A Physical Deformation Method Based on Shape Constraint for Product Shape Design

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Abstract. In this paper, a modeling method based on physical deformation is described. The proposed method is designed to solve the modeling of deformation products with requirements such as a local shape requirement or space assembly. It is utilized to designing products by analogy to balloon inflating, and the deformation process obeys physical laws of motion. First of all, the boundary of spatial constraint of deformable component is obtained according to design requirements and assembly environment. Secondly, component is initialized by a simplified mass-spring model, and the process of expansion is simulated by increasing the internal pressure until the deformation extent meets the needs. Finally, several examples are illustrated for verifying this method. Comparing to traditional design method, this method gets concept model under the condition of design constraint firstly, then regards the concept model as references for detail design. Therefore, the idea can avoid repeated modification of artificial design and shorten design period significantly.

Introduction

Shape design of product is a problem of space design under the action of multiple constraints. It is usually subjected by the factors of performance requirement, man-machine engineering, assembly environment and processing technic, etc. On the premise of meeting required functions, deformable component has no determined shape and great designing freedom. However, the design constraints of deformable component are usually very complicated. Hence, in order to acquiring satisfactory result, the design model must be modified repeatedly. However, compiling free-from surface based on existing three-dimensional modeling software is quite intricate, and the efficiency is not optimistic.

At present, the method based on physical deformation has been received widespread attention in product design [1, 2, 3, 4]. It discretizes consecutive deformable body into grid model, depends on principle of physics to compute the process of movement and deformation of the grid model, then renders the process through image display technology [5, 6]. The mass-spring model is one of the frequently-used methods, which has characteristics of smaller amount of calculation and higher efficiency, because the force of particle in mass-spring model is found out from force analysis directly. Thus, the method of mass-spring model not only overcomes the time-consuming deficiency of geometric modeling, but also solves the problem of final shape expression of unknown deformable body. Kilian and Ochsendorf modeled the
roof surface by means of mass-spring model, and simulated the force deformation of model to design the roofing structure with large span [7]. Igwe et al [3, 8, 9] used the mass-spring model for product conceptual design, but they merely took the mechanical deformation of single object into account, and it doesn't fit the deformation simulation of deformable component with complex constraints.

In this study, deformation simulation technology of flexible object is applied in the designing of deformable component among complex products, and a kind of design method for deformable component is put forward as well. Its main idea is using the spontaneous deformation of model under the constraining force for design, which can avoid the modeling process of tedious complex surface and improve the design efficiency superbly.

**Design Method of Inflation Modeling for Product Shape**

**Extraction of Spatial Constraint**

The designing of deformable component in this method is obtained by simulating the procedure that analogy to the expansion of balloon under the limit of constraint boundary. Before extracting the constraint boundary, the following situations need to be handled due to the differences of shape and working condition for the components in actual product:

Convex hull of model: the characteristics such as lug bosses, grooves on component surfaces. Simply regarding the model surfaces as constraint boundary will affect the assembly performance and simulation efficiency of deformation. While using the three-dimensional convex hull of model as constraint boundary can avoid this problem efficaciously. The geometric model and three dimensional convex hull of engine are shown in Fig. 1 and Fig. 2 respectively.

Motion component: it refers to the movable components in practical operation. The constraint boundaries of those components should be considered synthetically according to their shapes and motion.

Assembly clearance: the assembly clearances between components generally need to be reserved. When extracting the constraint boundaries, the boundary surface of the model must be offset along with its outside normal direction.

![Figure 1. Geometric Model of Engine.](image1)

![Figure 2. 3D Convex Hull of Engine.](image2)

**Expansion Modeling Design of Deformable Component**

As shown in Fig. 3, (a), (b) and (c) display the initial stage, the middle stage and the end stage of expansion deformation process for component E severally. The constraint boundary of component E is composed by component A, B, C and D, as well as the assembling shell.
Stage (a): Define constraint boundary. The main work of this stage is initializing component E into a small ball, dividing its surface through triangle mesh generation technique and defining the topological structure of its mass-spring.

