POT Model of Operation Vehicle Load under Toll-by-weight Mode

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ABSTRACT: The phenomenon of overload operation of Expressway vehicles is particularly serious in China recently. Research on the extreme value of the vehicle load and its development trend is an urgent problem in the design of toll-by-weight mode. On the basis of statistical analysis of a highway WIM data under toll-by-weight mode in Guangdong Province, the vehicle load according to the models of the gross vehicle weight of the statistical analysis by the extreme value analysis theory to establish the pot model, gross vehicle weight distribution in the tail of the distribution function is obtained, and scientifically predict future arbitrary reproduce that may arise during the period of vehicle load value. Results show that the high speed sections of large tonnage vehicles is not accidental, in the future may also appear more large tonnage vehicle load and to pose a serious threat to the safe operation of the bridges and roads. Finally, based on the research results, the toll-by-weight standards proposed improvement suggestions and comments. It provided a useful reference for effectively controlling overloading vehicle loads.

KEY WORDS: Vehicle load; Toll-by-weight; Extreme value theory; POT model

INTRODUCTION

With the continuous development of China's economy and society and the automobile industry, the growing overload transportation, road vehicle loading operations produce significant variation, new research has become one of the hot topics. In order to effectively curb the increasingly serious overload situation, better guarantee the smooth flow of traffic safety, weight management fee model came into being. Vehicle weight generally take the dynamic weighing technology mostly (Weight In Motion, referred WIM), has been successfully applied in our country freeways. In the weight charging mode, whether the vehicle is overloaded overweight has been effectively controlled? What happens to vehicles operating load characteristics? As under the current weight fees policy, the development trend of operating vehicle loads what is? How can I do more scientific and rational prediction, and in order to improve weight charges policy? And so on these issues, some researchers explored analysis [1-6], but in general, these studies are preliminary, the conclusions are very different, in addition to operating the vehicle load feature itself, there are obvious regional, causing the problem more complicated. In this view, the
paper use Guangshao section of Beijing-Zhuhai Expressway data measured on the basis of a toll station, the use of extreme value theory. Building operational vehicle load model, under the weight of the charging status of vehicle loading operations, the development trend more systematic and comprehensive discussion.

VEHICLE LOAD STATISTICAL ANALYSIS

Beijing-Zhuhai Expressway is a capital radioactive highway, connecting Beijing and southern city such as Guangzhou, Zhuhai and Hong Kong, is the China's north-south traffic artery, from the skeleton of the road network in the Pearl River Delta. Guangdong Province, with effect from November 1, 2009 on the territory of the high-speed Beijing-Zhuhai highway truck weight began to implement charging policy paper in which randomly selected representative weight of a toll station in April 2014 for a period of 30 days of 80181 vehicle load data analysis. Collation of measured traffic data in accordance with "Guangdong Provincial Expressway Toll Vehicle Classification Standard" is divided into five types.

Type one vehicle is the most, occupying 49.3% of the total traffic, mainly cars, jeeps, taxis head trucks, motorcycles, etc., a total of 39,533 vehicles measured, the average total weight of 1.8 tons, more concentrated distribution, discrete data small.

The Following is the type five vehicle, 32.4% of the total traffic, including heavy goods vehicles, heavy trailers (hanging) car, 40 feet container trucks and other vehicles, were collected to 25,952 vehicles, the average vehicle weight 48.8 tons. The maximum total weight distribution, vehicle weight distribution from 10 to 110 tons, has two peaks, respectively, in the vicinity of 20 tons and 55 tons, maximum vehicle weight 107.9 tons.

Then the type three vehicle, occupying the total traffic of 13.5 percent, the third type of the cars include medium-sized passenger car, large passenger cars, medium-sized truck-based, a total of 10,826 vehicles measured, the average vehicle weight 11.2 tons. The total weight distribution is quite dispersed, total weight distribution two peaks, respectively, in the vicinity of 3.5 tons and 14.5 tons. Type two vehicles and type four vehicles are the minimal, occupying the total traffic accounted for 2.0% and 2.8% respectively.

Results show that type one vehicle is more, but the main is the sedan, its vehicle load is relatively small; type three vehicle and type five vehicle has heavy car based, which is the focus of this paper needs to study. Type three vehicle, type five vehicle weight histogram are shown as Fig.1 and Fig.2.

