Energy and Spectral Efficient Designs for a Multihop CC-HARQ Relay Network

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Abstract — Over block Rayleigh fading channels, three optimum designs to achieve energy efficiency (EE) and spectral efficiency (SE) in a multihop relay network with Chase-combining based hybrid automatic repeat request (CC-HARQ) are investigated in this paper. By adopting a new log-domain linear threshold approximation method, we first formulate our optimization problem with respect to EE and SE, in which the cross-layer parameters concerning frame length and transmit power are considered; Relative optimal schemes are then proposed to maximize EE and SE individually, joint optimization of frame length and transmit power is identified superior to single-parameter optimization and conventional scheme. Additionally, due to the conflicts between EE and SE, a fundamental EE-SE tradeoff metric to optimize both EE and SE is introduced, which can be applied to adjust EE and SE flexibly. Numerical simulation results verify the correctness of the analytical optimum schemes.

I. Introduction

Energy efficiency (EE) and spectral efficiency (SE) are two of the most significant design metrics in wireless communication systems, energy efficient communication has received increasing attention due to the requirement of lower energy consumption and green communication, while SE systems focus on higher data rate over scarce spectrum resource. On the other hand, HARQ protocol has emerged as an effective approach to enhancing the reliability and robustness of wireless networks in fading channels. Lots of contributions are also made to the EE or SE analysis in HARQ systems. The optimization scheme of power allocation to minimize the total energy consumption per information bit was studied in [1], researchers in [2] proposed a joint power and rate adaptation method to maximize the SE of HARQ systems. However, most of previous works are based on outage probability method for its simplicity without considering the practical system parameters such as modulation coding scheme (MCS) and frame length.

To analyze the influence of transmit power in PHY layer and frame length in MAC layer, a log-domain linear threshold approximation method for cross-layer design in Rayleigh fading channels was proposed in [3,4]. Specifically, [3] introduced an optimized packet length scheme to maximize the SE of uncoded HARQ systems, also the EE performance with coded HARQ was optimized through transmit power and frame length control in [4]. Considering that the EE and SE are often the conflicting metrics in a single network [5], therefore, making an EE-SE tradeoff to simultaneously optimize both EE and SE appears rather imperative. Currently new progress in [6] and [7] developed a EE-SE tradeoff metric for TI-HARQ and CC-HARQ protocols.

Meanwhile, multihop relay technology enables the improvement of system capacity and the extension of communication coverage, thus a multihop relay network operating with HARQ will gain more benefits. Compared with other HARQ schemes, CC-HARQ is characterized with its balanced complexity and performance, which retransmits the same frame in case of unsuccessfully decoding and adopts the maximum ratio combining (MRC) scheme at the receiver. In this paper, differ from previous works, we investigate the cross-layer designs of EE and SE in multihop relay networks with CC-HARQ over Rayleigh fading channels via a log-domain approximation model.
The rest of this paper is organized as follows. In Section II, a multihop relay model with CC-HARQ scheme is introduced, then we formulate our system optimization goal of EE and SE. Analytical solution and derivation of optimum transmit power and frame length is proposed to maximize EE and SE in Section III. Section IV considers an EE-SE tradeoff metric to optimize both EE and SE simultaneously. In Section V, simulation results are showed to verify the outperformance of the schemes. Finally, we present the relative conclusions in Section VI.

II. Model Design and Optimization Problem Formulation

We consider a linear M-hop relay network which contains a source(S), M-1 relays(R), and a destination(D). All relays are serially connected from node S to D. It is assumed that the signal transmission from the other nodes is negligible to neighboring nodes, and each node can either transmit or receive at one time. Over Rayleigh fading channels, channel gains is constant in one single round but are i.i.d in different rounds, the \( m \) th hop in \( l \) th round channel coefficient \( \{ h_{m,l} \} \) can be modeled as \( CN(0,\sigma^2_m) \). Considering the case that there is no limitation on the maximum number of retransmissions, with reference to the proposed log-domain linear threshold model in [6] and the average approximated frame error rate (FER) in [7] and [8], we can calculate the average number of retransmissions at \( m \) th hop as

\[
\bar{N}_m = 1 + \frac{\omega_m}{\gamma_m} 
\]

where \( \omega_m \approx k \ln(L + L_o) + b \) is modeled as a threshold value that relates to the applied MCS and frame length, \( k \) and \( b \) are the values that depend on the system MCS, \( L \) is the information bit length per frame, \( L_o \) is the number of overhead bits per frame. Furthermore, the average received signal-to-noise ratio (SNR) at receiver in \( m \) th hop is expressed as \( \gamma_m = P_m \sigma_m^2 / N_0 \), where \( P_m \) denotes the transmit power of the \( m \) th hop.

