Application of Mathematical Model Based on Fuzzy Comprehensive Evaluation

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Abstract. The paper introduces a fuzzy comprehensive evaluation method model, the formulation of the level pollution grade standard, and using the fuzzy mathematics were calculated by the regions for different pollution levels of membership, membership and corresponding grade product obtained the regional pollution comprehensive evaluation index, through the accurate evaluation regional pollution degree of heavy metal. This model advantage of the region's total potential ecological risk coefficient obtaining, and the model advantage of accurate evaluation of a regional heavy metal pollution degree.

Introduction

How to determine in different areas of the city of heavy metal pollution levels, the main reason for the heavy metal pollution and how to through the analysis of heavy metal pollution to determine the propagation characteristics of pollution sources is the position of the three problems is studied in this paper.

For one of the problems in different areas of the city of heavy metal pollution levels, research approach. This article through the use of SPSS software processing experimental data to solve the first eight major of the heavy metal element in the urban spatial distribution, intuitive reflects various metal elements in urban geographical location and concentration of the size distribution. A model to solve different area of heavy metal pollution levels, first for the soil heavy metal pollution degree level evaluation criteria, and then find out each function area all the sample points of membership and establish judgment matrix R and normalized, finally draw comprehensive evaluation index.

Fuzzy Comprehensive Evaluation Method

Variables Definition

<table>
<thead>
<tr>
<th>S</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>The n functional areas all the sample points to the level i of membership</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Membership matrix</td>
</tr>
<tr>
<td>$k$</td>
<td>A functional area the k sample points</td>
</tr>
<tr>
<td>$n$</td>
<td>The n function zone sample points total</td>
</tr>
<tr>
<td>$c_i$</td>
<td>A functional area sample sites j elements in the level i measured the score</td>
</tr>
<tr>
<td>$R_i$</td>
<td>The n function of the i level membership</td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>Judgment matrix</td>
</tr>
<tr>
<td>$\tilde{V}_i$</td>
<td>The n function area i level normalization of the membership</td>
</tr>
<tr>
<td>$\tilde{V}$</td>
<td>normalization membership matrix</td>
</tr>
<tr>
<td>$C_i$</td>
<td>The n functional areas of the comprehensive evaluation index</td>
</tr>
<tr>
<td>$C$</td>
<td>Pollution levels in the level i standard score</td>
</tr>
</tbody>
</table>
Set up Model

Definition the Degree of Soil Heavy Metal Pollution Level Evaluation Standard. Using the method of fuzzy comprehensive evaluation of soil heavy metal pollution degree level set criteria[3]. The first I level by the city of various soil heavy metal background value; The first II level using the background value added 2 S or background signal is multiplied by $S^2$ (when the background value sample for logarithmic normal distribution). The rest of the level 3 using <<Beijing and soil and crop irrigation pollution survey and its prevention and control way research>> provide the data is shown in table1.

Table 1. Soil Heavy Metal Content and Grading Standard Table.

<table>
<thead>
<tr>
<th>Elements</th>
<th>I level</th>
<th>Measured</th>
<th>II level</th>
<th>Measured</th>
<th>III level</th>
<th>Measured</th>
<th>IV level</th>
<th>Measured</th>
<th>V level</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>As (μg)</td>
<td>2</td>
<td>background value=3.6</td>
<td>4</td>
<td>22.0</td>
<td>6</td>
<td>32.0</td>
<td>8</td>
<td>32.0</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Cd (ng/g)</td>
<td>2</td>
<td>background value=130</td>
<td>4</td>
<td>1000</td>
<td>6</td>
<td>1000</td>
<td>8</td>
<td>1000</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Cr (μg)</td>
<td>2</td>
<td>background value=31</td>
<td>4</td>
<td>500</td>
<td>6</td>
<td>1000</td>
<td>8</td>
<td>1000</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Cu (μg)</td>
<td>2</td>
<td>background value=3.2</td>
<td>4</td>
<td>125</td>
<td>6</td>
<td>400</td>
<td>8</td>
<td>400</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Hg (ng)</td>
<td>2</td>
<td>background value=35</td>
<td>4</td>
<td>700</td>
<td>6</td>
<td>1500</td>
<td>8</td>
<td>1500</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Ni (μg)</td>
<td>2</td>
<td>background value=12.3</td>
<td>4</td>
<td>120</td>
<td>6</td>
<td>390</td>
<td>8</td>
<td>390</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Pb (μg)</td>
<td>2</td>
<td>background value=31</td>
<td>4</td>
<td>200</td>
<td>6</td>
<td>700</td>
<td>8</td>
<td>700</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Zn (μg)</td>
<td>2</td>
<td>background value=69</td>
<td>4</td>
<td>300</td>
<td>6</td>
<td>1000</td>
<td>8</td>
<td>1000</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Calculation of Functional Areas All The Sample Points Membership. In the table 1 for the standard, the function of the sample points all the elements of measured score, use the equation, and membership:

