A New Linear Programming Approach for TDGCS Layout Problem

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Abstract. Currently, considering the layout effect, the linear programming method based on column generation is generally proposed to solve the large-scale TDGCS layout problem, but layout scheme which is obtained by this method is the need to get multiple layout, a large number of calculations and a lot of time. However, there must be a balance between the material utilization ratio and computing time in the actual production process. Therefore, in this paper, a new linear programming method, which is based on two aspects of the minimum and maximum of objective function about the layout problem, is proposed. Firstly, a preliminary layout is finished from the aspect of the minimum optimization, then, a precise layout based on the preliminary layout is achieved from the aspect of maximum optimization. Compared with the original algorithm, theoretically speaking, this method can significantly reduce the amount of calculation through the calculation process, guaranteeing the material utilization.

Introduction

Generally, two-dimensional layout problem can be departed into two types of layout problem, namely, two-dimensional guillotine-cutting stock and two-dimensional non-guillotine cutting stock. In this paper, a linear programming based on column generation is adopted to solve the large-scale TDGCS Layout problem. This method, which was first proposed by Gilmore and Gomory (1961), is an accurate algorithm, so far, it has been widely used in many manufacturing. Later, dynamic programming based on tree search and integer programming algorithms were proposed by Christofide (1977) and Farley (1988) for solving rectangular guillotine-cutting layout problems, requiring a lot of computation time. To solve a general class of large-scale linear programming problems, Muter, Ibrahim (2013) developed a simultaneous column-and-row generation algorithm, and applied this method to the multi-stage cutting stock and the quadratic set covering problems. In addition, G Young Gun(2001), R Morabito (1998) and Guntram (1999) proposed some heuristic algorithms, which segment materials and arrange needed pieces on it. Another heuristic algorithms based on linear programming, for the large-scale layout problem, were proposed. For example, a hybrid genetic algorithm based on linear programming was proposed by Kulturel-Konak (2013) to solve the facility layout problem that the layout had the large number of binary decision variables as well as the lack of tight lower bounds. Although calculation time is less, the heuristic algorithms often easily fall into local optimal solution in a limited period of time, so, it is very important how to get appropriate calculation time and higher material utilization.

The main motivation of this paper is to reduce the amount of calculation through the improved mathematical model, particularly for the large-scale TDGCS layout. Usually, considering
the cost, efficiency and quality of the production, there must get a balance between the material utilization ratio and computing time in the actual production process. In this paper, when satisfy the material utilization, reducing the computing time by improved mathematical model.

The Original Mathematical Model of Layout Problems

The Mathematical Model of Layout Problems

Generally, method of linear programming based on column generation is used to solve the large-scale TDGCS layout problem. The mathematical model of the TDGCS layout problems are presented here:

\[
\min \sum_{i=1}^{n} c_i x_j \quad j = 1, \ldots, n. 
\]

\[
a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq b_1
\]

\[
a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq b_2
\]

\[\vdots
\]

\[
a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq b_m
\]

\[x_1, x_2, \ldots, x_n \geq 0
\]

The Improved Mathematical Model of Layout Problems

This improved model is proved to be feasible by considering the limited resources that is AND is not more than \( B \). Mathematical model as shown below:

\[
\min \left\{ \sum_{i=1}^{m} c_i x_j ; x_j \in N ; \sum_{j=1}^{m} a_{ij} x_j \geq b_i ; i = 1, \ldots, m ; b_i = \min(b_1, \ldots, b_m) \right\}. 
\]

\[
\max \left\{ \sum_{i=1}^{m} k_i p_i ; k = s ; \sum_{i=1}^{m} x_i p_i \leq h ; h = \sum_{i=1}^{m} (b_i - \sum_{k=i}^{m} a_{ik} x_k) ; x_i p_i \leq b_i - \sum_{k=i}^{m} a_{ik} x_k \right\}. 
\]

\[
\max \sum_{i=1}^{m} k_i p_i \leq S . 
\]

\[
\sum_{i=1}^{m} P_i \geq P_0 . 
\]

\[w = \max\left( \sum_{i=1}^{m} v_i p_i \right), \quad v_i = CA^{-1}.
\]

\[L \times W - w \leq 0 .
\]
In the case of cutting stock, the parameters are as follows. 