We define \( E_{s,m} \) as the average transmit energy per information bit at \( m \) th hop, and \( E_s \) is the energy per bit consumed by hardware, which is assumed to be a constant, \( E_{s,m} \) and \( E_s \) compose the energy consumption per information bit, combining the previous discussion, then we can formulate the EE as

\[
\eta_{EE} = \frac{L}{\sum_{m=1}^{M} (E_s + E_{s,m})(L + L_o)\bar{N}_m} 
\]

In [6], we observe that \( E_{s,m} = G_{d,m}N_0\gamma_m \), where \( G_{d,m} = G_md_m^\kappa M_m \) being the global channel gains(subscript \( m \) means the \( m \) th hop unless specially mentioned), \( G_m \) being the gain factor in unit distance, \( d_m \) being the distance between neighboring nodes, \( \kappa \) being the path-loss exponent, \( M_m \) being the link margin to compensate the background noise and other interference.

Hence, we can expand the EE of CC-HARQ relay networks as

\[
\eta_{EE} = \frac{L}{\sum_{m=1}^{M} (E_s + G_{d,m}P_m\sigma_m^2)(L + L_o)(1 + \frac{(k \ln(L + L_o) + b)N_0}{P_m\sigma_m^2})} 
\]

where \( \eta_{EE} \) is a closed-form expression integrating PHY-layer \( P_m \) and MAC-layer \( L \), the unit of \( \eta_{EE} \) is bits/Joule. Thus the optimization problem of EE can be described as

\[
\max_{P_m} \eta_{EE} \quad s.t. \quad L > 0, P_m > 0 \quad 1 \leq m \leq M 
\]

where \( P = [P_1, P_2, \ldots, P_M, P_o] \) is the array of the optimum transmit power. Similarly, the formulation of SE in multihop CC-HARQ systems can be written as
\[ \eta_{se} = \frac{L \eta_M}{\sum_{m=1}^{M} (L + L_m) N_m} = \frac{L \eta_M}{\sum_{m=1}^{M} (L + L_m)(1 + (k \ln(L + L_m) + b) N_m)} \]  

(5)

where \( \eta_M \) denotes the maximum available SE of the adopted MCS, the unit of \( \eta_M \) is bps/Hz, thereby the optimization goal of SE can be expressed as

\[
\max_{P_m} \eta_{se} \quad s.t. \ L > 0, P_m > 0 \quad 1 \leq m \leq M
\]

(6)

### III. Optimization Analysis for EE and SE

This section proposes two design schemes to solve the optimum values of \( L \) and \( P_m \) to maximize the EE and SE according to (3) and (5).

#### A. The optimal scheme for EE

Before the actual optimization, aiming to maximize the EE, we consider three schemes to derive the optimum \( L \), \( P \) and the joint optimum values of \( L \) and \( P \).

**Scheme 1:** Assuming that \( P_m \) is fixed with equal transmit power in every hop, then to solve the optimum \( L \) to maximize EE of CC-HARQ relay networks.

With attempt to obtain the EE maximum in Scheme 1, the optimization problem (4) can be simplified as follows.

\[
\max_{L} \eta_{EE} \quad s.t. \ L > 0, P_m > 0 \quad 1 \leq m \leq M
\]

(7)

Based on the previously proposed expression of \( \eta_{EE} \), by solving the formula \( \frac{\partial \eta_{EE}}{\partial L} = 0 \), we can calculate the optimum \( L \) as

\[
L' = -L_0 W \left(-1 \exp\left(-\frac{\sum_{m=1}^{M} (E_c + E_{b,m})}{\sum_{m=1}^{M} \frac{(E_c + E_{b,m}) N_0}{\sigma^2_{m} P_m^2} + k + b}\right) - L_0 \right)
\]

(8)

where \( W(x) \) is the Lambert function defined in [9], the branch of \( W_{-1} \) is expected to be applied in this case. The proof of the optimum solution to \( L' \) is in Appendix A.

**Scheme 2:** Assuming that \( L \) is fixed and the average total transmit power \( P \) is constrained, then to solve the optimum \( P \) to maximize EE of CC-HARQ relay networks.

