Analysis of the Annular Rotor Component Dynamic Characteristics Test System

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Abstract. The annular rotor component is the core of the rotating equipment, its performance is particularly important especially under the condition of high acceleration load condition. In this paper, a test system is designed which can measure the dynamic characteristics of the annular rotor component. The dynamic characteristics of the test system are analyzed and the influence on the results of the system is revealed. Some of the test results can be used to analyze by the grey forecast model to get the predictive value of system dynamic characteristics, which provides the beneficial reference for the selection of the annular rotor components.

Introduction

With the rapid development of the aerospace, military, logistics fields, mechanical devices often works in high acceleration load conditions. The acceleration load value of some components have reached to 150000g (where g is the gravitational acceleration), which makes the high acceleration load capacity test become very important. The annular rotor components are widely used in the designed high acceleration load system, especially in the super-high acceleration load system. This trend has become more obvious when the support components are made of damping alloy. The dynamic characteristics (stiffness coefficient, damping coefficient, loss factor) of the annular rotor component are very important, which will directly affect the working process of the system. Therefore the design of the annular rotor component becomes very important. The recent researches show that the dynamic characteristics are influenced by the operating frequency, temperature, and external loads, and the relationship between the factors and dynamic characteristics is usually nonlinear, which makes the dynamic characteristics difficult to analyze by simulation. In order to obtain the dynamic characteristics, the actual tests need to be done by which the data can be measured and calculated [1, 2, 3].

Analysis of Dynamic Characteristics of the Annular Rotor Component Test Principle

The proposed in the United States NASA-Lewis Research Center report base excitation resonance mass method (Abbrev. BERM method) can measure a wide frequency range of rubber component’s dynamic characteristics parameter values, experimental principle as shown in Fig. 1[4].
M is the mass of the vibrating mass in the figure,

The equation of motion is:
\[ m \ddot{y} + c ( \dot{y} - \dot{x}) + k (y - x) = 0 \]  

(1)

Vibration table for incentive form is:
\[ x = x_0 e^{i \omega t} \]  

(2)

The vibration response is:
\[ y = y_0 e^{i(\omega t - \varphi)} \]  

(3)

The stiffness coefficient of elastic spring \( k \) is derived by the simultaneous equations:
\[ k = \frac{m \omega^2 (\alpha^2 - \alpha \cos \varphi)}{\alpha^2 - 2\alpha \cos \varphi + 1} \]  

(4)

The damping coefficient \( c \) is:
\[ c = \frac{m \omega \alpha \sin \varphi}{\alpha^2 - 2\alpha \cos \varphi + 1} \]  

(5)

The amplitude ratio \( \alpha \) is:
\[ \alpha = \frac{y_0}{x_0} = \frac{\ddot{y}_0}{\ddot{x}_0} \]  

(6)

The loss factor \( \beta \) is:
\[ \beta = \frac{c \omega}{k} \]  

(7)

If the measured amplitude of excitation amplitude \( x_0 \) and vibration mass \( m \) are known, the stiffness \( k \) and damping coefficient \( c \) can be calculated by the phase difference \( \varphi \), the amplitude difference between excitation signal amplitude \( x_0 \) and response signal amplitude \( y_0 \) and the vibration mass \( m \).
Research of Annular Rotor Component Dynamic Characteristics Test System Design

The basic principle of the BERM method is to simplify it for single degree of freedom linear model, so with the real condition of the annular rotor component has a certain difference. In order to make the study more specific, this paper involves the annular rotor component vibration and with supporting, mainly for the metal material and high polymer material two kinds big, in order to improve the generality of the system, we use the idea of modular design, specific structure as shown in Fig. 2.

![Test System Structure Diagram](image)


Figure 2. Test System Structure Diagram.

The test system is divided into two parts, which can be connected to form the concluded through screw mounting holes. The tested parts are assembled on the shaft sleeve, and the sleeves are assembled on the test shaft. By changing the shaft sleeves, the dynamic characteristics of the tested parts made by different materials and different sizes can be tested without changing the size of the test shaft. As the large screw connection stiffness and low structure damping value of the threaded connection, the threaded connections in the system can be thought as rigid; In order to ensure the accurate positioning with the repeatedly assemblies, positioning pins are used in the upper and lower fixture fit surface connection. In order to test the influence of temperature on the dynamic characteristics of the tested parts, a temperature holding hood is designed and assembled at outside of the test system. By setting the initial work temperature and the blocking work temperature of the Quartz heating tube, the testing environment temperature can be controlled in a certain level. The dynamic characteristics of high polymer material made tested parts can be tested by changing the size of the ring structure on the shaft sleeve, as shown in Fig. 3.
Fig. 4 is the test system schematic diagram which shows the transmission of the signals in the test system. Tri-axial acceleration sensors are installed on the vibration table and the test shaft, the data acquisition instrument can collect the vibration acceleration signals of vibration table and the test shaft and transmit the data to the analyzing software which can analyze amplitude and phase information of the acceleration signals. Thus the dynamic characteristic parameters of test parts can be calculated. The type numbers and performance parameters of the vibration table, tri-axial acceleration sensors, data acquisition instrument and analyzing software are shown in Tab. 1.
Table 1. Type and Parameters of the Test Instrument and the Vibration Table.