![Figure 1. Type 3 vehicle weight frequency histogram.](image1)

![Figure 2. Type 5 vehicle weight frequency histogram.](image2)
POT MODEL BASED ON THE EXTREME VALUE ANALYSIS THEORY

At present, most researchers at home and abroad in the construction of vehicle load model, usually using the hypothesis test, analysis of the parameters of the technical route \[2,7,8\]. This trial and error analysis method has strong subjectivity, even through K-S, X^2, U^2 and inspection, can also simulate the distribution of central better, but failed to appropriately, accurately description operating the vehicle load distribution in the tail, and extreme value distribution of the right tail i.e. operation of vehicle load is the construction of the key and core of the running vehicle load model. Based on this, this paper intends to use the extreme value analysis theory to build the vehicle load model.

POT model \[10\]

\(F(x)\) is an arbitrary distribution function of random variable sequence \(\{X_i\}\), which is assumed to have independent identically distributed random variables, \(F(x)\) support at the top of the point \(x^*\), There is a value of \(X_i < \mu < x^*\), called Threshold (Threshold), \(X_i\) is a sequence of random variables, \((X_i - \mu)\) for out of sequence. \(F_{\mu}(y)\) is defined as the conditional distribution function of the random variable, which can be expressed as

\[
F_{\mu}(y) = P(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)} = \frac{F(x) - F(\mu)}{1 - F(\mu)} \tag{1}
\]

So

\[
F(x) = F_{\mu}(y)(1 - F(\mu)) + F(\mu) \tag{2}
\]

\(X_1, X_2, ..., X_n\) is a sample from the same distribution \(F(x)\) population, if the distribution function of the random variable \(X\) can be satisfied

\[
G(x; \mu, \sigma, \xi) = 1 - (1 + \frac{x - \mu}{\xi \sigma})^\frac{-1}{\xi}, x \geq \mu, 1 + \xi(x - \mu) / \sigma > 0 \tag{3}
\]

The \(X\) is said to obey the generalized Pareto distribution (General Pareto Distribution, referred to as GPD distribution). \(\mu \in R\) is a function of the position, \(\sigma > 0\) is the scale parameter, \(\xi \in R\) is the shape parameter.

If there exist constants \(a_n\) and \(b_n\), making when reaching \(F(x)\) at the upper end of the point, \(F_{\mu}(a_n + b_n)\) have continuous limit distribution limit distribution , the theorem is established.

\[
\lim_{\mu \to x^*} \sup_{0 \leq y \leq x} |F_{\mu}(y) - G(y; \mu, \sigma, \xi)| = 0 \tag{4}
\]

The theorem: for sufficiently large threshold \(\mu\), the majority of unknown distribution function \(F(x)\) beyond the volume distribution function \(F_{\mu}(y)\) available GPD distribution \(G(y; \mu, \sigma, \xi)\) approximation is \(F_{\mu}(y) \approx G(y; \mu, \sigma, \xi)\), the type of substitution (2):

\[
F(x) = G(y; \mu, \sigma, \xi)(1 - F(\mu)) + F(\mu) \tag{5}
\]

After the determination of the \(\mu\), you can get a large number of \(N_{\mu}\) than the threshold \(\mu\ in \{X_i\}\, according to the formula (5) with the frequency \((1 - N_{\mu}/n)\) instead of the \(F(\mu)\), you can get the expression of \(F(x)\).

\[
F(x) = F_{\mu}(y)(1 - F(\mu)) + F(\mu) = 1 - \frac{N_{\mu}}{n} (1 + \frac{\xi}{\sigma} (x - \mu))^\frac{-1}{\xi} \tag{6}
\]

Through parameter estimation can be estimated \(\hat{\sigma}\) and \(\hat{\xi}\), the type (6) to
\[ F(x) = 1 - \frac{N}{n} (1 + \frac{\xi}{\sigma} (x - \mu))^{\frac{1}{\tau}} \]  \hspace{1cm} (7)

The threshold model (Peaks Over Threshold Model, referred to as the POT model) to estimate the distribution of the tail of the vehicle load, the key is the choice of threshold \( \mu \). If threshold \( \mu \) is too large, there will be only a small amount of excess sample, the estimated amount of variance will be high; if threshold \( \mu \) is too small, then the amount of \( Y \) and GPD distribution difference is larger, the estimated amount to be biased estimate.

