Reliability of Passive Earth Pressure

Li Zhang¹ and Xianfeng Luo²

¹Associate professor, School of Civil Engineering, Hubei Polytechnic University, Huangshi 435003, China; 626205037@qq.com
²Associate professor, School of Civil Engineering, Hubei Polytechnic University, Huangshi 435003, China; 4550573@qq.com

ABSTRACT: Reliability of passive earth pressure calculations in geotechnical engineering are usually performed using the Rankine theories of earth pressure. This paper uses the elasto-plastic finite element method to calculate the passive earth pressure, which can reflect the stress-strain relationship of the earth when the retaining wall begins to overturn. Besides, the covariance matrix decomposition method is used to generate random fields of soil in the study. Thus, Monte-Carlo based random finite element method (RFEM) is adopted here to perform the reliability of passive earth pressure in the study.

INTRODUCTION

The reliability of passive earth pressure calculations in geotechnical engineering are usually performed using the Rankine theories of earth pressure. However, the Rankine theories cannot appropriately reflect the stress-strain relationship of the earth when the retaining wall begins to overturn. Therefore, the elastic-plastic finite element method is used to perform the stability analysis of the retaining wall. On the other hand, the soil in nature has spatial variability, it is necessary to use the random field model to simulate the spatial variability of the soil when calculating passive earth pressure. Therefore Monte-Carlo based random finite element method (RFEM) is adopted here to perform the reliability of passive earth pressure [1].

Elasto-plastic finite element method

The constant stiffness algorithm is implemented in the elasto-plastic finite element method. The (usually elastic) global stiffness matrix $\mathbf{K}$ in the algorithm is formed only once and then kept unchanged throughout the whole process, where loads are applied incrementally. The iterative process is based on repeated solutions of the equilibrium equations:

$$\mathbf{[K]} \{\mathbf{U}^i\} = \{\mathbf{F}^i\}$$

where $i$ represents the iteration number, $\{\mathbf{U}^i\}$ the global displacements increments, and $\{\mathbf{F}^i\}$ the global external and internal loads.

The element displacement increments $\{\mathbf{u}^i\}$ are extracted from $\{\mathbf{U}^i\}$, and these lead to strain increments $\{\Delta \mathbf{\varepsilon}^i\}$ via the element strain-displacement relationships:

$$\{\Delta \mathbf{\varepsilon}^i\} = [\mathbf{B}] \{\mathbf{u}^i\}$$

where $[\mathbf{B}]$ denotes the strain-displacement matrix.

Assuming the material is yielding, the strains will contain both elastic stresses and (visco) plastic components, thus
It is only the elastic strain increments \( \{ \Delta e \}' \) that generate stresses through the elastic stress-strain matrix \([D']\), (the plastic strain increments \( \{ \Delta e^p \}' \) can be obtained by the viscoplastic algorithm) hence

\[
\{ \Delta \sigma \}' = [D'] \{ \Delta e \}'
\]  
(4)

These stress increments are added to stresses already existing from the previous load step and the updated stresses can be substituted into the failure criterion. For soil with both frictional and cohesive components of shear strength, the Mohr-Coulomb criterion is undoubtedly appropriate and given as follows:

\[
F = \sigma_m \sin \varphi + \sigma \left( \cos \frac{\theta}{\sqrt{3}} \sin \theta \sin \varphi \right) - c \cos \varphi
\]  
(5)

where \( F \) denotes a failure function, \( \varphi \) friction angle, \( c \) cohesion, \( \theta \) Lode angle, \( \sigma_m \) mean stress and \( \bar{\sigma} \) equivalent stress.

If stress redistribution is necessary (\( F > 0 \)), this is done by altering the load increment vector \( \{ F' \} \) in equation. In general, this vector consists of two types of load, as given by

\[
\{ F' \} = \{ F_a \} + \{ F_b \}'
\]  
(6)

where \( \{ F_a \} \) is the actual applied external load increment and \( \{ F_b \}' \) is the body loads vector that varies from one iteration to the next. Here the initial stress method is used for generating body loads.

In the initial strain method, elasto-plasticity is described by

\[
\{ \Delta \sigma \} = [D^p] \{ \Delta e \}
\]  
(7)

where

\[
[D^p] = [D'] - [D']^T [D']
\]  
(8)

For perfect plasticity in the absence of hardening or softening, the stress state should satisfy

\[
\left\{ \begin{array}{c} \frac{\partial F}{\partial \sigma}^T \\ \frac{\partial \sigma}{\partial \sigma} \end{array} \right\} \{ \Delta \sigma \} = 0
\]  
(9)

Considering the possibility of non-associated flow, plastic strain increments are normal to a plastic potential surface, thus

\[
\{ \Delta e^p \}' = \lambda \left\{ \frac{\partial Q}{\partial \sigma} \right\}
\]  
(10)

