Correcting Method of the Effect in Radar Range-rate Measurement by 6 DOF Movement of Sea-based TT&C Platform

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Abstract. In order to solve the issue that the range rate measurements of the sea-based tracking radar cannot be used effectively in the orbit determination of space targets, a new correction method is proposed based on the investigation of error sources and the traditional methods. Numerical results using mission tracking data show that the proposed method can suppress the random and system errors of the range rate measurements effectively. The precision and accuracy of the corrected measurements sent to the control center are greatly improved.

Introduction

In the development of major space powers, sea-based mobile platforms have been used more or less, and it is the same for China without exception till today. For the radar installed on the sea-based mobile platform, its antenna will shift and sway with the motion of its base platform. Since the antenna of the radar has a relative velocity with the instantaneous local horizontal frame, the radar range rate measurements contain the velocity component of its base platform. The range rate measurements provided to user are described in the instantaneous local horizontal frame (ILHF), so that the data includes the motion of the radar itself. The motion has two parts: the first part is 3-DOF attitude motion, namely pitch, yaw and heading. The second part is 3-DOF translational motion. In order to remove the disturbance by the platform, correction for range rate measurement is necessary.

Conventional Correction Method

To guarantee the accuracy of the tracking, Inertial navigation system (INS) and satellite navigation system are installed the center of the sea-based tracking platform, namely the origin of the Inertial Deck Frame (IDF), to obtain the translational and attitude motion of the platform. Furthermore, deformation measuring system is also equipped to obtain the flexure between the radars and the platform [1]. INS installed on the platform has two accelerometers in both north and east direction, but the velocity component along the zenith cannot be obtained. Furthermore, Limited by the computing capacity, the simplified model was used in the past correction method

\[ \rho'_s \approx \rho + v_s \cos h \cos A + \frac{1}{\rho} (x_o \hat{x} + y_o \hat{y}), \]

where \( v_s \) is the horizontal velocity of the platform, \( \vec{n}_o = [x_o \ y_o \ z_0]^T \) is the intersection between the azimuth axis and elevation axis of radar antenna in the IDF, and the vector is provided by the parameter calibration. \( \hat{\mathbf{v}} = [\hat{x} \ \hat{y} \ \hat{z}]^T \) is the velocity vector of the target under the measurement coordinate frame.

The equation (1) has two large sources of errors. The first is from the neglect of up-down movement of the platform. The second is from the horizontal movement of the platform with the velocity \( v_s \). In fact \( v_s \) is the velocity of the platform relative to the Earth. It can be approximated as the sum of the velocity of ocean current and the velocity of the platform relative to the current. The
direction of the vector is along the tangent of the ship trajectory. There is an angle between the vector and the longitudinal axis of the platform, and it is close correlated with the ocean current and the heading.

**Radar Correction Method for Six DOF Sea-based Platform**

**A. Radar Correction Method for Three DOF Attitude Motion**

The attitude motion of the platform can be measured by the INS. The attitude motion of the platform makes the radar antenna to move in the dynamic ILHF. After the correction, the relative velocity between spacecraft and the ILHF $\rho$ can be obtained.

1. The velocity vector of spacecraft in radar measurement frame (RMF)

   The position vector of the spacecraft in the RMF can be expressed as

   $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \rho \cos h \cos A \\ \rho \sin h \\ \rho \cos h \sin A \end{pmatrix}.$  \hfill (2)

   Its velocity vector can be expressed as

   $\mathbf{\dot{r}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \cos h \cos A & -\rho \sin h \cos A & -\rho \cos h \sin A \\ \sin h & \rho \cos h & 0 \\ \cos h \sin A & -\rho \sin h \sin A & \rho \cos h \cos A \end{pmatrix} \begin{pmatrix} \dot{\rho} \\ \dot{h} \\ \dot{A} \end{pmatrix}.$  \hfill (3)

   where $A$, $h$, $\rho$, $\dot{\rho}$ are azimuth, elevation, range and range rate after series of data processing e.g. sorting, system error correction, etc. $\dot{A}$, $\dot{h}$ are the derivative of relative variables.

