Model and Solution Algorithm of Bus Network Design Problem under Clusters Urban Structure

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ABSTRACT

Based on clusters urban structure, model and algorithm for bus network design problem was researched. To minimize passenger bus travel distance, passenger bus transfer times and total length of bus lines, a multi-objective programming model was built. Combined with artificial immune algorithm, solving steps of model is proposed, and then the design description of encoding antibodies and antibody update operator. Finally, the effectiveness of the model and algorithm was verified in the case study.

INTRODUCTION

Bus network design problem is an important part of public transport planning system, which also is the focus and difficulty in the field of bus research[1-5]. Study of existing bus network design problems mostly apply to the bus service area based on single group or a given area, and did not consider the feature of clusters urban structure. Layout of urban space of a single center changes to multi-center cluster gradually, which also leads to the changes of resident trip characteristics. Urban bus network optimization method under the structure of one-single center cities can not reflect the structural features of cluster bus network. The purpose of this study is to propose a bus network design method with clusters urban structure characteristics.

PROBLEM ANALYSIS

The bus network of clusters urban consists of two types of bus lines: ones serve for inner group (region), ones serve for between groups (region). A schematic diagram of bus network in clusters urban structure is shown in figure 1. It consists of A1, A2, A3 and A4 group, and bus lines link each pair of adjacent group. Wherein, b1, b2, b3 and b4 are bus hub station inside each group, which serves for the passenger transfer of bus lines inside and between groups.
Bus passengers travel costs and operating costs are two important optimization objects in bus network design. In the study, departure interval decision is not considered, and the chosen bus travel distance and transfer times of passengers are based on bus travel cost. Operating cost is closely related to the length of the bus lines, and the chosen total length of bus lines are based on bus operating cost. In summary, the model takes the minimization of total bus travel distance of passengers, total number of bus transfer of passengers and total length of bus lines into account.

**MATHEMATICAL MODELS**

**Assumptions**

(1) Public demand is the fixed demand patterns
(2) Driving speed of buses is constant, and not be affected by road congestion.
(3) Bus station collections are known, candidate bus lines collection are known.
(4) When there is multiple bus travel route between a given OD, passengers would choose the one with minimum bus travel distance.

**Symbol Definition**

\( i, j \)—Bus station set, also the set of bus demand points;
\( A \)—Number set of groups;
\( d_{ij} \)—Bus demand from bus station (demand point) \( i \) to bus station (demand point) \( j \);
\( R \)—Candidate bus line set;
\( l_r \)—Line length of candidate bus line \( r \in R \);
\( \theta_r \)—When candidate bus line \( r \in R \) is selected into bus network, 1. Otherwise, 0;
\( K \)—Bus travel route set. \( k \in K \), given one specific bus travel route;
\( \varphi_{kr} \)—When bus travel route \( k \in K \) contain candidate bus line \( r \in R \), 1. Otherwise, 0;
\( \mu_{ijk} \)—When bus travel route \( k \in K \) is chosen from bus station (demand point) \( i \) to bus station (demand point) \( j \) in passenger travel decision, 1. Otherwise, 0;
\( c_{ijk} \)—When bus station (demand point) \( i \) and bus station (demand point) \( j \) are both in the bus travel route \( k \in K \), and bus travel route \( k \in K \) contain candidate bus line \( r \in R \), it represents the travel distance of passengers in the candidate bus line \( r \in R \) from bus station (demand point) \( i \) to bus station (demand point) \( j \).
**Multi-objective Programming Model**

The formula of total bus travel distance of passenger shows as:

\[ z_1 = \varepsilon_1 \sum_{i} \sum_{j} \sum_{k} \sum_{a} \sum_{b} \theta_i d_{ij} \mu_{ijk} \varphi_{kr} c_{ijk} r + \varepsilon_2 \sum_{i} \sum_{j} \sum_{k} \sum_{a} \sum_{b} \theta_i d_{ij} \mu_{ijk} \varphi_{kr} c_{ijk} r \]

(1)

Among them, the bus lines between and inside groups can be divided into the trunk and branch of the bus in the actual planning. And set two different weights \(\varepsilon_1\) and \(\varepsilon_2\) in the first two items in the formula (1), \(\varepsilon_1+\varepsilon_2=1\), and \(\varepsilon_1>\varepsilon_2\).

