Based on Nonlinear Adaptive Kalman Filter of Underwater Robot Space Attitude PID Control

Jing YU

Jilin Teachers Institute of Engineering and Technology, Changchun Jilin, China
dongnana860901@163.com

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Abstract. This article talks about the space attitude algorithm and PID control system of the underwater robot. This system uses nonlinear adaptive Kalman filter combined with PID control method to study, and the simulation analysis is carried out on the control effect to improve the space attitude control of underwater robot.

Introduction

Now the application fields of underwater robot expands continuously, such as ocean research, ocean exploitation and underwater engineering. It is a high technology-intensive, systematic strong technology. In the increasingly complex motion control, especially in environments with strong interference noise, the system uses a conventional PID controller is difficult to adjust its parameters to reach the desired control effect. Therefore PID algorithm of Kalman filter has been proposed, but in the system itself due to the instability of the system components, the uncertainty in the external environment as well as the processing of simplified linear model of factors, the ordinary algorithm of Kalman filtering can not meet the accuracy requirements of the system. Finally, this paper proposes PID control of underwater robot attitude based on space nonlinear adaptive Kalman filter.

Discussion on Nonlinear Adaptive Kalman Filter Algorithm

In discrete stochastic nonlinear systems

\[ X_k = f\left[X_{k-1}, k-1\right] + W_{k-1} \]
\[ Z_k = h\left[X_k, k\right] + V_k \]

In the equations (1), \( f\left[X_{k-1}, k-1\right] \) is the \( n \times 1 \)-dimensional differentiable vector function. \( h\left[X_k, k\right] \) is the \( m \times 1 \)-dimensional differentiable vector function. \( V_k \) and \( W_{k-1} \) are independent white noise, noise statistics are as follows:

\[ E\{W_k\} = q_k \]  
\[ E\{V_k\} = r_k \]
\[ E\left[[W_k - q_k][W_k - q_k]']\right] = Q_k \delta_k \]
Suppose the mean \( q, r \) and covariance matrix \( Q, R \) is unknown. If it has been filtered \( X_{k-1} \) at time \( k \) estimated value \( \hat{X}_{k-1} \), if develop into a Taylor series.

\[
f[X_{k-1},k-1] = f[\hat{X}_{k-1},k-1] + \frac{\partial f}{\partial X_{k-1}}(X_{k-1} - \hat{X}_{k-1}) + H.O.T + W_{k-1}
\]

H.O.T on behalf of Taylor expansion in all higher-order terms, \( X_i \) can be written as a linear equations, as follows:

\[
X_k = \frac{\partial f}{\partial X_{k-1}}X_{k-1} + u_{k-1} + \xi_{k-1}
\]

(7)

\[u_{k-1} = f[\hat{X}_{k-1},k-1] - \frac{\partial f}{\partial X_{k-1}}\hat{X}_{k-1} + \xi_{k-1} \] is a virtual model noise, \( \xi_{k-1} = W_{k-1} + H.O.T \). It compensates for the linear model error, that Taylor expansion unknown higher order H.O.T, it becomes noise with unknown statistics.

\[
E\{\xi_k\} = q_k
\]

(8)

\[
E\left[\left(\xi_k - q_k\right)\left(\xi_k - q_k\right)^T\right] = Q_k \delta_k
\]

(9)

Similarly, \( h \) forecast in the next valuation \( \hat{X}_{k/k-1} \) at \( X_k \) has expanded.

\[
Z_k = h[\hat{X}_{k/k-1},k] + \frac{\partial h}{\partial X_{k/k-1}}(X_k - \hat{X}_{k/k-1}) + H.O.T + V_k
\]

(10)

Of linear measurement equation as follows:

\[
Z_k = \frac{\partial h}{\partial X_{k/k-1}}X_k + y_k + \eta
\]

(11)

\[y_k = h[\hat{X}_{k/k-1},k] - \frac{\partial h}{\partial X_{k/k-1}}\hat{X}_{k/k-1} \] is a virtual measurement noise, \( \eta_{k-1} = V_k + H.O.T \). It is unknown when the band becomes noise statistics as follows:

\[
E\{\eta_k\} = r_k
\]

(12)

\[
E\left[\left(\eta_k - r_k\right)\left(\eta_k - r_k\right)^T\right] = R_k \delta_k
\]

(13)

