Analytical Modeling of Soft Soil Stabilized by Deep Mixed Columns Combined with Vertical Drains

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ABSTRACT: A combined method involving the use of deep mixed (DM) columns and prefabricated vertical drains (PVDs) was proposed to reinforce the soft soils. To obtain analytical solution of the composite foundation with DM columns and PVDs, the drainage system was converted into a cylindrical drain wall with the equivalent properties, and only the radial drainage was considered. An axi-symmetric unit cell was established and the solution of average degree of consolidation was derived. Based on the proposed solution a design chart was developed and was applied to a case history of an embankment on soft soil with short DM columns and long PVDs. A good agreement was found between the predicted average degree of consolidation and the field data.

INTRODUCTION

Various techniques (for example, lightweight materials, stone columns, DM columns and vertical drains) have been used to improve soft soil, when constructing roadway embankments on such unfavorable geotechnical condition. Ye et al. (2008) proposed a ground improvement method combined DM columns and PVDs. Figure 1 presents the profile of the ground improved by the combined technique. In the combined method, DM columns are used to increase the bearing capacity and reduce the settlement of embankments on soft soil, meanwhile PVDs are used to accelerate the consolidation of soft soil. Thus, in practice the length of PVD is generally longer than that of DM column, which is beneficial to an acceleration of consolidation rate in deep thick soft soil.

Due to the complexity of the combined method, so far people have not well understood the consolidation behavior of a ground improved by the combined method yet (Ye et al. 2008, 2012). Ye et al. (2012) proposed a simplified calculation for the composite foundation available only for the case when length of PVD is equal to that of DM column. However, as indicated previously, in application of the technique, especially for the subsoil consisting of a deep thick layer of soft soil, the length of DM columns is usually shorter than that of PVDs. Therefore, it is necessary to develop a method to calculate the soil consolidation of soft soil improved by short DM columns and long PVDs.
The objective of this study is to develop an analytical solution to investigate the consolidation characteristics of a composite foundation with short DM columns and long PVDs. Based on the simplified method of drainage system proposed by Ye et al. (2012) an axi-symmetric consolidation model was established and the solution of the average degree of consolidation was derived. A design chart was proposed and it was applied to predict the consolidation behavior of a foundation improved by the combined method in a case history.

SIMPLIFIED CONCEPT OF THE COMPOSITE FOUNDATION

The use of cement reduces the permeability of the in situ soil (Han et al. 2002). PVDs greatly shorten the drainage path, and the permeability of soil in horizontal direction is commonly greater than that in vertical direction. In application, length of PVDs is much longer than diameter of influence zone for a single DM column. Therefore, most excess pore water pressure dissipates via PVD. For simplicity, it is assumed that the excess pore water pressure in soil dissipates only in a radial direction.

In the composite foundation, the DM columns and PVDs are commonly installed in a triangular pattern. Based on the characteristics of the plan view distribution (see Fig.2), Ye et al. (2012) proposed a simplified method for the drainage system, in which the three discrete PVDs in the boundary of the unit cell were converted into a continuous cylindrical drain wall with the equivalent permeability, as shown in Fig. 3. Smear effect was taken into account but well resistance was neglected in the conversion method due to the large discharge capacity of current PVD products. The equivalent permeability of soil or DM column for the cylindrical drain wall model was expressed as
\[ k_{h}^{eq} = \frac{k_{h}}{4 \left( \ln n + \kappa \ln s - \frac{3}{4} \right)} \]

where \( k_{h}^{eq} \) = equivalent horizontal permeability of soil or DM column for the model of cylindrical drain wall; \( k_{h} \) = horizontal permeability of soil or DM column; \( \kappa \) = ratio of horizontal permeability of undisturbed soil to permeability in the smear zone; \( n = \frac{r_{c}}{r_{w}} \), \( s = \frac{r_{e}}{r_{w}} \), \( r_{c} \) = radius of influence zone, \( r_{w} \) = equivalent radius of vertical drain, \( r_{e} \) = radius of smear zone.