The formula of total bus transfer times of passenger shows as:

\[ z_2 = \sum_{i} \sum_{j} \sum_{r} \sum_{k} \sum_{a} \sum_{b} d_{ij} (\theta_i \mu_{ijk} \varphi_{kr} - 1) \]

(2)

Bus operating cost is measured by the total length of bus lines, the specific formula shows as:

\[ z_3 = \sum_{r} \theta_i l_r \]

(3)

Through multi-objective weights, Multi-objective programming model shows as:

\[
\min z = \alpha_1 (\varepsilon_1 \sum_{i} \sum_{j} \sum_{k} \sum_{a} \sum_{b} \theta_i d_{ij} \mu_{ijk} \varphi_{kr} c_{ijk} r + \varepsilon_2 \sum_{i} \sum_{j} \sum_{k} \sum_{a} \sum_{b} \theta_i d_{ij} \mu_{ijk} \varphi_{kr} c_{ijk} r ) \\
+ \alpha_2 (\sum_{i} \sum_{j} \sum_{k} \varphi_{kr} (\theta_i \mu_{ijk} \varphi_{kr} - 1) + \alpha_3 \sum_{r} \theta_i l_r )
\]

(4)

\[
\theta_i l_{\text{min}} \leq \theta_i l_r \leq \theta_i l_{\text{max}}
\]

(5)

\[
\sum_{r} \sum_{k} (\theta_i \mu_{ijk} \varphi_{kr} - 1) \leq 2 \ \forall i \neq j
\]

(6)

\[
\sum_{k} \mu_{ijk} = 1 \ \forall i \neq j
\]

(7)

\[
\theta_i \in \{0,1\} , \ \mu_{ijk} \in \{0,1\} , \ \varphi_{kr} \in \{0,1\}
\]

(8)

Among them, \(\alpha_1\), \(\alpha_2\) and \(\alpha_3\) represent the object weight parameters of multi-objective programming model in the formula (4), \(0<\alpha_1<1, 0<\alpha_2<1, 0<\alpha_3<1\), and \(\alpha_1+\alpha_2+\alpha_3=1\). Constraint (5) represents a single bus line length constraint, \(L_{\text{min}}\) and \(L_{\text{max}}\) represent the minimum and maximum values allowed of a single bus lines. Constraint (6) indicates the number of transfers for each passenger could not be greater than 2. Constraint (7) shows the selection constraint of passenger travel route. Passengers can select only one travel route based on assumptions (4). Constraint (8) is 0-1 variable.

**Solution Algorithms**

Effective solution algorithm for multi-objective programming model is proposed based on artificial immune algorithm.
For decision variables $\theta_r$ is 0-1 integer variable, so antibody coding form is also 0-1 coded, each candidate bus line corresponds to a code. When code is 1, it indicates that the candidate bus line corresponded to this antibody has been incorporated into the network, otherwise, not selected.

Algorithm steps as follows:

**Step1**: Generate candidate bus trunk set between service groups. Select two different regional groups and two bus stops $i$ and $j$ belong to this two groups. Generate candidate bus trunk between bus stops $i$ and $j$ using the shortest path algorithm;

**Step2**: Generate candidate bus brunch set inside service group. Select each regional group, then select bus stops $i$ and $j$ within this group. Generate candidate bus brunch between bus stops $i$ and $j$ using the shortest path algorithm;

**Step3**: Merge candidate bus trunk set served between groups and candidate bus brunch set served inside groups into one set, and generate the final set of candidate bus lines

**Step4**: generate initialization antibody populations according to the set encoding rules;

**Step5**: Calculate affinity of all antibodies sequentially;

**Step6**: Operate the antibody variation and cross to antibody populations;

**Step7**: Inhibit and promote antibodies using information entropy approach. And it aims to prevent premature convergence of antibodies, and to obtain local optimal solution;

**Step8**: Select antibodies using roulette manner;

**Step9**: Judge whether abort conditions of algorithm are met, if met, output the optimal antibody to decode. If not, turn to step5 to continue the calculation.

**CASE STUDY**

Figure 2 is a schematic diagram of case. Nodes 1-12 represent bus demand (bus station), node 1-2 represent preselected transfer hub sites. There are two regional groups, group 1 contains nodes 1,3,4,5,6 and 7, group 2 includes nodes 2,8,9,10,11 and 12. The connection between nodes represents the roads, and the number indicates the length of road. Table 2 is the OD travel demand. $\varepsilon_1$ and $\varepsilon_2$ were taken 0.7 and 0.3.

Related parameters of algorithm: antibody population size takes 40, crossover and mutation probability were 0.7 and 0.1, and the maximum number of iterations is 200.

![Figure 2. Example of numerical example.](image)
When $\lambda$ is set to 0, the optimization results would be composed of four lines, 3-5-1-2-9, 7-1-6-4, 8-2-11 and 2-10-12. When $\lambda$ is 0.3, the optimization results contains seven lines, 7-1-2-10-12, 7-1-6-4, 1-5-3, 7-5-6, 8-2-11, 11-2-9 and 8-9-10. When $\lambda$ is 0, the optimal result is to cover bus network with minimum total bus lines distance, regardless of the rationality of passengers’ bus travel distance. The composition of bus network when $\lambda$ is 0.3 is more complex than 0, the number of bus lines increases from 4 to 7. So the higher $\lambda$ is, the more reasonable of when passenger bus travel distance.

**SUMMARY**

This paper considers the characteristics of bus network structure in cluster urban, and proposes network optimization method. And build a multi-objective programming model to minimize passenger bus travel distance, passenger bus transfer times and total length of bus lines. Artificial immune algorithm is used to solve it. Finally, the case shows the effectiveness of the proposed model and algorithm.

**REFERENCES**