c_i is the value of a single material and the matrix is \( C = [c_1, c_2, \ldots, c_n]^T \), \( c_i = L \times W \), \( x_j \) is the number of material of the layout of the corresponding way and the matrix is \( x = [x_1, x_2, \ldots, x_n]^T \), \( a_{ij} \) is number of the pieces of material the corresponding layout way, and \( b_i \) is the limited number of the pieces of the corresponding way and the matrix is \( b = [b_1, b_2, \ldots, b_m]^T \). \( p_i \) is The current layout and the matrix is \( p = [p_1, p_2, \ldots, p_n]^T \), \( k_i \) is the square meter of the pieces and \( S \) is the square meter of each material, \( P_i \) is the utilization of every material and \( P_0 \) is the utilization of all material, \( v_i \) is current value vector of the pieces.

The Calculation Steps of the Improved Mathematical Model

In this paper, the minimum value of objective function problem can be transformed to a maximum optimization problem by layout scheme. The calculation process of this method as shown in flowchart. Flowchart of method is illustrated in Fig.1.

![Flowchart](image)

**Figure 1. Flowchart of the Improved Method.**
The step description of the flowchart is as follows:

① The solution of the standard size
Choosing an appropriate layout from existing layout types and obtaining the standard size of the corresponding layout types; Calculating the number of pieces of strip cutting pattern.

② The determination of the initial basic feasible solution
Make $A$ for a non singular matrix. The first line of $A$ is $b$ that is the minimum. $A$ is as follows:

$$ A = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix} $$

③ Calculating the corresponding variable values of the current minimum of the objective function, as shown in (3); Getting a corresponding layout through area calculation of all pieces of current material, as shown in (4). Area of all pieces of current material is not more than area of the current material, as shown in (5). Whether layout of the last material is completed, if not, Repeat step ③ until all material layout are made. ④ If $\sum_{i=1}^{P} \ge b_i$, the effect of layout meets the requirement and save the current result, stopping calculation, that means, layout is completed, if not, go to step ⑤. ⑤ Calculating the value vector of the current pieces, that is $v_i$. ⑥ Calling a layout scheme and obtaining the current optimal layout of the current maximum of the objective function, calculating the corresponding variable values, as shown in (7). ⑦ If $L \times W \le 0$, go to step ⑥ if not, save the current result, stopping calculation and output integer value of the current optimal solution.

The Validation of the Improved Mathematical Model

Using the improved linear programming method based on the column generation to solve the layout scheme through a simple example, this algorithm is a same-shape strip straight cutting layout type.

Size of the material, the number of pieces and requirement see Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Pieces 1</th>
<th>Pieces 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>10×7</td>
<td>2×1</td>
</tr>
<tr>
<td>Requirement</td>
<td>1200</td>
<td>1000</td>
</tr>
</tbody>
</table>

The effect of the calculation for this method is as shown in Figure 2.

Figure 2. The Effect of the Layout Scheme.
Through the calculation, The number of pieces 1 and pieces 2 in material are respectively 15 and 3, the another material has only 5 pieces 2. Thus, the numbers of the two materials are respectively 80 and 152 and the value of objective function is minimum, so, the result can be accepted. Meanwhile, in calculation process, A-1 is need not to calculate many times and don't even need to calculate it when the preliminary layout has meet requirements. Therefore, amount of calculation is reduced.

In conclusion, the improved mathematical model is feasible.

**Summary**

In this paper, a new linear programming method based on the column generation was established by the minimum and maximum of objective function. For the original approach, the inverse matrix must be calculated many times and takes a lot of computation time, however, the improved mathematical model need not to calculate it many times, and don't even need to calculate it when result of the preliminary layout has meet requirements, especially for the large-scale TDGCS layout problem. Since this method does not have to validate the large-scale TDGCS layout problem and the later research is underway, theoretically speaking, the improved mathematical model is adapted to calculate the large-scale TDGCS layout problem.

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**References**