Put it to compensate for linear error term.
Virtual noise $\xi_k$ and $\eta_k$ are compensated for the error of linear state model and linearization measurement equation. We order terms of the Taylor expansion into virtual noise to go in order to turn nonlinear system into a linear Kalman filtering problem. However, the general nonlinear Kalman filter algorithm can not meet the accuracy requirements, we propose a nonlinear adaptive Kalman filter.

$$
\begin{align*}
\frac{\partial f}{\partial \hat{X}_{k-1}} &= 
\begin{bmatrix}
\frac{\partial f_1[\hat{X}_{k-1}, k-1]}{\partial x_{1,k-1}} \\
\frac{\partial f_2[\hat{X}_{k-1}, k-1]}{\partial x_{2,k-1}} \\
\frac{\partial f_3[\hat{X}_{k-1}, k-1]}{\partial x_{3,k-1}}
\end{bmatrix} \\
&\quad \rightarrow 
\begin{bmatrix}
\frac{\partial f_1[\hat{X}_{k-1}, k-1]}{\partial x_{1,k-1}} \\
\frac{\partial f_2[\hat{X}_{k-1}, k-1]}{\partial x_{2,k-1}} \\
\frac{\partial f_3[\hat{X}_{k-1}, k-1]}{\partial x_{3,k-1}}
\end{bmatrix}
\end{align*}
$$

(14)

$$
\begin{align*}
\frac{\partial h}{\partial \hat{X}_{k-1}} &= 
\begin{bmatrix}
\frac{\partial h_1[\hat{X}_{k-1}, k]}{\partial x_{1,k}} \\
\frac{\partial h_2[\hat{X}_{k-1}, k]}{\partial x_{2,k}} \\
\frac{\partial h_3[\hat{X}_{k-1}, k]}{\partial x_{3,k}}
\end{bmatrix} \\
&\quad \rightarrow 
\begin{bmatrix}
\frac{\partial h_1[\hat{X}_{k-1}, k]}{\partial x_{1,k}} \\
\frac{\partial h_2[\hat{X}_{k-1}, k]}{\partial x_{2,k}} \\
\frac{\partial h_3[\hat{X}_{k-1}, k]}{\partial x_{3,k}}
\end{bmatrix}
\end{align*}
$$

(15)

Time-varying noise statistics estimators as follows:

$$
\begin{align*}
\hat{X}_k &= f[\hat{X}_{k-1}, k-1] + \hat{q}_{k-1} + K_k e_k \\
e_k &= Z_k - h[\hat{X}_{k-1}, k] \\
K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_{k-1})^{-1} P_{k|k-1} H_k^T \hat{R}_{k-1}^{-1} \\
P_{k|k-1} &= \Phi_{k|k-1} P_{k|k-1}\Phi_{k|k-1}^T + \hat{Q}_{k-1} \\
P_k &= (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T + K_k R_k K_k^T \\
&= (I - K_k H_k) P_{k|k-1}
\end{align*}
$$

(16)

Improved adaptive noise estimation algorithm is as follows:
The above algorithm is extended to nonlinear systems, and then combined with Kalman filter algorithm of nonlinear systems, we can get the nonlinear adaptive Kalman filter algorithm. As follows:

\[
X_k = f \left[ \hat{X}_{k-1}, k-1 \right] + \hat{q}_{k-1} + K_k e_k
\]

\[
e_k = Z_k - h \left( f \left[ \hat{X}_{k-1}, k-1 \right] + \hat{q}_{k-1}, k \right)
\]

\[
K_k = P_{x,k-1}H^T_k \left( H_kP_{x,k-1}H^T_k + R_{k-1} \right)^{-1} = P_kH^T_k R_{k-1}^{-1}
\]

\[
P_{x,k-1} = \Phi_{k-1,k}P_{x,k-1} + Q_{k-1}
\]

\[
P_k = (I - K_kH_k) P_{x,k-1} (I - K_kH_k)^T + K_k R_k K^T_k
\]

\[
= (I - K_kH_k) P_{x,k-1}
\]

(18)

Time-varying noise statistics estimators as follows:

\[
d_{k-1} = 1 - b \frac{1 - b^T}{1 - b^T}
\]

\[
\hat{q}_k = (1 - d_{k-1}) \hat{q}_{k-1} + d_{k-1} \left[ \hat{X}_k - \Phi_{k-1} \hat{X}_{k-1} \right]
\]

\[
\tilde{r}_k = (1 - d_{k-1}) \tilde{r}_{k-1} + d_{k-1} \left[ Z_k - H_k \hat{X}_{k|k-1} \right]
\]

\[
\hat{Q}_k = (1 - d_{k-1}) \hat{Q}_{k-1} + d_{k-1}K_k e_k e_k^T K_k^T
\]

\[
\hat{R}_k = (1 - d_{k-1}) \hat{R}_{k-1} + d_{k-1} \left[ e_k e_k^T - H_kP_{x,k|k}H_k^T \right]
\]