**Threshold selection**

The method of selecting threshold \( \mu \) is mainly illustrated by graphic method and calculation method. Graphic method is based on the average of the amount of \( e(\mu) \) linear change or to determine the threshold of change caused by the parameter estimation of the change in the selection of threshold \( \mu \). The calculation method of main is Hill method, the kurtosis method \({}^{[11-13]}\). Although the application of graphical method is widely used, there is the existence of strong subjectivity. Therefore, in this paper, the kurtosis method for threshold selection, calculation steps are as follows:

1. Calculate the sample kurtosis \( K_n \);
   \[ K_n = \frac{E(X_i - \bar{X}_n)^4}{[E(X_i - \bar{X}_n)^2]^2} \cdot i = 1, 2, \cdots, n \]

2. To judge if the kurtosis, \( Kn \geq 3 \), the maximum value is selected to make \( X_i \) in \( |X_i - \bar{X}_n| \), which was removed from the sample;

3. Repeat the first step, second step, until the kurtosis is less than 3;

4. Select the largest \( X_i \) in the left sample point, which is the threshold.

**Parameter estimation**

The parameter estimation is based on the existing vehicle load data to estimate the unknown parameters of the POT model, which mainly include Maximum Likelihood Estimation, Moment Estimation, Bayes Estimation and so on\(^{[14]}\). After comparative analysis, the selection of the maximum likelihood method is used to estimate the parameters.

Assumed observed vehicle load sample sequence \( X_1, X_2, \ldots, X_n \), method of hypothesis and inference on the parameters of \( \xi \) and \( \sigma \) with Maximum Likelihood Estimation. By the formula (3) on both sides of the derivative can be GPD distribution density function

\[ g(x) = \frac{1}{\sigma} (1 + \frac{\xi x}{\sigma})^{-\frac{1}{\xi} - 1} \]  \hspace{1cm} (8)

The log maximum likelihood estimation function of the GPD distribution for the upper and the natural logarithm.

\[ L(\sigma, \xi) = -n \ln \sigma - (1 + \frac{1}{\xi}) \sum_{i=1}^{n} \ln(1 + \frac{\xi x_i}{\sigma}) \] \hspace{1cm} (9)

Make the partial derivation of \( \sigma \) and \( \xi \) separately, then make it equal to 0. The \( \sigma \) and \( \xi \)'s maximum likelihood estimates is \( \hat{\sigma} \) and \( \hat{\xi} \).
\[
\begin{align*}
\frac{1}{\xi^2} \sum_{i=1}^{n} \ln(1 + \frac{\xi}{\sigma} x_i) - (1 + \frac{1}{\xi}) \sum_{i=1}^{n} \frac{x_i}{\sigma + \xi x_i} &= 0 \\
-n + (1 + \xi) \sum_{i=1}^{n} \frac{x_i}{\sigma + \xi x_i} &= 0
\end{align*}
\]

EXAMPLE

A site in April 2014 for a total of 30 days of the vehicle load are selected for the study of the five vehicles, using a total of 25952 sets of data for example analysis. The kurtosis method selection threshold, the type 5 car data through the MATLAB R2010b \([15-16]\) program can be obtained for the 76.8 as threshold \(\mu\).

Table 1. Threshold of kurtosis.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Kurtosis</th>
<th>(N\mu)</th>
<th>(N\mu/n)</th>
<th>Suitable</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.8</td>
<td>2.9938</td>
<td>921</td>
<td>3.55%</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Then, according to the formula (10), use MatlabR2010b program on GPD distribution parameters \(\sigma\) and \(\xi\) to make the estimation. The estimated results are shown in table 2.

Table 2. Results of parameter estimate.