**Figure 3. Conversion of analysis model (after Ye et al. 2012).**

**Figure 4. Unit cell of the consolidation model.**

**CONSOLIDATION OF THE COMPOSITE FOUNDATION**

**Consolidation Problem**

Based on the above analyses, the unit cell of the consolidation model can be established as shown in Fig.4, in which, \( P \) = external load; \( r_{c} \) = radius of DM column; \( h_{1} \) and \( h_{2} \) = thickness of DM column-improved soil and soil below the base of DM column, respectively (thus the total thickness of subsoil \( h \) equals \( h_{1} + h_{2} \)); \( k_{s1h}^{eq} \), \( k_{s2h}^{eq} \), and \( k_{ch}^{eq} \) = equivalent permeability of subsoil in the DM column-improved zone, subsoil below the base of DM columns, and DM columns. The equivalent permeability of soil and DM column was calculated based on Eq.(1) by substituting the original permeability of
surrounding soil or DM column in it. \( E_{s1} \), \( E_{s2} \), and \( E_c \) = compression moduli of subsoil in the DM column-improved zone, subsoil below the base of DM columns, and DM columns, respectively. The drainage boundaries are defined as: (1) the top surface and interface of drainage wall are drained; and (2) the bottom surface is impermeable.

The main assumptions made in the analysis are summarized as: (1) equal strain assumption (small strain) and only vertical strain is considered; (2) the soil is fully saturated and the permeability of soil is assumed to be constant during consolidation; (3) well resistance is neglected due to the sufficient discharge capacity of the drain, thereby the pore pressure at the drain interface is assumed to be zero; (4) it is assumed that the cylindrical drain wall has a negligible thickness; (5) only radial flow is permitted; (6) the external load is applied instantaneously and keeps constant during the period of soil consolidation.

**Analytical Solution**

Since only radial drainage is considered, the terms related to the vertical drainage are ignored. The partial differential equations in the DM column-improved zone (i.e., \( 0 \leq z \leq h_1 \)) can be expressed as

\[
\frac{k_{ch}^{eq}}{\gamma_w} \left( 1 \frac{\partial u_c}{\partial r} + \frac{\partial^2 u_c}{\partial r^2} \right) = \frac{1}{E_{sc}} \left( \frac{\partial \bar{u}_1}{\partial t} \right) \quad (0 \leq r \leq r_c) \quad (2a)
\]

\[
\frac{k_{sh}^{eq}}{\gamma_w} \left( 1 \frac{\partial u_{s1}}{\partial r} + \frac{\partial^2 u_{s1}}{\partial r^2} \right) = \frac{1}{E_{sc}} \left( \frac{\partial \bar{u}_1}{\partial t} \right) \quad (r_c < r \leq r_e) \quad (2b)
\]

The partial differential equation in the subsoil below the base of DM columns (i.e., \( h_1 < z \leq h \)) can be expressed as

\[
\frac{k_{sh}^{eq}}{\gamma_w} \left( 1 \frac{\partial u_{s2}}{\partial r} + \frac{\partial^2 u_{s2}}{\partial r^2} \right) = \frac{1}{E_{sc}} \left( \frac{\partial \bar{u}_2}{\partial t} \right) \quad (0 \leq r \leq r_c) \quad (2c)
\]

in which, \( u_{s1} \), \( u_c \), and \( u_{s2} \) = excess pore water pressures of the subsoil in DM column-improved zone, DM columns, and subsoil below the base of DM column, respectively; \( \bar{u}_1 \) and \( \bar{u}_2 \) = average pore water pressures within the DM column-improved zone and subsoil below the base of DM columns, respectively; \( \gamma_w \) = unit weight of water; \( E_{sc} \) = composite modulus of DM column-improved zone (i.e., \( E_{sc} = E_{s1}(1-m)+E_cm \)), \( m \) = replacement ratio of DM column. The other notations are defined previously.

The average excess pore water pressures in the foundation can be defined as follows: in the DM column-improved zone, i.e., \( 0 \leq z \leq h_1 \),

\[
\bar{u}_1 = \frac{1}{\pi r_c^2} \left( \int_0^{r_c} u_c \cdot 2\pi r dr + \int_{r_c}^{r_e} u_{s1} \cdot 2\pi r dr \right) \quad (3a)
\]

below the base of DM columns, i.e., \( h_1 < z \leq h \),

\[
\bar{u}_2 = \frac{1}{\pi r_c^2} \int_0^{r_c} u_{s2} \cdot 2\pi r dr \quad (3b)
\]