2. Coordinate transition and the attitude correction of velocity vector

   The transition matrix between measurement frame and the ILHF can be written as \[2\]

   $\mathbf{\vec{r}}_{\text{ILHF}} = \mathbf{B}(\frac{b}{c}) \mathbf{B}(\frac{b}{c}) \mathbf{\vec{r}} + \mathbf{\vec{r}}_{\text{ILHF}}.$  \hfill (4)

   where $\mathbf{B}(\text{b}), \mathbf{B}(\text{c})$ are deformation and swaying euler angles transition matrix, respectively. They can be written as

   \[5\]

   $\mathbf{B}(c) = \begin{bmatrix} \cos K_c & 0 & -\sin K_c \\ 0 & 1 & 0 \\ \sin K_c & 0 & \cos K_c \end{bmatrix} \begin{bmatrix} \cos \psi_c & -\sin \psi_c & 0 \\ \sin \psi_c & \cos \psi_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \theta_c & -\cos \theta_c \\ 0 & \cos \theta_c & \sin \theta_c \end{bmatrix}.$

   \[6\]

   $\mathbf{B}(b) = \begin{bmatrix} \cos K_{b'} & 0 & -\sin K_{b'} \\ 0 & 1 & 0 \\ \sin K_{b'} & 0 & \cos K_{b'} \end{bmatrix} \begin{bmatrix} \cos \psi_{b'} & -\sin \psi_{b'} & 0 \\ \sin \psi_{b'} & \cos \psi_{b'} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \theta_{b'} & -\cos \theta_{b'} \\ 0 & \cos \theta_{b'} & \sin \theta_{b'} \end{bmatrix}.$

   where $K_c$, $\psi_c$, $\theta_c$ are attitude angles of heading, pitch and roll, respectively. The angle are measured by INS. $K_{b'}$, $\psi_{b'}$, $\theta_{b'}$ are equivalence deformation Euler angles of heading, pitch and roll, respectively. The angles are expressed by
where \( K_b, \psi_b, \theta_b \) are deformation Euler angles of heading, pitch and roll, respectively.

The derivative of Eq. Error! Reference source not found. can be written as

\[
\dot{\mathbf{x}}_{\text{gd}} = \mathbf{B}(c)[\mathbf{B}(b) \dot{\mathbf{r}} + \dot{\mathbf{r}}_0] + \mathbf{B}(c)[\mathbf{B}(b) \dot{\mathbf{r}} + \mathbf{B}(b) \dot{\mathbf{r}}]
\]

(12)

where \( \mathbf{B}(b) \) is the first-order derivative of deformation transition matrix, \( \dot{\mathbf{B}}(c) \) is the first-order derivative of the attitude transition matrix. They can be expressed as

\[
\mathbf{B}(b) = \begin{bmatrix}
\cos K_b \cdot \cos \psi_b & -\cos K_b \cdot \sin \psi_b \cdot \cos \theta_b - \sin K_b \cdot \sin \theta_b & \cos K_b \cdot \sin \psi_b \cdot \sin \theta_b - \sin K_b \cdot \cos \theta_b \\
\sin \psi_b & \cos \mu_b \cdot \cos \theta_b & -\cos \psi_b \cdot \sin \theta_b \\
\sin K_b \cdot \cos \psi_b & -\sin K_b \cdot \sin \psi_b \cdot \cos \theta_b + \cos K_b \cdot \sin \theta_b & \sin K_b \cdot \sin \psi_b \cdot \sin \theta_b + \cos K_b \cdot \cos \theta_b
\end{bmatrix}
\]

\[
\dot{\mathbf{B}}(c) = \begin{bmatrix}
 b11 & b12 & b13 \\
 b21 & b22 & b23 \\
 b31 & b32 & b33
\end{bmatrix}
\]

(13)

where

\[
b11 = -\sin K_{b'} \cdot \psi_{b'} \cdot \dot{K}_{b'} - \cos K_{b'} \cdot \sin \psi_{b'} \cdot \dot{\psi}_{b'}
\]