Differ from the frame length optimization case, the total transmit power constraints are considered in Scheme 2, then the optimization goal (4) can be written as

\[
\max_{P} \eta_{EE} \quad s.t. \ L > 0, P_m > 0, \sum_{m=1}^{M} P_m \leq P_0 \quad 1 \leq m \leq M
\]

(9)

To simplify the analysis complexity, the optimization problem (4) can be transformed to follows.

\[
\min_{P} f(P) = \min_{P} \sum_{m=1}^{M} (E_c + G_{d,m} P_m \sigma^2_{m})(1 + \frac{\alpha_0 N_0}{P_m \sigma^2_{m}})
\]

\[
\quad s.t. \ P_m > 0, \sum_{m=1}^{M} P_m \leq P_0 \quad 1 \leq m \leq M
\]

(10)

We observe that the maximization problem of \( \eta_{EE} \) in (9) is equivalent to minimize \( f(P) \) for the given constraints. If the given total transmit power \( P_0 \) is high enough, then let \( \frac{\partial f(P)}{\partial P_m} = 0 \) we can have the optimum \( P \) shown as

\[
P_m^* = \sqrt{\frac{N_0 E_c \alpha_0}{G_{d,m} \sigma^2_{m}}} \quad 1 \leq m \leq M
\]

(11)
The proof of (11) is similar to that of (8). However, there is a case that \( P_0 \) is insufficient to guarantee the \( P^*_m \) as (11), which means the case of \( \sum_{m=1}^{M} P^*_m > P_0 \), so we establish a Lagrange equation to solve this optimization problem.

\[
L(P, \lambda) = \sum_{m=1}^{M} (E_m + G_{d,m} P_m \sigma_m^2)(1 + \frac{\eta_m N_0}{P_m \sigma_m^2}) + \lambda(\sum_{m=1}^{M} P_m - P_0)
\]  

(12)

Similarly, when \( \frac{\partial L(P, \lambda)}{\partial P_m} = 0 \), we can obtain

\[
\lambda = \frac{N_0 \eta \sigma_m^2}{P_m^2 \delta_m^2} - G_{d,m} \delta_m^2
\]

(13)

\[
P^*_m = \frac{N_0 \eta \sigma_m^2}{\sqrt{G_{d,m} \delta_m^2 + \lambda \delta_m^2}}, \quad 1 \leq m \leq M
\]

(14)

Considering the convex property of (10) and the given total transmit power constraints in this case, the \( P^*_m \) can be derived in the constraint boundary \( \sum_{m=1}^{M} P^*_m = P_0 \), but substituting (14) into \( \sum_{m=1}^{M} P^*_m = P_0 \) is hard to solve the \( \lambda \), thus, we propose an iterative search algorithm that is briefly presented in Algo 1 to find the \( P^*_m \).

**Algo 1** The iterative search algorithm for the optimum transmit power scheme.

1) Set the value of frame length \( L \), total power constraint \( P_0 \) and maximum error tolerance \( \epsilon \)
2) Calculate \( P^*_m \) for \( 1 \leq m \leq M \) through (11)
3) If \( \sum_{i=1}^{M} P^*_i = P_0 \), jump to 7), otherwise move to 4)
4) Set the initial values of \( \lambda \) as
   \[
   \lambda_{\text{min}} = \max_{1 \leq i \leq M} \frac{N_0 \eta \sigma_i^2}{P_i^2 \delta_i^2} - G_{d,i} \delta_i^2 \quad \text{and} \quad \lambda_{\text{max}} = \max_{1 \leq i \leq M} \frac{N_0 \eta \sigma_i^2}{(P_0 / M)^2 \delta_i^2} - G_{d,i} \delta_i^2
   \]
5) Let \( \lambda = (\lambda_{\text{min}} + \lambda_{\text{max}}) / 2 \), and calculate \( P^*_m \) for \( 1 \leq m \leq M \) through (14)
6) If \( \sum_{i=1}^{M} P^*_i - P_0 \leq \epsilon \), move to 7), otherwise
   a. if \( \sum_{i=1}^{M} P^*_i - P_0 < 0 \), set \( \lambda_{\text{min}} = \lambda \) and move back to 5)
   b. if \( \sum_{i=1}^{M} P^*_i - P_0 > 0 \), set \( \lambda_{\text{max}} = \lambda \) and move back to 5)
7) Output the optimum power array \( P^* = [P^*_1, P^*_2, \ldots, P^*_M] \)

**Scheme 3**: Assuming that the available total transmit power is \( P_0 \), then to solve the joint optimum values of \( L \) and \( P \) to maximize EE of CC-HARQ relay networks.

