tab. 4 Distribution Amount of Requirement Points for Each Time [Units: tank]

<table>
<thead>
<tr>
<th>Delivery Quantity</th>
<th>Fuel 1</th>
<th>Fuel 2</th>
<th>Fuel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Station 1</td>
<td>1.77</td>
<td>4.04</td>
<td>2.56</td>
</tr>
<tr>
<td>Fuel Station 2</td>
<td>8.07</td>
<td>3.45</td>
<td>16.24</td>
</tr>
<tr>
<td>Fuel Station 3</td>
<td>5.51</td>
<td>2.86</td>
<td>11.52</td>
</tr>
<tr>
<td>Fuel Station 4</td>
<td>4.73</td>
<td>6.89</td>
<td>13.00</td>
</tr>
<tr>
<td>Fuel Station 5</td>
<td>6.89</td>
<td>5.12</td>
<td>2.76</td>
</tr>
<tr>
<td>Fuel Station 6</td>
<td>8.96</td>
<td>7.38</td>
<td>17.33</td>
</tr>
<tr>
<td>Fuel Station 7</td>
<td>2.86</td>
<td>2.66</td>
<td>13.19</td>
</tr>
<tr>
<td>Fuel Station 8</td>
<td>1.28</td>
<td>8.37</td>
<td>5.51</td>
</tr>
<tr>
<td>Fuel Station 9</td>
<td>2.76</td>
<td>1.28</td>
<td>13.09</td>
</tr>
<tr>
<td>Fuel Station 10</td>
<td>2.46</td>
<td>9.84</td>
<td>19.79</td>
</tr>
<tr>
<td>Order Quantity</td>
<td>45.29</td>
<td>51.89</td>
<td>114.99</td>
</tr>
</tbody>
</table>

The Analysis of the Results

Judging from the results of the program running 15 times, there is no obvious rule between order quantity and the total transportation cost. When the order quantity is 212 tankers, the total cost minimum is 22296.7 thousand yuan, which saves 207.2 thousand yuan compared to the maximum of the cost. Thus, the integrated optimization of inventory and transportation has a greater practical significance in reducing the total cost of the fuel supply rail.

Summary

The paper studied the problem of inventory and transportation of railway fuel supply. It estab-
lished the rail fuel inventory - transportation Integrated optimization model whose goal is the total cost of the minimum and uses iterative genetic algorithm to solve it. Firstly, the initial optimal order quantity is calculated without considering the transportation cost. And then calculate the corresponding transportation cost according to the order quantity and bring the order cost to re calculate the optimal order quantity considering the transportation cost.

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