An Improved Probabilistic Data Association Algorithm in Wireless Sensor Networks

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Abstract. Data association algorithm is considered as an important part in modern target tracking systems. Although traditional probabilistic data association algorithms are used in various areas and perform well, there are numbers of disadvantages. In this article we introduce an advanced algorithm that perform differently depends on measurements of tracking gates, filtering out weak correlated or uncorrelated data to reduce the expanse while improve the accuracy. In the last part, the algorithm is compared with probabilistic data association algorithms. The comparison shows that the advanced algorithm reduces the number of relative position error as well as the computation time at a certain point of level, which proves the effectiveness of the algorithm.

Introduction

At present, with the development of wireless communication and sensor technology, wireless sensor networks (WSNs) have been widely used and developed for their strong computing and communication ability [1]. Target tracking is one of the many applications in WSNs [2, 3]. WSNs have many characteristics, such as self-organization, wireless communication and so on, so it is very suitable for target tracking. However, due to the limitation of communication bandwidth, storage capacity [4] and sensor's precision and the impact of environment noise and measurement noise, the research on target tracking is facing severe challenges in WSNs.

Data association [5] is not only the core content, but also a difficult point in the process of target tracking. Data association is the process of the association between the target data and the observed data when the sensor detects a number of observations of the target. At present, there are many kinds of data association algorithms, such as Bayesian theory, evidence theory, and so on, in which the Probability Data Association Algorithm [6] is widely used. However, in the actual work environment, due to the impact of measurement noise and environmental noise, the error of measurement data [7] will be relatively large, and the resource consumption is large.

To overcome the defects of poor robustness and large resource consumptions in big noise, an improved probabilistic data association algorithm in WSNs is proposed. Different processing methods are adopted according to the number of measured data in tracking gate. Compared with the probabilistic data association algorithm, the average computation time is significantly reduced, and it can achieve very good tracking performance for the variable acceleration moving target.

IPDA Algorithm

The probabilistic data association algorithm considers all points in the tracking gate, whose computing load is relatively large in a big noise environment. Especially when there are no points in the tracking gate, the predictive value is used as the measurement value. As a result, it produces many errors. Therefore, considering these problems of the algorithm, the IPDA algorithm will take into
account the M points whose statistical distance is smallest. If the number of points falling into the tracking gate is less than M, the algorithm considers all measured data. If no tracking points fall into the track gate, select the tracking point which has the shortest statistical distance as the trace point at current time. Thus, the effective measure considered by IPDA algorithm is simplified, reducing the amount of computation. The accuracy of the algorithm is not affected, because M points with minimum statistical distance are chosen rather than the number of observation data decreases. The accuracy and stability of the algorithm has been improved, because the algorithm considers the case that the predicted value is used to replace the measured value when no trace points fall into the tracking gate.

Due to ambient noise, the sensor receives many measurement data at the k moment. Let N be the number of measurement data. The vector difference between the measured value and the predicted value is defined as the residual vector [8]. Formula is expressed as follows:

\( \mathbf{v}_i(k) = \mathbf{Y}_i(k) - \mathbf{H}(k) \hat{\mathbf{X}}(k|k-1) \)  

In equation (1), \( \mathbf{Y}_i(k) \) is the ith observation value, \( \mathbf{H}_i(k) \) is observation matrix and \( \hat{\mathbf{X}}(k|k-1) \) is predicted value at the k moment. The statistical distant [8] of residual vector needs to be expressed as follows:

\( g_i(k) = \mathbf{v}_i^T(k) \mathbf{S}^{-1}(k) \mathbf{v}_i(k) \quad i = 1, 2, \ldots, N \)  

In equation (2), \( \mathbf{S}(k) \) is covariance matrix of residual vector. Tracking gate [8] is defined as follows:

\[ \mathbf{V}_k(\gamma) = \{ \mathbf{Y}(k) : g_i(k) \leq \gamma \} \]  

In equation (3), \( \gamma \) is the threshold. At time k, the number of measurement data falling into the tracking gate is recorded as \( m_k \). Set the constant M. If \( m_k > M \), chose M measurements that have the shortest statistical distance. If \( 0 < m_k \leq M \), chose all measurements. If \( m_k = 0 \), mark the measurement that has the shortest distance as the track point to the target. Considering all circumstances, formula is expressed as follows:

\[ \begin{align*}
\phi_i(k) &= \{ \mathbf{Y}_i(k) \text{ is considered to derive from the target, when } M < m_k \text{ at } k \text{ time.} \} \\
\phi_i(k) &= \{ \mathbf{Y}_i(k) \text{ is considered to derive from the target, when } 0 < m_k \leq M \text{ at } k \text{ time.} \} \\
\varphi_i(k) &= \{ \text{There is no measurement inside the tracking gate at the time } k \} \\
\end{align*} \]  

The conditional probability based on \( \mathbf{Y}_i^k \) is expressed as follows:

\[ \alpha_i(k) = p\{\phi_i(k)|\mathbf{Y}_i^k\} \quad i = 1, 2, \ldots, m_k \]  

\[ \beta_i(k) = p\{\varphi_i(k)|\mathbf{Y}_i^k\} \quad i = 1, 2, \ldots, m_k \]  