In Scheme 3, aiming at optimizing the EE performance, (8) and Algo 1 can be jointly referred to figure out the joint optimum combination \( (\hat{L}, \hat{P}) \) via an alternate iteration algorithm, more specifically, the proposed joint optimum algorithm is introduced in Algo 2.

**Algo 2** The alternate iteration algorithm for the joint optimal scheme.

1) Set the initial total power constraint \( P_0 \), iteration rounds \( i = 0 \) and maximum error tolerance \( \epsilon \)
2) Initialize the transmit power array \( P^{(i)} = [P_1^{(i)}, P_2^{(i)}, \ldots, P_M^{(i)}] \) with \( P_m^{(i)} = P_0 / M \) for \( 1 \leq m \leq M \), compute \( L^{(i)} \) via (8) and \( \eta_{\text{EE}}^{(i)} \) through (3)
3) While(1)
   i. Substitute \( L^{(i)} \) into Algo 1 to obtain \( P^{(i+1)} \)
   ii. Substitute \( P^{(i+1)} \) into (8) to obtain \( L^{(i+1)} \)
   iii. Compute \( \eta_{\text{EE}}^{(i+1)} \) through (3) and let \( i = i + 1 \)
4) If \( |\eta_{\text{EE}}^{(i)} - \eta_{\text{EE}}^{(i-1)}| \leq \epsilon \), then stop, or move to 3)
5) Output the joint optimum combination \( (\hat{L}, \hat{P}) \).
B. The optimal scheme for SE

Based on the previous analysis for EE optimization, similar optimizing approaches can be adopted for the maximization of SE in multihop CC-HARQ system. Therefore, we also propose three schemes to improve the SE performance in three different cases.

Scheme 1: When \( m_P \) is fixed with equal transmit power for \( 1 \leq m \leq M \), based on (5), let \( \partial \eta_{SE} / \partial L = 0 \), we can obtain the optimum \( L \) as

\[
L' = -L_0 W \left( -\frac{1}{L_0} \exp\left( -\frac{M}{L_0} \sum_{m=1}^{M} \frac{N_m}{P_m \sigma_m^2} + k + b \right) \right) - L_0
\]

(15)

Scheme 2: When \( L \) is a fixed value, it is easy to observe that \( \eta_{SE} \) is a strictly increasing function with \( m_P \). Hence, we can derive the maximum \( \eta_{SE,max} = L \eta_{EU} / (M(L + L_0)) \) as \( P_n = \infty \) for \( 1 \leq m \leq M \). However, the total power of practical systems is always limited, with reference to the analyzing method of Scheme 2 in part A, we can easily calculate the optimum \( P_n \) for SE as

\[
P_n^* = \left( \frac{\alpha_0 N_0}{\sigma_n^2} / \sum_{m=1}^{M} \left( \frac{\alpha_0 N_m}{\sigma_m^2} \right) P_0 \right) \left( \frac{1}{\sigma_m^2} / \sum_{m=1}^{M} \frac{1}{\sigma_m^2} \right) P_0, \quad 1 \leq m \leq M
\]

(16)

which can be found that \( P_n^* \) has no association with \( L \), but only relates to the channel exponents.

Scheme 3: The joint optimal scheme is expected to be designed with respect to \( L \) and \( P \) in this part. we can observe that the available maximum SE is \( \eta_{SE,max} = \eta_{EU} / M \), which requires that \( L = \infty \) and \( P_n = \infty \) for \( 1 \leq m \leq M \), actually, towards the power-constrained network, similar to Algo2, an alternate iterative method between (15) and (16) can be used to compute the maximum SE under total transmit power constraints.

IV. A EE-SE Tradeoff Metric

Ideally, maximizing EE and SE simultaneously is desirable. However, the optimal EE performance often contradicts with the optimal SE performance, which can be observed via the analysis in Section III with respects to power optimization, when \( P_n = \infty \), we have \( \eta_{SE} = L \eta_{EU} / (M(L + L_0)) \) while \( \eta_{EE} = 0 \). Therefore, in this Section, we propose a EE-SE tradeoff metric to optimize both EE and SE.