(14)

\[
b12 = (\sin K_{b'} \cdot \sin \psi_{b'} \cdot \cos \theta_{b'} - \cos K_{b'} \cdot \sin \theta_{b'}) \cdot \dot{K}_{b'} - \cos K_{b'} \cdot \cos \psi_{b'} \cdot \cos \theta_{b'} \cdot \dot{\psi}_{b'}
\]

+ (\cos K_{b'} \cdot \sin \psi_{b'} \cdot \sin \theta_{b'} - \sin K_{b'} \cdot \cos \theta_{b'}) \cdot \dot{\theta}_{b'}
\]

(15)

\[
b13 = (\sin K_{b'} \cdot \sin \psi_{b'} \cdot \sin \theta_{b'} - \cos K_{b'} \cdot \cos \theta_{b'}) \cdot \dot{K}_{b'} + \cos K_{b'} \cdot \cos \psi_{b'} \cdot \sin \theta_{b'} \cdot \dot{\psi}_{b'}
\]

+ (\cos K_{b'} \cdot \sin \psi_{b'} \cdot \cos \theta_{b'} + \sin K_{b'} \cdot \sin \theta_{b'}) \cdot \dot{\theta}_{b'}
\]

(16)

\[
b21 = \cos \psi_{b'} \cdot \dot{\psi}_{b'}
\]

(17)

\[
b22 = -\sin \psi_{b'} \cdot \cos \theta_{b'} \cdot \psi_{b'} - \cos \psi_{b'} \cdot \sin \theta_{b'} \cdot \dot{\theta}_{b'}
\]

(18)

\[
b23 = \sin \psi_{b'} \cdot \sin \theta_{b'} \cdot \psi_{b'} - \cos \psi_{b'} \cdot \cos \theta_{b'} \cdot \dot{\theta}_{b'}
\]

(19)
\[ b_{31} = \cos K_{b'} \cdot \cos \psi_{b'} \cdot \dot{K}_{b'} - \sin K_{b'} \cdot \sin \psi_{b'} \cdot \dot{\psi}_{b'} . \] (20)

\[ b_{32} = ( - \cos K_{b'} \cdot \sin \psi_{b'} \cdot \sin \theta_{b'} - \sin K_{b'} \cdot \sin \theta_{b'} ) \cdot \dot{K}_{b'} - \sin K_{b'} \cdot \cos \psi_{b'} \cdot \cos \theta_{b'} \cdot \dot{\psi}_{b'} \\
+ ( \sin K_{b'} \cdot \sin \psi_{b'} \cdot \sin \theta_{b'} + \cos K_{b'} \cdot \cos \theta_{b'} ) \cdot \dot{\theta}_{b'} . \] (21)

\[ b_{33} = ( \cos K_{b'} \cdot \sin \psi_{b'} \cdot \sin \theta_{b'} - \sin K_{b'} \cdot \cos \theta_{b'} ) \cdot \dot{K}_{b'} + \sin K_{b'} \cdot \cos \psi_{b'} \cdot \sin \theta_{b'} \cdot \dot{\psi}_{b'} \\
+ ( \sin K_{b'} \cdot \sin \psi_{b'} \cdot \cos \theta_{b'} - \cos K_{b'} \cdot \sin \theta_{b'} ) \cdot \dot{\theta}_{b'} . \] (22)

where \( \dot{K}_{b'} , \dot{\psi}_{b'} , \dot{\theta}_{b'} \) is the first-order derivative of deformation transition matrix, which is obtained from the derivative smoothing of the attitude angles.

\[
\mathbf{B}(c) = \begin{bmatrix}
\cos K_e \cdot \cos \psi_c & - \cos K_e \cdot \sin \psi_c \cdot \cos \theta_c - \sin K_e \cdot \sin \theta_c & \cos K_e \cdot \sin \psi_c \cdot \sin \theta_c - \sin K_e \cdot \cos \theta_c \\
\sin \psi_c & \cos \psi_c \cdot \cos \theta_c & - \cos \psi_c \cdot \sin \theta_c \\
\sin K_e \cdot \cos \psi_c & - \sin K_e \cdot \sin \psi_c \cdot \cos \theta_c + \cos K_e \cdot \sin \theta_c & \sin K_e \cdot \sin \psi_c \cdot \sin \theta_c + \cos K_e \cdot \cos \theta_c \\
\end{bmatrix}. \tag{23}
\]