These events are mutually exclusive and completed. Therefore, the following formula holds:

\[ \sum_{i=0}^{m_k} \alpha_i(k) + \sum_{i=1}^{m_k} \beta_i(k) = 1 \]
Based on Total Probability Theorem, at time $k$, conditional mean of state is expressed as follows:

$$
\hat{X}(k|k) = E\left[ X(k) | Y^k \right]
= \sum_{i=0}^{m} E\left[ X(k) | \phi_i(k), Y^k \right] p\left( \phi_i(k) | Y^k \right) + \sum_{i=1}^{m} E\left[ X(k) | \phi_i(k), Y^k \right] p\left( \phi_i(k) | Y^k \right)
= \alpha_0(k) \hat{X}_0(k) + \sum_{i=1}^{m} \left[ \alpha_i(k) + \beta_i(k) \right] \hat{X}(k|k)
$$

(8)

In equation (8), $\hat{X}_i(k|k)$ is the state estimation in the conditions of events $\phi_i(k)$ and $\phi_i(k)$ as follows:

$$
\hat{X}_i(k|k) = \hat{X}(k|k-1) + K(k)v_i(k) \quad i=1,2,\ldots,m_i
$$

(9)

Depending on equation (2), calculate the statistical distance of all the measurements. Suppose that when $i=s$, $g_i = \min(g_i)$. In the condition of $\phi_0(k)$, select the measurement which has the shortest statistical distance as the track point associated with the target. At this time, state estimation is expressed as follows:

$$
\hat{X}_0(k|k) = \hat{X}(k|k-1) + K(k)v_i(k)
$$

(10)

The state equation of the IPDA algorithm as follows can be derived from equation (8), (9) and (10).

$$
\hat{X}(k|k) = \hat{X}(k|k-1) + K(k)\left[ v(k) + \alpha_0(k)v_i(k) \right]
$$

(11)

In equation (11):

$$
v(k) = \sum_{i=1}^{m_i} \left[ \alpha_i(k) + \beta_i(k) \right] v_i(k)
$$

(12)

Operation process of the algorithm is introduced through detailed description of the flow chart in Figure 1.

Step 1: Set the number of all measurements at time $k$ as $N$, the constant $M$, and the threshold $\gamma$.

Step 2: Suppose the measurement data from sensor are $Y_i(k)$ at time $k$. Based on equation (1), the residual vector $v_i(k)$ is calculated with the vector difference between $i$th measurement and trajectory predicted value. Then calculate the statistical distance of residual vector based on equation (2).

Step 3: Compare $g_i(k)$ with $\gamma$. If $g_i(k) \leq \gamma$, the $i$th measurement is inside the tracking gate; if $g_i(k) > \gamma$, the $i$th measurement is not. After filtering out the incorrect data, we have the number of measurement inside the gate as $m_k$.

Step 4: Select the corresponding algorithm based on $m_k$. If $m_k > M$, go to step 5. If $0 < m_k \leq M$, go to step 6. If $m_k = 0$, go to step 7.

Step 5: Select $M$ measurements that have the shortest statistical distance to the target value, go to step 8.

Step 6: Select all measurements, go to step 8.

Step 7: Mark the measurement that has the shortest distance as the track point to the target, go to step 8.

Step 9: If the target tracking is completed, the computation is over. Otherwise, go back to step 2.
Figure 1. The flow of track searching algorithm.

Simulation and Analysis

In order to test the effectiveness of the algorithm, we have done a simulation study on the tracking performance of the moving target.

Table 1. Simulation parameter.

<table>
<thead>
<tr>
<th>algorithm</th>
<th>$\sigma_x^2, \sigma_y^2, \sigma_z^2$ [m]</th>
<th>T [s]</th>
<th>$P_d$</th>
<th>$P_s$</th>
<th>$\gamma$ [m]</th>
<th>num</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDA</td>
<td></td>
<td>20</td>
<td>1</td>
<td>0.9</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>IPDA (M=3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The simulation parameters are shown in Table 1. From the table we can see that $\sigma_x^2$, $\sigma_y^2$ and $\sigma_z^2$ are random measurement errors of different sensors respectively; $num$ is the number of interference echoes which has a uniform distribution; $p_d$ is detection probability of target; $p_s$ is the probability of a correct measurement falling into the tracking gate; $\gamma$ is the threshold.

We assume that the target is moving in the two-dimensional plane of XOY. The target does the uniform motion that the initial velocity is 200m/s during the 1 to 15 cycles; during the 16 to 28 period, the target does the variable acceleration motion.

In this paper, we use the PDA algorithm and the IPDA algorithm of M=3 to carry out 50 simulation experiments on the moving target. The simulation results of different algorithms are shown in Fig. 2 and 3, respectively. Table 2 and Table 3 are the results of the average computation time and the relative position error for the data association.
From Fig. 2 and 3, we can see that the PDA algorithm and the IPDA algorithm have good tracking performance for the target tracking of the uniform motion and the uniformly variable motion. However, while the number of errors of the PDA algorithm is relatively large, the IPDA algorithm can be very good to track target with variable accelerated motion. Table 2 and Table 3 show that, compared to the PDA algorithm, IPDA algorithm greatly reduces the average computation time and the computational cost of data association. The simulation results show that the IPDA algorithm has high-accuracy and low computational overhead.

**Summary**

In this paper, by integrating the algorithm of nearest neighbor and probabilistic data association, an improved probabilistic data association in WSNs is introduced. The algorithm performs differently...
depending on the number of measurements inside the tracking gate, filtering out weak correlated or uncorrelated data. The algorithm can improve the accuracy of target tracking by decreasing the amount of system error and reduce the computation overhead through decreasing the number of effective measurements. The experiment results show that in comparison with the PDA algorithm, the method can improve the robustness as well as reduce the system error and computation consumption.

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**References**