Owing to the difficulty to unified evaluate the performance of EE and SE, we utilize a normalized method to establish the EE-SE tradeoff metric, which can be modeled as

\[
\eta_{\text{tradeoff}} = \eta_{EE}^{a} \eta_{SE}^{1-a}
\]

(17)

where \( \eta_{EE} \) and \( \eta_{SE} \) are the normalized metrics of EE and SE, which is separately defined as \( \eta_{EE} = \eta_{EU} / \eta_{EE,max} \) and \( \eta_{SE} = \eta_{EU} / \eta_{SE,max} \), \( \eta_{EE,max} \) and \( \eta_{SE,max} \) are the constant maximum EE and SE in specific cases, therefore, the EE-SE tradeoff metric is expanded to

\[
\eta_{\text{tradeoff}} = \frac{L}{(L + L_0) K_1 K_2} \frac{\eta_{EU}^{1-a} \eta_{SE,max}^{1-a}}{\eta_{EE,max}^{a} \eta_{SE,max}^{1-a}}
\]

(18)

where \( K_1 = \sum_{m=1}^{M} (E_1 + E_\infty) (1 + \alpha_0 N_0 / P_m \sigma_m^2) \) and \( K_2 = \sum_{m=1}^{M} (1 + \alpha_0 N_0 / P_m \sigma_m^2) \). Owing to the combining consideration of both EE and SE, we design the optimization objective as

\[
\max_{L,P} \eta_{\text{tradeoff}} \quad \text{s.t.} \quad L > 0, P_n > 0, \sum_{m=1}^{M} P_m \leq P_0, \quad 1 \leq m \leq M
\]

then similar to the analysis of EE and SE, let \( \partial \eta_{\text{tradeoff}} / \partial L = 0 \) and \( \partial \eta(P) / \partial P_n = 0 \) we can have
\[
L_0 - kI[\alpha] \sum_{n=1}^{M} \left( \frac{1}{\sum_{n=1}^{M} (E_c + E_{n,m})N_0} + \frac{1}{\sum_{n=1}^{M} P_n \sigma_n^2 + \rho_0} \right) + (1-\alpha) \frac{1}{\sum_{n=1}^{M} N_0 + \rho_0} = 0
\] (19)

\[
\alpha \frac{K_2}{K_1} \left( G_{\delta,m} \delta_m^2 - \frac{N_0 E_{\delta,m}}{P_m^2 \delta_m^2} \right) - (1-\alpha) \frac{N_0 \rho_m}{P_m^2 \delta_m^2} = 0
\] (20)

By solving (19) and (20), we can obtain individual optimum \(L\) and \(P\), here we mainly consider the joint optimum of \(L\) and \(P\) with the given total power constraint, due to the high complexity of presenting the close-form solution to (19) and (20), we propose the following algorithm to solve the optimum \((\hat{L}, \hat{P})\), which is proved to converge at most cases.

Algo 3 The alternate iteration algorithm for the joint optimal scheme

1) Set the initial total power constraint \(P_0\), iteration rounds \(i=0\) and maximum error tolerance \(\varepsilon\).
2) Initialize the transmit power array \(P^{(0)}=[P_1^{(0)}, P_2^{(0)}, \ldots, P_M^{(0)}]\) with \(P_m^{(0)} = P_0 / M\) for \(1 \leq m \leq M\).
3) Search \(L^{(i)}\) via (19) and compute \(\eta_{\text{tradeoff}}^{(i)}\) via (18).
4) Substitute \(L^{(i)}\) into (20) to search \(P^{(i+1)}\).
5) Substitute \(P^{(i+1)}\) into (19) to search \(L^{(i+1)}\).
6) Compute \(\eta_{\text{tradeoff}}^{(i+1)}\) through (18) and let \(i = i + 1\).
7) If \(|\eta_{\text{tradeoff}}^{(i)} - \eta_{\text{tradeoff}}^{(i-1)}| \leq \varepsilon\), then stop, or move to 3).
8) Output the joint optimum combination \((\hat{L}, \hat{P})\).

V. Numerical Results and Discussion

A multihop CC-HARQ relay network is simulated in this section, we assume the hop number as \(M=2\), other simulation parameters are set with \(L_0 = 20\) bits, \(G_1 = 30\) dB, \(G_2 = 25\) dB, \(d_1 = 100\) m, \(d_2 = 200\) m, \(\kappa = 3.5\), \(M_1 = 40\) dB, \(M_2 = 45\) dB, \(N_0 = -171\) dBm/Hz. The MCS used in this system is a rate 1/2 convolutional code with QPSK modulation, thereby we can set \(k = 0.207, b = 0.140\) [6].