(19)

The main advantages of this algorithm are as follows: Firstly, it avoids the adaptive estimation formula appears in the subtraction, and it can avoid the influence of noise due to the statistical properties of instability brought about; Secondly, the algorithm can estimate the noise statistical properties online, small rely on priori statistical noise characteristics and mathematical models, and it has a wide range of adaptability.

**PID Control Based on Kalman Filter and the Simulation Analysis**

Based on nonlinear adaptive Kalman filtering algorithm of underwater robot space attitude PID control system structure is shown in figure 1.

![Figure 1. Control System Structure.](image)
On the simulation, the controlled object of simulation is the spatial attitude of an underwater robot, which is a third-order system, and the discrete transfer function which is obtained by identification is

$$ G(z) = \frac{8.702 \times 10^{-1} z^{-1} - 7.337 \times 10^{-6} z^{-2} - 3.878 \times 10^{-4} z^{-3}}{1 - 1.302 \times z^{-1} + 0.616 \times z^{-2} - 0.313 \times z^{-3}} $$

(21)

We adopt ordinary PID control and PID control based nonlinear adaptive Kalman filter to simulate. In the actual system process the noise \( \nu_k \) covariance takes 0, and we can obtain the covariance \( 4.355 \times 10^{-5} \) by measuring the noise \( \nu_k \), that is \( Q = 0, R = 4.355 \times 10^{-5} \). In Matlab simulation we use the random function \( \text{randn}() \) to simulate the white noise. The initial value \( p(0) = BB^T \).

When we take different values of PID parameters, the simulation results are as follows:

1. sampling time \( T_s = 0.001 \text{ms} \), PID parameters respectively are \( K_p = 50, K_i = 0, K_d = 0.2 \).

A. Without Kalman filter, the simulation results of ordinary PID control are shown in figure 2a, overshoot \( M_p = 1.978\% \); peak time \( t_p = 0.61\text{s} \); time to adjust \( t_s = 1.978 \).

B. With nonlinear adaptive Kalman filter, the simulation results of ordinary PID control are shown in figure 2b, overshoot \( M_p = 0.0015\% \); peak time \( t_p = 0.793\text{s} \); time to adjust \( t_s = 0.263 \).

![Figure 2. (a)Simulation Diagram Without Kalman Filter (b)Simulation Diagram With Nonlinear Adaptive Kalman Filter.](image1)

![Figure 3. (a)Simulation Diagram Without Kalman Filter (b)Simulation Diagram With Nonlinear Adaptive Kalman Filter.](image2)
(2) sampling time $T_s = 0.001\, ms$, PID parameters respectively are $K_p = 80, K_i = 0, K_d = 0.2$.

A. Without Kalman filter, the simulation results of ordinary PID control are shown in figure 3a, overshoot $M_p = 2.638\%$; peak time $t_p = 0.344\, s$; time to adjust $t_s = 0.615\, s$.

B. With nonlinear adaptive Kalman filter, the simulation results of ordinary PID control are shown in Figure 3b, overshoot $M_p = 0.002\%$; peak time $t_p = 0.56\, s$; time to adjust $t_s = 0.163\, s$.

From the simulation results:

1. ordinary PID control, the disturbance by noise is very large, overshoot is large and settling time is longer.

2. PID control based on nonlinear adaptive Kalman filter is almost no overshoot, settling time is short, indicating that it is very good to suppress the interference of white noise.

As the actual noise environment is too complex, therefore, based on this method, there are still some errors. But in terms of the control of underwater robot it has reached a very significant effect.

**Conclusions**

This paper explores spatial attitude algorithm and PID control system of the underwater robot. On account of a third-order system, which uses underwater robots as the controlled object, in case of changing the scale factor $K_p$ conditions, compares the nonlinear adaptive Kalman Filter PID control method and the ordinary PID control method. The simulation results show that the the nonlinear adaptive Kalman Filter PID control method has a smaller overshoot rate, is adaptable, good control effect, etc., the quality control compared with ordinary PID control quality has been significantly improved.

**References**