Stage (b): Deform component E by dint of the ever-increasing air pressure.

Stage (c): Decide to stop deformation or not by means of judging whether the deformation of component E reaches a set standard.

![Figure 3. Deformation Process of Component.](a) (b) (c)

**Deformation Method of Product Shape Based on Mass-spring Physical Model**

**Mass-spring Model**

The object in mass-spring model is comprised by discrete particles with mass and massless springs that connect particles [1]. For improving the simulation efficiency, a simplified mass-spring model is adopted in this research. The particles are distributed only on the surface of component, and only the two adjacent particles are connected through springs. Compared with complex models, such as cloth model and skin model, the simplified mass-spring model reduces the number of particles and springs greatly. Besides, in order to maintain the model at a certain space volume, relative gas pressure is introduced into the system of mass-spring model. In addition, the initial value of relative pressure is set as 0, and it increases following time step to drive the deformation of component.

The load condition of particle on deformable component is shown in Fig. 4. In this work, force of gravity and friction are left out of account, as they have no actual meaning for the design of deformable component. So, the system exerted by the spring force, the damping force of particle motion and the internal gas pressure. Its equation of motion can be represented as:

\[
M \frac{\partial^2 X}{\partial t^2} = F_s + F_d + F_p
\]

Where \( M \) is the mass matrix of particle system; \( X \) is the spatial location of particle; \( F_s \) is the spring force matrix; \( F_d \) is the damping force matrix; and \( F_p \) is the gas pressure matrix.
Assuming that the described spring is ideal linear spring and ignores the deformation damping itself. Supposing the number of particles which are connected with particle \( i \) adjacently by spring is \( j \), the resultant force of particle \( i \) exerted by spring is represented as,

\[
F'_{si} = \sum_{\forall j \in E(i)} k_{ij} \left( \frac{x_i - x_j}{|x_i - x_j|} \right) \left( |x_i - x_j| - l_{ij}^0 \right)
\]

Here, \( E \) is the point set of the adjacent particles which are connected with particle \( i \) by spring; \( k_{ij} \) is the spring stiffness between particle \( i \) and \( j \); \( x_i \) is the place of particle \( i \) at time \( t \); \( x_j \) is the place of particle \( j \) at time \( t \); and \( l_{ij}^0 \) is the original length between spring \( i \) and \( j \).

The damping force of particle is proportional to its moving velocity. Assuming the damping coefficient is \( C \); i.e.,

\[
F'_{di} = -Cv'_i
\]

At time \( t \), the gas pressure that the particle \( i \) sufferers is the sum of gas pressure acting on the triangles adjacent to particle \( i \). The pressure can be expressed as:

\[
F'_{pi} = \sum_{\forall j \in \text{Tri}(i)} A_{ij} \left( P_t + \Delta P \right) \vec{n}_i
\]

Where, \( \vec{n}_i \) is the external normal vector of triangle patch; \( \text{Tri} \) is the set of triangle patches which have the common particle \( i \); \( A_{ij} \) is the square of triangle \( j \) that connect with particle \( i \).

**Solution of Motion Equation**

Numerical integration algorithm is used to solve the motion equation of particle at every time step. The integral method of Verlet [10,11] is adopted in this text, whose basic idea is utilizing the Taylor formula to expand the position item \( x(t + \Delta t) \) and \( x(t - \Delta t) \) of particle respectively. Considering the Eq. 1, that is:

\[
x(t + \Delta t) = 2x(t) - x(t - \Delta t) + M^\dagger R(x(t), v(t))\Delta t^2
\]

The integral method of Verlet has advantages of high efficiency and good stability because it doesn't involve the calculation of particle velocity [10, 12], while the damping force does.
Therefore, the average velocity in time step \( \Delta t \) is regarded as the particle velocity, and the particle velocity can be expressed as:

\[
v = \frac{x(t) - x(t - \Delta t)}{\Delta t}
\]  

(6)