<table>
<thead>
<tr>
<th>Instruments</th>
<th>performance parameters</th>
<th>Type numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration table</td>
<td>Maximum working frequency of 2000 Hz, three to the vibration at the same time, the sine thrust 9.8 kN</td>
<td>Suzhou SUSHI test instrument co., LTD MAV-1000-4H</td>
</tr>
<tr>
<td>Acceleration sensor</td>
<td>Three axes to measure at the same time, 1-4000 Hz frequency range</td>
<td>China Orient Institute of Noise &amp; Vibration INV9832</td>
</tr>
<tr>
<td>Data collecting</td>
<td>Support four-way signal input, support the BNC connector input</td>
<td>China Orient Institute of Noise &amp; Vibration INV3018A</td>
</tr>
<tr>
<td>instrument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis software</td>
<td>A sampling oscilloscope, time domain analysis, the spectrum analysis, the basic function such as functional analysis</td>
<td>China Orient Institute of Noise &amp; Vibration DASO-V10</td>
</tr>
</tbody>
</table>

The axial and radial characteristic parameters of the annular rotor component can be observed by calculate the vibration signals of the vibration table and the test shaft. And assembly relations in the system make the annular rotor component is in the similar environment with the actual using situation make the test with strong practical reference value.

Influence Factors Analysis for System Test

Even though the main consideration is the dynamic characteristic parameters of the annular rotor component, the possible influence factors in the testing process need to be considered and objectively evaluated the actual effect to the testing process.

Analysis of Natural Frequency of the System

Actual test system is a typical multi degree-of-freedom vibration system. Based on the key components appropriately simplified, the system three-dimensional dynamic model is established by any software as shown in figure 5.

In the finite element model, the radial structure stiffness of annular supporting parts is simulated by 24 combin14 elements, and the axial radial structure stiffness of annular supporting parts is simulated by 2 combin14 elements. The constraints of the finite element model are added to the system base as constrains of all DOFs.

By setting the stiffness of the combin14 elements which simulate the radial structure stiffness, the influence of different radial structure stiffness level of natural frequency is analyzed. Set the axial structure stiffness as $1.0 \times 10^7$ N/m and the radial structure stiffness as $1.0 \times 10^7$ N/m. the first eight order vibration shape diagram of the system are shown in Fig.6 as follows.

Figure 5. The System Three-dimensional Dynamic Model.
Figure 6. The System First Eight Order Vibration Shape Diagram.
The descriptions of vibration of each mode shape are shown in Tab.2:

<table>
<thead>
<tr>
<th>Order of system model shape</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>the axial vibration along the axial direction of the shaft</td>
</tr>
<tr>
<td>2</td>
<td>the translational vibration in the horizontal plane of the shaft</td>
</tr>
<tr>
<td>3</td>
<td>the translational vibration in the vertical plane of the shaft</td>
</tr>
<tr>
<td>4</td>
<td>the swing vibration in the horizontal plane of the shaft</td>
</tr>
<tr>
<td>5</td>
<td>the swing vibration in the vertical plane of the shaft</td>
</tr>
<tr>
<td>6</td>
<td>the translational vibration along the axial direction of the base</td>
</tr>
<tr>
<td>7</td>
<td>the translational vibration along the axial direction of the base</td>
</tr>
<tr>
<td>8</td>
<td>the torsional vibration of the base</td>
</tr>
</tbody>
</table>

The natural frequency of the system under the conditions of different radial structure stiffness calculation results are shown in Fig. 7.