\[ c_{11} = - \sin K_e \cdot \cos \psi_c \cdot \dot{K}_e - \cos K_e \cdot \sin \psi_c \cdot \dot{\psi}_c . \] (24)

\[ c_{12} = ( \sin K_e \cdot \sin \psi_c \cdot \cos \theta_c - \cos K_e \cdot \sin \theta_c ) \cdot \dot{K}_e - \cos K_e \cdot \cos \psi_c \cdot \cos \theta_c \cdot \dot{\psi}_c \\
+ ( \cos K_e \cdot \sin \psi_c \cdot \sin \theta_c - \sin K_e \cdot \cos \theta_c ) \cdot \dot{\theta}_c . \] (25)

\[ c_{13} = ( - \sin K_e \cdot \sin \psi_c \cdot \sin \theta_c - \cos K_e \cdot \cos \theta_c ) \cdot \dot{K}_e + \cos K_e \cdot \cos \psi_c \cdot \sin \theta_c \cdot \dot{\psi}_c \\
+ ( \cos K_e \cdot \sin \psi_c \cdot \cos \theta_c + \sin K_e \cdot \sin \theta_c ) \cdot \dot{\theta}_c . \] (26)

\[ c_{21} = \cos \psi_c \cdot \dot{\psi}_c . \] (27)

\[ c_{22} = - \sin \psi_c \cdot \cos \theta_c \cdot \dot{\psi}_c - \cos \psi_c \cdot \sin \theta_c \cdot \dot{\theta}_c . \] (28)

\[ c_{23} = \sin \psi_c \cdot \sin \theta_c \cdot \dot{\psi}_c - \cos \psi_c \cdot \cos \theta_c \cdot \dot{\theta}_c . \] (29)

\[ c_{31} = \cos K_e \cdot \cos \psi_c \cdot \dot{K}_e - \sin K_e \cdot \sin \psi_c \cdot \dot{\psi}_c . \] (30)

\[ c_{32} = ( - \cos K_e \cdot \sin \psi_c \cdot \cos \theta_c - \sin K_e \cdot \sin \theta_c ) \cdot \dot{K}_e - \sin K_e \cdot \cos \psi_c \cdot \cos \theta_c \cdot \dot{\psi}_c \\
+ ( \sin K_e \cdot \sin \psi_c \cdot \sin \theta_c + \cos K_e \cdot \cos \theta_c ) \cdot \dot{\theta}_c . \] (31)

\[ c_{33} = ( \cos K_e \cdot \sin \psi_c \cdot \sin \theta_c - \sin K_e \cdot \cos \theta_c ) \cdot \dot{K}_e + \sin K_e \cdot \cos \psi_c \cdot \sin \theta_c \cdot \dot{\psi}_c \\
+ ( \sin K_e \cdot \sin \psi_c \cdot \cos \theta_c - \cos K_e \cdot \sin \theta_c ) \cdot \dot{\theta}_c . \] (32)

where \( \dot{K}_e , \dot{\psi}_c , \dot{\theta}_c \) are the first-order derivative of ship attitude angles, which are obtained from the derivative smoothing of the attitude angles.

Therefore, transformation from the measurement frame to the ILHF are done and the error of velocity vector caused by the platform swaying is also fixed.
B. Radar Correction Method for Three DOF Shift Motion

There are usually two accelerometers installed in the INS along north and east directions, which can only provide the measure of horizontal movement without the measurement along zenith movement \[3\]. Furthermore, due to the coarse integral of the accelerometers, the accuracy of the measurements are relatively low. The satellite navigation system of the platform provides another velocity measurements for the platform and it has a higher precision and accuracy.

(1) The calculation of the velocity of the satellite navigation system antenna.