Assuming stress changes are caused by elastic strain components gives

\[
\{ \Delta \sigma \} = [D'] \left( \{ \Delta e \} - \lambda \left\{ \frac{\partial Q}{\partial \sigma} \right\} \right)
\]  
(11)

Substitution of Eq. (11) into Eq. (9) leads to

\[
[D'] = \frac{[D'] \left\{ \frac{\partial Q}{\partial \sigma} \right\} \left[ \frac{\partial F}{\partial \sigma} \right]^T [D']} {\left[ \frac{\partial F}{\partial \sigma} \right]^T [D'] \left\{ \frac{\partial Q}{\partial \sigma} \right\}}
\]  
(12)
Random field model

The covariance matrix decomposition method is used in the study to produce a random field directly[2]. If the random field \( Z(t) \) is meshed into \( n \) grid points, its covariance matrix \( C_{n\times n} \) consists of elements \( C_{i,j} = C(t_i, t_j), \) \( i = 1,2,...,n \), and \( j = 1,2,...,n \). Therefore, the Gaussian stationary random field can be generated as follows

\[
C_{n\times n} = L_{n\times n} L_{n\times n}^T \tag{13}
\]

\[
G_{n\times 1} = L_{n\times n} w_{n\times 1} \tag{14}
\]

in which \( L L^T \) is the Cholesky decomposition of the covariance matrix \( C \), \( w \) a vector of independent, \( N(0,1) \) distributed random numbers, \( G_{n\times 1} \) is the vector of random variables \( g_i, i = 1,2,...,n \) that constitutes the discretized Gaussian stationary random field.

In the passive earth pressure problem, the friction angle \( \phi'(t) \), and unit weight \( \gamma(t) \) are considered to be spatially random fields, where \( t \) is the spatial position. The correlation function for the \( \ln(\tan \phi') \), and \( \ln \gamma \), both normally distributed, is assumed to be Markovian[3],

\[
\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \tag{15}
\]

where \( \theta \) is the correlation length beyond which two points, \( |\tau| \) is the distance between the two points.

In the present study, the friction angle \( \phi'(t) \), and unit weight \( \gamma(t) \) are assumed to be independent. Thus, two independent standard normal random fields \( g_i^1 \) and \( g_i^2 \), \( i = 1,2,...,n \)

\[
\gamma_i = \exp(\mu_{\ln\gamma} + \sigma_{\ln\gamma} g_i^1) \tag{16}
\]

\[
\tan \phi'_i = \exp(\mu_{\ln\tan\phi'} + \sigma_{\ln\tan\phi'} g_i^2) \tag{17}
\]

where \( \mu \) and \( \sigma \) are the mean and standard deviation of the subscripted variable.

Passive earth pressure reliability

In this section, the proposed methodology is used to perform the probabilistic analysis of a retaining wall with the friction angle \( \phi' \), and unit weight \( \gamma \) as random fields. The statistic properties of the soil parameters are listed in Table 1. Other deterministic soil parameter are given as cohesion \( c = 0 \), Young’s modulus \( E = 10^5 \text{kPa} \), Poisson’s ratio \( \nu = 0.3 \). The geometry of the retaining wall system used in this study is shown in Fig 1, which represents a smooth wall that is translated into a bed of sand. The bed of sand consists of 2048 elements, also shown in Fig 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi' ) friction angle</td>
<td>Lognormal</td>
<td>30 (°)</td>
<td>3</td>
</tr>
<tr>
<td>( \gamma ) unit weight</td>
<td>Lognormal</td>
<td>20 (kN/m^3)</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1. Statistical properties of soil parameters.
The probability of failure of the retaining system is defined as the true lateral load $P_t$, exceeds the resistance $R$.

$$p_f = P[P_t > R] = P[P_t > FP_p]$$  \hspace{1cm} (18)

in which

$$P_p = \frac{1}{2} \gamma H^2 K_p$$  \hspace{1cm} (19)

$$K_p = \tan^2 (45^\circ + \phi / 2)$$  \hspace{1cm} (20)

$$R = FP_p$$  \hspace{1cm} (21)

where $F$ is a factor of safety that is equal to 1 in this study.

The result of Monte Carlo simulation for this study is $p_f = 0.38$. The passive earth displacements for two different possible soil friction angle field realizations are shown in Fig 2 and Fig 3 respectively. From the figures, the different deformation of meshes which indicates the different displacement of the soil in each realization is given, which leads to the overturning risk of retaining wall.
CONCLUSIONS

This paper uses the elasto-plastic finite element method to calculate the passive earth pressure, which can reflect the stress-strain relationship of the earth when the retaining wall begins to overturn. Besides, the covariance matrix decomposition method is used to generate random fields of soil in the study. Thus, Monte-Carlo based random finite element method (RFEM) is adopted here to calculate the failure probability of passive earth pressure.

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