![Figure 1](image1.png)  
Figure 1. EE versus \(P_0 / N_0\) of the proposed three optimization schemes and conventional scheme when \(\sigma_1^2 = 1, \sigma_2^2 = 2\).

![Figure 2](image2.png)  
Figure 2. SE versus \(P_0 / N_0\) of the proposed three optimization schemes and conventional scheme when \(\sigma_1^2 = 1, \sigma_2^2 = 2\).

Fig. 1 shows the EE performance of the three proposed schemes in Subsection A compared with the conventional fixed frame length and equal power scheme with different \(P_0 / N_0\). Three optimal designs all outperform the conventional scheme in terms of EE, Scheme 3 achieves superior EE performance to Scheme 1 and 2. As the power increases, EE of all schemes increase at low \(P_0\) level, but the EE of Scheme 2 and 3 eventually reaches the saturated maximum EE with 0.48bits/Joule EE gap, while Scheme 1 and Conv-scheme significantly degrade at high \(P_0\) without power adaptation.
We can also observe that the optimization for power allocation leads to higher EE gains than frame length optimization.

In Fig. 2, SE performance of Scheme 1, 2 and 3 in Subsection B compared with the Conv-scheme with different $P_t / N_o$ is shown, where $\eta_{\text{w}}$ equals 1bps/Hz under the applied MCS. The three proposed schemes outperform the Conv-scheme. As the power increases, SE of all schemes increases and reaches the saturated $\eta_{\text{w}}$. Meanwhile, Scheme 2 yields similar SE performance to Conv-scheme but is inferior to that of Scheme 1 and 3. Furthermore, we can observe that the optimization for SE is more sensitive to frame length instead of the transmit power.

![Figure 3. EE, SE and EE-SE tradeoff under optimal scheme with different tradeoff factor $\alpha$.]

Fig. 3 shows the EE, SE and EE-SE tradeoff relation with $\alpha$, we can note that a tradeoff relation exists between EE and SE, normalized EE presents an increasing trend with the increase of $\alpha$ while normalized SE decreases. The proposed optimal scheme leads to maximum EE with $\alpha = 1$ but to a maximum SE with $\alpha = 0$, furthermore, approximately 8% loss of EE can get over 22% rewards of SE when $\alpha$ varies from 1.0 to 0.6. Therefore, $\alpha$ can be flexibly adjusted to achieve the balanced preference between EE and SE based on the proposed EE-SE tradeoff metric.

VI. Conclusion

In this letter, we investigated the cross-layer optimization design of multihop relay network with CC-HARQ operating over block Rayleigh fading channels. Based on the log-domain approximation model, we obtained the optimum frame length and transmit power for EE and SE in various scenarios, and proceeded to the tradeoff of EE-SE metric. It can be shown that the proposed optimal schemes are feasible and effective to improve the system EE or SE performance. In addition, transmit power optimization is more crucial to maximize the EE than frame length, whereas SE is more sensitive to the frame length optimization. Meanwhile, the proposed EE-SE tradeoff design can be regarded as the guideline for the practical communication network.

References

Appendix A

Proof: According to (3), we can formulate that

$$\frac{\partial \eta_{EE}}{\partial L} = \frac{L_0 \sum_{m=1}^{M} K_m - kL \sum_{m=1}^{M} \frac{(E_x + E_{b,m})N_0}{P_n \sigma_m^2}}{(L + L_0)^2 \left( \sum_{m=1}^{M} K_m \right)^2}$$

where

$$K_m = (E_x + E_{b,m})(1 + \alpha_b N_0 / P_n \sigma_m^2)$$

Let $\frac{\partial \eta_{EE}}{\partial L} = 0$, we can have

$$\sum_{m=1}^{M} [L_0 (P_n \sigma_m^2 / N_0 + \alpha_b) - kL (E_x + E_{b,m}) / P_n \sigma_m^2] = 0$$

with the assistance of the Lambert function, we can obtain the solution as (8). We assume that

$$g(L) = \sum_{m=1}^{M} [L_0 (P_n \sigma_m^2 / N_0 + \alpha_b) - kL (E_x + E_{b,m}) / P_n \sigma_m^2]$$

it is easy to obtain that

$$g'(L) = \frac{-kL \sum_{m=1}^{M} (E_x + E_{b,m}) / P_n \sigma_m^2}{L + L_0} < 0$$

considering that $g(L) = 0$, so when $L < L', g(L) > 0$ and

$L > L', g(L) < 0$, therefore, $L'$ in (8) is the optimal point of $\eta_{EE}$ in (3).