The satellite navigation system antenna is commonly installed near the center of the tracking platform, so that the deformation is very small in acrseconds. The error caused by the deformation can be neglected, the antenna position vector in the instantaneous INS local horizontal frame can be written as

\[
\vec{r}_{GPS} = \begin{pmatrix} x_{GPS} \\ y_{GPS} \\ z_{GPS} \end{pmatrix} = B(c) \cdot \vec{r}_{INS}.
\]  

(33)

where \( \vec{r}_{INS} \) is the antenna position vector in the instantaneous INS deck frame. \( B(c) \) is the rotation matrix of the tracking platform, refer to Eq. (5).

The first order derivative of \( \vec{r}_{GPS} \), \( \dot{\vec{r}}_{GPS} \) can be written as

\[
\dot{\vec{r}}_{gps} = \begin{pmatrix} \dot{x}_{GPS} \\ \dot{y}_{GPS} \\ \dot{z}_{GPS} \end{pmatrix} = \dot{B}(c) \cdot \vec{r}_{INS}.
\]  

(34)

where \( \dot{B}(c) \) is the first order derivative of the rotation matrix of the tracking platform.

(2) The calculation of the velocity of the tracking platform

The velocity of the tracking platform \( \dot{\vec{r}}_{GD} \) can be calculated by the difference between the velocity vector \( \dot{\vec{r}}_{GPS} \) expressed in the east-north frame and the velocity vector \( \dot{\vec{r}}_{ins} \) expressed in the instantaneous INS local horizontal frame

\[
\dot{\vec{r}}_{GD} = \begin{pmatrix} \dot{x}_{GD} \\ \dot{y}_{GD} \\ \dot{z}_{GD} \end{pmatrix} = \dot{\vec{r}}_{GPS} - \dot{\vec{r}}_{INS}.
\]  

(35)

Based on Eqs. (35)(30), the following correction model can be constructed

\[
\dot{\vec{r}}_{gd} = \begin{pmatrix} \dot{x}_{gd} \\ \dot{y}_{gd} \\ \dot{z}_{gd} \end{pmatrix} = \dot{\vec{r}}_{gd} + \dot{\vec{r}}_{GD}.
\]  

(36)
After the correction, the radar range rate can be written as [4]

$$
\dot{\rho}_g' = \frac{x_{gd}' \dot{x}_{gd} + y_{gd}' \dot{y}_{gd} + z_{gd}' \dot{z}_{gd}}{\rho_g}.
$$

(37)

where $x_{gd}$, $y_{gd}$, $z_{gd}$ are the position vector components in the ILHF. Range $\rho_g$ in the frame can be obtained by the following equation

$$
\rho_g = \sqrt{x_{gd}^2 + y_{gd}^2 + z_{gd}^2}.
$$

(30)

**Numerical Results**

In this section, the traditional method and the proposed method are used for the correction of radar range rate measurements of multiple circles in flying tests. The corrected results are compared with the positioning results of the air-born satellite navigation system. The statistical results comparison between the two methods is shown in Table 1. The comparison results for a circle of air flights are given in Fig. 1.

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Figure 1. Graph results comparison between the two methods.
The results show that the corrected range rate measurements using the proposed method achieve obvious improvement both in the fix of random errors and system errors than the traditional method. Though, from the perspective of mean values, the proposed method is slightly better than the old one. The difference is mainly from the ocean current, because the up-down movement of the platform has a disturbance to range rate measurements with zero average value. But from the perspective of standard values, the proposed method generates obvious improvement than the traditional method, so that total errors are reduced.

**Conclusion**

The full application of range rate measurements of sea-based radar data for the spacecraft orbit determination has been our technical challenge for a long time. Based on past results, the adaptive weights of range rate measurements are usually less than 0.01, while the adaptive weights of ranging data are usually greater than 0.8. In the orbit determination process, range rate measurements pay little contribution to the accuracy and precision of the final results. The proposed method in this paper can effectively reduce the impact of platform motion, and the processed ranging rate data are improved. In the orbit determination process, the effect of range rate is becoming obvious. Thus, the improvement of range rate measurements and its application for the orbit determination is an important measure to improve the accuracy of the orbit determination. Furthermore, more research should be focused on the weighted statistical orbit determination.

**References**