![Figure 7. System First Eight Natural Frequencies.](image)

By setting the stiffness of the comb14 elements which simulate the axial structure stiffness, the influence of different axial structure stiffness level of natural frequency is analyzed. The axial stiffness mainly influences the shaft axial vibration. The natural frequency results are shown as Fig.8.
The calculation results show that the vibration of the test system itself has important effect on the result of the test. The test system has stiffness value both in the rotor radial, axial and circumferential direction. In the event of a low resonance conditions, there will be a corresponding displacement in both three directions, which make the test system movement presents the unstable state. The structural damping on the mating surface of the test system also needs to be considered. The influence of the structure damping is difficult to make quantitative analysis and evaluation from the current conditions, especially under the condition of the small vibration excitation signal, the test system gets uncertainty. In high frequency testing, for example, in the case of test frequency of 1800 Hz, if vibration excitation signal control system under the maximum acceleration value of 5 g, the vibration amplitude is less than 1 micron, the response speed has been involved in the structure of the relaxation phenomenon, and thus to produce the problem such as fever, makes the problem further complicated.

Analysis of the Influence of Annular Parts cooperate Relationship

From the perspective of the actual use of the annular rotor component, the test system can simulate the actual conditions of use to the greatest extent. In the process of testing, test results are influenced by the actual cooperate relationship. The axial stiffness of rotor is much affected by the cooperate relationship except its dynamic characteristics. In order to guarantee the stability of system test in the testing process, small interference fit is selected to be the fitting mode of the system, which equivalent load on the pre-tightening force of the annular rotor component, and will affect the axial stiffness. It can be analyzed from the actual use process, if the annular rotor component is used as vibration absorbing parts, its performance parameters are based on its mechanical performance parameters, under the influence of assembly condition and the using environment at the same time. Considering from the assembly condition, because of the need to guarantee to position in the process of installation and use, small interference fitting mode need to be selected to be the hole and shaft fitting.

![Figure 8. The Axial Structure Natural Frequency.](image)
mode. The minimum interference value should be larger than the vibration amplitude in order to prevent the occurrence of positioning relationship failure in use process problems.

The above analysis shows though the testing scheme can simulate the actual running environment of the annular rotor component, the decoupling analysis is difficult to do to analyze the radial or axial dynamic characteristics’ mapping relation to the influencing factors. The main reason is the inherent dynamic characteristics of test system itself, the installation of the annular rotor cooperate relationship factors such as comprehensive effect. The main reason is the comprehensive effect of the inherent dynamic characteristics of test system itself and the assembly of the annular rotor cooperate relationship factors.

Forecast analysis of Stiffness Value Based on the Gray Method

As the data is affected by many factors in the testing process, it’s difficult to analyze the data by single factor. Forecast analysis by data processing based on the data recognition method can be used to analyze single direction dynamic characteristics, such as the axial structure stiffness and the radial structure stiffness.

In this article, the principle of data selection is based on the existing titanium alloy test results data. As the small amount of data, and data test points of discontinuity, range interval, the grey prediction method is selected to forecast the stiffness of titanium alloy made annular rotor component. Grey prediction method can forecast system dynamic characteristics which contains uncertainties. By identifying development trend of dissimilarity degree between the system factors, namely, correlation analysis, to generate the raw data to find the law of the system change, to generate strong regularity of data sequences, and then establish the corresponding differential equation model, the grey prediction method can predict the future development trend of system. The grey prediction method can built the grey prediction model by forecasting the characteristic parameters values using the observed response value to forecast the system characteristic parameters value at a certain time in the future or the certain time when the system characteristic parameters approach the expected value [5].

Establishment of Non-equidistance Grey Forecast Model.

Let the sequence $X^{(0)}(K_i) = \{X^{(0)}(K_1), X^{(0)}(K_2), \ldots, X^{(0)}(K_n)\}$, if the distance $\Delta K_i = K_i - K_{i-1}$, $i=2, 3 \ldots n$, is not a constant, $X^{(0)}(K_i)$ is a non-equidistance sequence.

Let $X^{(1)}(K_i) = \{X^{(1)}(K_1), X^{(1)}(K_2), \ldots, X^{(1)}(K_n)\}$, if $X^{(1)}(K_i) = \sum_{j=1}^{i} X^{(0)}(K_j) \Delta K_j$, $i=1, 2 \ldots n$, $X^{(1)}(K_i)$ is the one-accumulate sequence of non-equidistance sequence $X^{(0)}(K_i)$.

Establish the whitening differential equation of $X^{(1)}(K_i)$,

$$\frac{dx_1^{(1)}(t)}{dt} + ax_1^{(1)}(t) = u$$  \hspace{1cm} (8)

Where $i=1, 2 \ldots n$.