<table>
<thead>
<tr>
<th>Threshold (\mu)</th>
<th>Kurtosis</th>
<th>(\hat{\sigma})</th>
<th>(\hat{\xi})</th>
<th>JB test (P) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>76.8</td>
<td>6.87</td>
<td>6.28</td>
<td>-0.036</td>
<td>0.001</td>
</tr>
</tbody>
</table>

According to the results of parameter estimation, super threshold data kurtosis is much larger than that of the normal distribution of the kurtosis 3 and JB test \(p = 0.001\), can see threshold exceedance refused to accept from a normal distribution, with the peak thick tail. So we can use the POT model based on GPD distribution to fit the tail of the data. According to the formula (7), the tail can be any one of the type 5 vehicle’s weight:

\[
F_{w5}(x) = 1 - 0.0355(1 - \frac{0.036}{6.28}(x - 76.8))^{1/0.036}
\]

The tail of types 5 vehicle weight distribution is plotted in Figure 3, the figure shows the distribution tail measured value and fitting the tail of distribution results are consistent with the better.
For the estimation of the vehicle load, the distribution function \([17]\) of any heavy vehicle \(w\) can be got by the whole probability formula \(F_w(x)\).

\[
F_w(x) = a_3 F_{w3}(x) + a_4 F_{w4}(x) + a_5 F_{w5}(x), a_3 + a_4 + a_5 = 1
\]  
(12)

Based on the theory of extreme value analysis of multi modal variables\([1]\), the formula (12) is approximated by

\[
F_w(x) \approx \left[F_{w5}(x)\right]^\alpha
\]  
(13)

According to the Guangdong provincial capital management center in recent years, the observation data, type five vehicle accounted for all heavy vehicles (including type three or four, five) of the total flow of 66.5%, and therefore desirable \(a_5 = 0.665\).

If the analysis in the reference period \(T\) of the total flow is \(n\), then \(t\) vehicle load reproduce level \(U(T)\), that is \(p = 1 - 1/n\) gross vehicle weight distribution points digit probability value. And the maximum value of the vehicle load in the \(T\) (guaranteed rate is 95%) is the maximum weight distribution of 0.95 points \(W_{0.95}\), that is the maximum weight distribution of \(n\) vehicles is \(p^{1/a_5} = (0.95^{1/n})^{1/0.665} = 0.95^{1/0.665n}\). Based on this, the \(U(T)\) and the maximum value of \(W_{0.95}\) (\(T\)) and the maximum value of the vehicle load in different reference period were calculated, and the results were summarized in Table 3.

### Table 3. Return levels and maximum of truck gross weight.

<table>
<thead>
<tr>
<th>Time</th>
<th>Total flow</th>
<th>Type5 vehicle flow</th>
<th>(\mu=76.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>U(T)</td>
</tr>
<tr>
<td>1 month</td>
<td>39046</td>
<td>25952</td>
<td>114.8</td>
</tr>
<tr>
<td>1 year</td>
<td>468552</td>
<td>311424</td>
<td>126.5</td>
</tr>
<tr>
<td>5 years</td>
<td>2342760</td>
<td>1557120</td>
<td>133.5</td>
</tr>
<tr>
<td>10 years</td>
<td>4685520</td>
<td>3114240</td>
<td>136.4</td>
</tr>
<tr>
<td>50 years</td>
<td>23427600</td>
<td>15571200</td>
<td>142.9</td>
</tr>
</tbody>
</table>

From the estimated results, more than 114.8 tons of heavy vehicles will appear once a month, more than 126.5 tons of heavy vehicles on average once a year. Before the
implementation of the policy of heavy vehicle unloaded weight and frequency statistics contrast weight calculating charge, weight calculating charge policy of overweight vehicle gross weight, the frequency has a certain inhibitory effect, but did not effectively control the variation of overweight vehicles. 2008 should be the day of the Buddha to open the highway for statistical analysis, extrapolation of the maximum load of 156.5 years to reproduce the maximum value of 50 tons\[^{[18]}\], is very close to 153.9 tons in this paper. On the other hand, the real vehicle load is far beyond the design load standard, and the normal service of the road and Bridge constitutes a serious threat. In fact, in the 30 day of the measured data, it has been the 107.9 tons of vehicles, and, with the increase in the return period, there is sufficient reason to believe that a greater weight of the vehicle load. Therefore, it is necessary to continue to improve and improve the policy of weighing charges.

CONCLUSIONS

In this paper, based on the measured data of the WIM with toll model, The POT model of operating vehicle load is constructed by using the extreme value theory, which can describe the gross vehicle weight of the tail distribution characteristics better. On this basis, the development and variation trend of the operation vehicle load is predicted. The results show that the large tonnage vehicles are not accidental. In the future, there may be a greater tonnage of vehicle load, which make that the safety of existing roads and bridges constitute a serious threat to the operation of the bridge.

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