$$V_{ni} = \sum_{k=1}^{n} \sum_{j=1}^{i} c_{ij}$$

(n=1, 2, ..., 5; i=1, 2, ..., 5)  

(1)

Membership degree matrix can be built $\vec{V}$.

$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \\ v_{21} & v_{22} & v_{23} & v_{24} & v_{25} \\ v_{31} & v_{32} & v_{33} & v_{34} & v_{35} \\ v_{41} & v_{42} & v_{43} & v_{44} & v_{45} \\ v_{51} & v_{52} & v_{53} & v_{54} & v_{55} \end{bmatrix}$$

$$\vec{V} = \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \\ v_{21} & v_{22} & v_{23} & v_{24} & v_{25} \\ v_{31} & v_{32} & v_{33} & v_{34} & v_{35} \\ v_{41} & v_{42} & v_{43} & v_{44} & v_{45} \\ v_{51} & v_{52} & v_{53} & v_{54} & v_{55} \end{bmatrix}$$
From type can be seen, to a higher level of pollution in great weight, accord with the more exceeds bid, the greater the effect.

Permission Levels for Each Function Membership, Establish Judgment Matrix $R$. The sample points to function at all levels of all membership degree average as the functional areas of the membership. By next type calculation:

$$R_{ni} = \frac{V_{ni}}{k_n}$$

(n=1, 2……5; i=1,2……5) (2)

$R_{ni}$ Said the n function of the first I level membership, can establish judgment matrix $R$.

$$R = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\
    r_{21} & r_{22} & r_{23} & r_{24} & r_{25} \\
    r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\
    r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \\
    r_{51} & r_{52} & r_{53} & r_{54} & r_{55}
\end{bmatrix}$$

For Each Function At All Levels on The Membership to The Normalized. For a function zone membership at all levels and all levels of membership and ratio, for membership is normalized. By next type calculation:

$$\tilde{V}_{ni} = \frac{V_{ni}}{\sum V_{ni}}$$

(n=1,2……5; i=1,2……5) (3)

Can be set up to be a membership matrix $\tilde{V}$.

$$\tilde{V} = \begin{bmatrix}
    \tilde{V}_{11} & \tilde{V}_{12} & \tilde{V}_{13} & \tilde{V}_{14} & \tilde{V}_{15} \\
    \tilde{V}_{21} & \tilde{V}_{22} & \tilde{V}_{23} & \tilde{V}_{24} & \tilde{V}_{25} \\
    \tilde{V}_{31} & \tilde{V}_{32} & \tilde{V}_{33} & \tilde{V}_{34} & \tilde{V}_{35} \\
    \tilde{V}_{41} & \tilde{V}_{42} & \tilde{V}_{43} & \tilde{V}_{44} & \tilde{V}_{45} \\
    \tilde{V}_{51} & \tilde{V}_{52} & \tilde{V}_{53} & \tilde{V}_{54} & \tilde{V}_{55}
\end{bmatrix}$$

Calculation Comprehensive Evaluation Index

Comprehensive evaluation index

$$Q_n = \sum_{i=1}^{5} \tilde{V}_{ni} c_i$$

(n=1, 2……5; i=1,2……5) (4)
Pruning Model

According to for each function of all the sample points to the first level $i$ sum of membership $V_i$, establish the membership degree matrix for:

$$V = \begin{bmatrix} 10 & 20 & 180 & 0 & 0 \\ 14 & 48 & 306 & 16 & 0 \\ 22 & 76 & 144 & 0 & 0 \\ 46 & 184 & 402 & 16 & 0 \\ 8 & 20 & 156 & 0 & 0 \end{bmatrix}$$

according to for each function for the first level $I$ of membership $R_i$, including $k_i = (k_1, k_2, k_3, k_4, k_5) = (44, 36, 66, 138, 35)$ establish judgment matrix $\tilde{R}$.