Let $t = K_1$, $X^{(1)}(K_i) = X^{(0)}(K_1)$, the response function is

$$x_1^{(1)}(K_i) = \left[ X^{(0)}(K_1) - \frac{u}{a} \right] e^{-a(K_i - K_1)} + \frac{u}{a}$$  \hspace{1cm} (9)

Where $i=1, 2 \ldots n$. 
The model function is
\[ x_1^{(0)}(K_{i+1}) = \frac{1}{\Delta K_{i+1}} \left( 1 - e^{a\Delta K_{i+1}} \right) \left[ X^{(0)}(K_i) - \frac{u}{a} \right] e^{a(K_{i+1} - K_i)} \] (10)

The parameters \( a \) and \( u \) can be calculated by the following equation, \([a, u]^T = (B^T B)^{-1} B^T Y_N\), where
\[
B = \begin{bmatrix}
-Z^{(1)}(K_2) & 1 \\
-Z^{(1)}(K_3) & 1 \\
\vdots & \\
-Z^{(1)}(K_n) & 1
\end{bmatrix}
\] (11)

\( Z^{(1)}(K_i) \) is the background value of \( X^{(1)}(t) \) in the interval \([K_{i-1}, K_i]\), the background value is calculated by the two-point smoothing calculation formula of the traditional GM(1,1) model, \( Z^{(1)}(K_{i+1}) = \frac{X^{(1)}(K_{i+1}) + X^{(1)}(K_i)}{2} \), \( Y_N = [X^{(0)}(K_2), X^{(0)}(K_3), \ldots, X^{(0)}(K_n)]^T \).

The forecast analysis is based on the axial stiffness value of the test titanium alloy (TA4) shaft (the outer diameter= 40 mm; the thickness =2.7 mm) shown in the Tab.3.

### Table 3. Axial Stiffness Value of the Test Titanium Alloy.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>300</th>
<th>600</th>
<th>900</th>
<th>1900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness [N/m]</td>
<td>2309225</td>
<td>967073.7</td>
<td>325181.3</td>
<td>157372.2</td>
</tr>
</tbody>
</table>

The predictive value calculated by grey prediction method are shown in Tab.4.

### Table 4. Axial Stiffness Value Calculated By Grey Prediction Method.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>300</th>
<th>600</th>
<th>800</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness [N/m]</td>
<td>2309225</td>
<td>905859.192</td>
<td>417136.455</td>
<td>98756.854</td>
</tr>
</tbody>
</table>

The fitting line of the grey prediction data is shown in Fig.9.

![Figure 9. Grey Prediction Data Contrast Figure.](image)
**Accuracy Test on Grey Prediction**

To verify the accuracy and reliability of the prediction algorithm, model accuracy $P$ and the posterior error ratio $C$ are used to test the accuracy of the prediction algorithm. Let $x_1^{(0)}(K_i)$ is the prediction data of the sequence, $X^{(0)}(K_i)$ is the original data of the sequence. The calculate process of the accuracy test as follows.

The relative residuals is

$$
\varepsilon(k_i) = \frac{x_1^{(0)}(K_i) - X^{(0)}(K_i)}{X^{(0)}(K_i)} \times 100\% 
$$

(12)

The mean residual is

$$
\varepsilon_{avg} = \frac{1}{n-1} \sum_{i=2}^{n} |\varepsilon(k_i)| 
$$

(13)

The variance of error is

$$
S_1^2 = \frac{1}{n} \sum_{i=1}^{n} (\varepsilon(k_i) - \varepsilon_{avg})^2 
$$

(14)

The variance of the original data is

$$
S_2^2 = \frac{1}{n} \sum_{i=1}^{n} \left( X^{(0)}(K_i) - \frac{1}{n} \sum_{i=1}^{n} X^{(0)}(K_i) \right)^2 
$$

(15)

The accuracy of model is

$$
P = (1 - \varepsilon_{avg}) \times 100\% 
$$

(16)

The posterior error ratio is

$$
C = \frac{S_1}{S_2} 
$$

(17)

According to calculation, $P = 76.1\%$, $C = 1.159 \times 10^{-7}$, the accuracy of model is at the third grade, the grey forecast model is qualified.

**Conclusion**

The annular rotor component test systems based on the BERM method can simultaneously obtain the radial and axial characteristic parameter values of the annular rotor component, and maximize reflect practical application characteristics of the system. But the results of the test has uncertainty, which is affected by the structure of the annular rotor component itself and the cooperate relationship in the system. The quantitative relationship need to be further study. Some of the test results can be used to analyze by the grey forecast model to get the predictive value of system dynamic characteristics, which provides the beneficial reference for the selection of the annular rotor components.

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References