The boundary conditions and the initial conditions of Eq. (2a)(2b)(2c) are defined as:
Integrating Eq. (2a)(2b)(2c), and using the boundary conditions (a), (b), and (c), then the Eq. (2) can be solved as

in the DM column-improved zone, i.e., \( 0 \leq z \leq h_1 \)

\[
u_c = \frac{1}{4} \frac{\gamma_w}{k_{ch} E_{sc}} (r^2 - r_c^2) \frac{\partial \bar{u}_1}{\partial t} + \frac{1}{4} \frac{\gamma_w}{k_{s1h} E_{sc}} (r_c^2 - r_e^2) \frac{\partial \bar{u}_1}{\partial t} \quad (0 \leq r \leq r_c)
\] (4a)

\[
u_{s1} = \frac{1}{4} \frac{\gamma_w}{k_{s1h} E_{sc}} (r^2 - r_c^2) \frac{\partial \bar{u}_1}{\partial t} \quad (r_c < r \leq r_e)
\] (4b)

below the base of DM columns, i.e., \( h_1 < z \leq h \),

\[
u_{s2} = \frac{1}{4} \frac{\gamma_w}{k_{s2h} E_{s2}} (r^2 - r_e^2) \frac{\partial \bar{u}_2}{\partial t} \quad (0 \leq r \leq r_e)
\] (4c)

Substituting Eq. (4a)(4b)(4c) into Eq. (3a)(3b), Eq. (3a)(3b) can be rewritten as

in the DM column-improved zone, i.e., \( 0 \leq z \leq h_1 \),

\[ar{u}_1 = -r_c^2 \left[ \frac{1}{8} \frac{\gamma_w}{k_{ch} E_{sc}} - \frac{1}{8} \frac{\gamma_w}{k_{s1h} E_{sc}} \right] \left( \frac{\partial \bar{u}_1}{\partial t} \right) - \frac{1}{8} \frac{\gamma_w r_c^2}{k_{s2h} E_{s2}} \frac{\partial \bar{u}_2}{\partial t}
\] (5a)

below the base of DM columns, i.e., \( h_1 < z \leq h \),

\[ar{u}_2 = \frac{1}{8} \frac{\gamma_w r_c^2}{k_{s2h} E_{s2}} \frac{\partial \bar{u}_2}{\partial t}
\] (5b)

Using the initial condition (d), the solutions of the average excess pore water pressures can be obtained. According to the definition of overall average degree of consolidation in terms of stress, the solution of the overall degree of consolidation can be obtained:

\[
\bar{U} = \frac{h \bar{U}_1}{h} + \frac{h_2 \bar{U}_2}{h}
\] (6a)

\[
\bar{U}_1 = 1 - e^{-\frac{8}{\mu_1}} ; \quad \bar{U}_2 = 1 - e^{-\frac{8}{\mu_2}}
\] (6b)

\[
T_{h1} = \frac{E_{sc}}{r_c^2 \gamma_w m^2 k_{s1h} + (1-m^2) k_{ch}} ; \quad T_{h2} = \frac{k_{s2h} E_{s2}}{r_c^2 \gamma_w}
\] (6c)

\[
\mu_i = 4 \left( \ln \frac{n}{s} + \kappa \ln s - \frac{3}{4} \right) \quad (i=1,2)
\] (6d)

in which \( \bar{U}_1 \) and \( \bar{U}_2 \) = average degree of consolidation of DM column-reinforced zone and below the base of DM columns; \( k_{s1h}, k_{s2h}, \) and \( k_{ch} = \) horizontal permeability of subsoil in the DM column-improved zone, subsoil below the base of DM columns, and DM columns; and \( T_{h1} \) and \( T_{h2} \) = dimensionless time factor of the DM column-improved
zone and subsoil below the base of DM columns. Other notations are defined previously. The value of $\mu_i$ within a range of 1 to 40 can cover most of PVD spacing in design (Hansbo, 1987; Bergado et al, 1991; Mesri et al., 1994). Based on the above analysis, the design chart was plotted as shown in Fig. 5. The dimensionless time factors can be calculated according to preloading time, soil properties and layout of the foundation with DM column and PVD. Then the average degree of consolidation (i.e., $\bar{U}_1$ and $\bar{U}_2$) can be obtained from the design chart. The overall degree of consolidation can be determined substituting $\bar{U}_1$ and $\bar{U}_2$ into Eq. (6a).

![Figure 5. Average degree of consolidation versus time factor.](image)

The subsoil consists of stratified soil layers with different properties in the real geotechnical condition. In addition, the DM column quality is not uniform due to non-uniformity of soil and cement (Horpibulsuk et al. 2011). When using the proposed method, the non-uniform properties of soil layers and DM column are necessary to be considered. The DM column-reinforced zone and the unreinforced zone with PVD only are divided into several sublayers according to the variation of properties of soil layers and DM columns. The interfaces of soil strata and the base of DM column are the natural stratified positions. Substituting the pertinent properties of each sublayer into Eq.(6c), the time factors for each sublayer can be determined and then the average degree of consolidation for each sublayer can be gained from Fig. 5. The overall degree of consolidation can be determined based on the thickness-weighted average of the degree of consolidation for each sublayer.