$$\tilde{R} = \begin{bmatrix} 0.227 & 0.818 & 4.091 & 0 & 0 \\ 0.389 & 1.333 & 8.500 & 0.444 & 0 \\ 0.333 & 1.152 & 2.182 & 0 & 0 \\ 0.333 & 1.333 & 2.913 & 0.116 & 0 \\ 0.229 & 0.571 & 4.457 & 0 & 0 \end{bmatrix}$$

according to to membership of normalized, to establish a membership matrix $\tilde{V}$.

$$\tilde{V} = \begin{bmatrix} 0.044 & 0.159 & 0.797 & 0 & 0 \\ 0.036 & 0.125 & 0.797 & 0.042 & 0 \\ 0.091 & 0.314 & 0.595 & 0 & 0 \\ 0.071 & 0.284 & 0.620 & 0.025 & 0 \\ 0.044 & 0.109 & 0.848 & 0 & 0 \end{bmatrix}$$

according to calculation comprehensive evaluation index type:

$$Q_1 = 5.506$$
$$Q_2 = 5.69$$
$$Q_3 = 4.946$$
$$Q_4 = 5.198$$
$$Q_5 = 5.612$$

Model Analysis and Discussions

The analysis of data summarizes the metal in the soil of the main causes of the pollution.

<table>
<thead>
<tr>
<th>City each function pollution levels membership</th>
<th>Sample</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Comprehensive evaluation index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Living areas</td>
<td>44</td>
<td>0.044</td>
<td>0.159</td>
<td>0.797</td>
<td>0</td>
<td>0</td>
<td>5.506</td>
</tr>
<tr>
<td>2 Industrial zone</td>
<td>36</td>
<td>0.036</td>
<td>0.125</td>
<td>0.797</td>
<td>0.042</td>
<td>0</td>
<td>5.69</td>
</tr>
<tr>
<td>3 Mountains</td>
<td>66</td>
<td>0.091</td>
<td>0.314</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
<td>4.946</td>
</tr>
<tr>
<td>4 Traffic area</td>
<td>138</td>
<td>0.071</td>
<td>0.284</td>
<td>0.62</td>
<td>0.025</td>
<td>0</td>
<td>5.198</td>
</tr>
<tr>
<td>5 Park Green area</td>
<td>35</td>
<td>0.044</td>
<td>0.109</td>
<td>0.848</td>
<td>0</td>
<td>0</td>
<td>5.612</td>
</tr>
</tbody>
</table>
Because $Q_1 > Q_2 > Q_3 > Q_4 > Q_5$, namely the city for, soil heavy metal pollution degree for industrial park green area traffic area > mountain living areas.

Summary
Firstly, the metal in the soil pollution the main reason is the industrial pollution:
   The atmosphere heavy metal settlement. The atmosphere mainly come from the heavy metal industry production of heavy metals in a lot of the harmful gas and dust etc.
   Metal acidic mining waste water pollution. The metal mine mining and smelting, heavy metal tailing, smelting slag and slag piled up, can directly or indirectly cause soil heavy metal pollution.
Secondly, secondary causes of agricultural pollution:
   Pesticides, fertilizers and plastic film use. Application of contains lead, mercury, cadmium, as the pesticide and unreasonably chemical fertilizers, can lead to soil of heavy metals in pollution.
   Sludge fertilization. Sludge contain a lot of organic matter, nitrogen, phosphorus, potassium and other nutrients, but at the same time also in sewage sludge contains a large amount of heavy metal, with a large number of municipal sludge into farmland, make the heavy metal content in the farmland have increased.
   Third, once again to reason for containing heavy metal waste accumulation. Living garbage containing heavy metal waste and also can cause heavy pollution.
   Last, traffic pollution mainly for car emissions and car tier wear a lot of heavy metals from the harmful gas and dust etc.

Conclusions
The advantages of the fuzzy comprehensive evaluation model evaluation is it can solve fuzzy boundary problem effectively; and the fuzzy comprehensive evaluation model of weakness: should pay attention to the weights of the evaluation factors, and the use of area of level sample points of membership average of the level as regional membership, cause the certain error. This article through to establish fuzzy comprehensive evaluation model, effectively solve the problem of fuzzy border.

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References