**Volume calculation**

For the sake of obtaining the volume of deformable component, volumes of the discrete tetrahedrons composed by the triangle patches together with origin of coordinates are integrated. The volume of deformable component can be represented as:

\[
V = \frac{1}{6} \sum_{i=0}^{k-1} (V_{i_x} - V_{i_y})(V_{i_z} - V_{i_z}) - (V_{i_x} - V_{i_y})(V_{i_x} - V_{i_y}) (V_{i_x} + V_{i_y} + V_{i_z})
\]  

(7)

**Collision Detection and Response**

Collision detection is an unavoidable problem in the deformation process of component with spatial constraints. The collision detection method based on axis aligned bounding boxes (AABB) bounding box tree is used to deal with the collision problem in this paper. This method has the peculiarities of simple structure and updating easily [13], whose algorithm procedures are as follows:

**Step1:** Establishing the structure of hierarchical bounding box tree of deformable component and its constraint boundaries via the Octree space partition method;

**Step2:** Performing a simulation time step, working out \( x(t + \Delta t) \) according to \( x(t) \), and updating the bounding box structure of current component;

**Step3:** Do intersection tests of bounding box for deformable component and its constraint boundaries, in order to find the underlying bounding box pairs that intervene with others;

**Step4:** Performing accurate detections for the internal geometric units of underlying bounding box pairs until finding all the particles and triangular units that collided with others.

The particle location at time \((t + \Delta t)\) should be amended after the collision units are found, as shown in Fig. 5.

![Figure 5. Amendment of Particle Location.](image-url)
Processing Flow of Expansion Deformation for Deformable Component

The flow chart of deformable component design is shown in Fig. 6, which can be decided whether to terminate deformation in various ways based on different design objects. For deformable component in complex assembly system, it can be judged through whether the volume of model is reaching the standard, while the products with high standard of ergonomics can be judged by monitoring the deformation process visually.

Method Validation

The simulation system is established with the help of VC ++ programming language. Meanwhile, the OpenGL graphics library is used to render the deformable component during the simulation of expansion deformation.

Expansion Deformation for Single Object

In this case, the constraint boundary is derived from three-dimensional convex hull of engine mentioned above, and the initial volume of the convex hull is 170756 cm³. The initial model is a ball with a diameter of 200 mm and 3526 triangular patches. The target volume of the ball is set to 110000 cm³. What’s more, the other parameters such as the spring constant $k_s$, the time step $\Delta t$, the mass of deformable component $m$ and the increment of gas pressure $\Delta p$ are 1000 N/m, 0.01 s, 1 kg and 1 N/m².

The Fig. 7 and Fig. 8 provide the initial status and ending status separately. From the pictures, it is easy to find the component experienced a large deformation process and its ultimate shape is similar to the shape of constraint space.
Shape Design of Man-machine Engineering

In the following part, we will take a mouse as the object for verifying this method. Using the mouse designed via traditional way for a long time is easy to cause fatigue, as it can't fit human palms preferably. In order to obtain a better solution, the method proposed in this paper is used for conceptual design of mouse. Firstly, palm model under relaxed state is extracted by three dimension scanner. Secondly, palm and desktop are defined as the constraint boundaries and the remaining boundaries are replenished for the purpose of forming a closed space in accordance with the design idea. Finally, initializing the deformable ball and making it expanded. The deformation process is shown in Fig. 9. The mouse designed on the basis of the final model enables users to reduce fatigue significantly. The Fig. 10 shows the physical object of vertical mouse.

Conclusion

A new modeling method is used to design the shape of deformable product under the circumstance of complicated spatial constraint in this research. The force and deformation of particles obeys the physical laws of motion. The main idea of this method is converting the design requirements and assembly restrictions of product into the boundary conditions and designing deformable component via the expansion deformation of the mass-spring model in constrained space. The cases show that this method can obtain the concept model of complex deformable component easily and quickly. Not only is it suitable for the design of complex
assembly within complicated environment, but it is appropriate for designing the products with special requirements of ergonomics.

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