**Case Application**

The selected project is a bridge-approach embankment on soft soil improved by DM columns and PVDs in the northern suburb of Shanghai, China. The detailed description of the case history was given by Ye et al. (2013). The geotechnical conditions are shown in Table 2. The groundwater level was at a depth of 0.5 m.
Table 2. Soil properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>( d ) (m)</th>
<th>( \gamma ) (kN/m(^3))</th>
<th>( w ) (%)</th>
<th>( e_0 )</th>
<th>( k_{sv} ) (m/d)</th>
<th>( k_{sh} ) (m/d)</th>
<th>( \phi' ) (°)</th>
<th>( c' ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crust</td>
<td>1.0</td>
<td>19.3</td>
<td>19.3</td>
<td>0.73</td>
<td>0.0005</td>
<td>0.0015</td>
<td>25</td>
<td>12</td>
</tr>
<tr>
<td>Silty clay</td>
<td>1.5</td>
<td>18.9</td>
<td>28.4</td>
<td>0.83</td>
<td>0.0007</td>
<td>0.0040</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>Soft silty clay</td>
<td>4.0</td>
<td>18.5</td>
<td>31.3</td>
<td>1.10</td>
<td>0.0002</td>
<td>0.0003</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Very soft clay</td>
<td>12.5</td>
<td>16.8</td>
<td>49.5</td>
<td>1.39</td>
<td>0.0003</td>
<td>0.0007</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>Clay</td>
<td>5.5</td>
<td>17.6</td>
<td>40.2</td>
<td>1.14</td>
<td>0.0003</td>
<td>0.0004</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Silty clay</td>
<td>4</td>
<td>19.5</td>
<td>24.6</td>
<td>0.72</td>
<td>0.0028</td>
<td>0.0060</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>Sand silt</td>
<td>--</td>
<td>18.7</td>
<td>28.5</td>
<td>0.84</td>
<td>0.0028</td>
<td>0.0060</td>
<td>32</td>
<td>5</td>
</tr>
</tbody>
</table>

Note: \( d \) = thickness of soil layer; \( \gamma \) = unit weight, \( w \) = water content, \( e_0 \) = initial void ratio, \( k_{sv} \) = vertical permeability of subsoil, \( k_{sh} \) = horizontal permeability of subsoil, \( \phi' \) = effective friction angle, \( c' \) = effective cohesion.

The combined method was used to reinforce the soft soil. DM columns with a diameter of 0.7 m and a length of 9.0 m were installed in a triangular pattern at a spacing of 2.5 m, and PVDs with a length of 25.0 m were installed in a triangular pattern at a spacing of 2.5 m. A 0.5 m thick sand blanket was placed at the ground surface. The embankment had a crest width of 42 m, a slope of 1.5H:1V, and height of 3.0 m. The embankment was constructed to a height of 0.7 m in 35 days, followed by a 22 day rest for soil consolidation. Then, the embankment was constructed to the final height of 3.0 m in 64 days and was maintained for 100 days for soil consolidation.

Figure 6. Measured degree of consolidation versus calculated results.

In this section, a comparison of the average degrees of consolidation based on the proposed solution and measured settlements was conducted. The method based on the field data by Asaoka (1978) was adopted herein to predict the final settlement (i.e., 536 mm). The average degree of consolidation was obtained by the measured settlement on the embankment base at a given time divided by the predicted final settlement by Asaoka’s method. As shown in Fig 6, the predicted average degree of consolidation by the proposed method agreed with that computed based on the field data. The average degrees of consolidation at the end of preloading period were 94.6% and 97.0% by the measured data and proposed method, respectively.
CONCLUSIONS

In this study, a simplified method to calculate the average degree of consolidation of a composite foundation with short DM columns and long PVDs was proposed and the design chart was developed. A case history of an embankment on soft soil reinforced by the combined method was selected to verify the proposed analytical method. The predicted average degree of consolidation by the proposed method agreed with that computed based on the field data. Most settlement of soft soil was completed during the preloading period.

ACKNOWLEDGEMENTS

The authors appreciate the financial supports provided by the National Natural Science Foundation of China (NSFC) (No. 51508408).

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