Single Light Beam Coherent Optical Communication with Stokes Parameters

Chunhui Huang
College of Physics and Information Engineering, Fuzhou University, Fuzhou 350002, China
hchuang@fzu.edu.cn

Keywords: Continuous variable coherent optical, Stokes parameters, Electro-optical modulator, Balance homodyne detection.

Abstract. A new mode of coherent optical communication for QKD was proposed in this paper, there are two quadrature Stokes parameters, one is used as observed physical quantity, the other as the carrier wave. The EOM of 45° of azimuth is changed by continuously rotating the HWP, which indirectly complete the linear modulation of $S_2$ and $S_3$ parameters. The receiver adopts the Q-Q-H wave plate rotation mode to select the measured $S_2$ or $S_3$. The detection system is built up in combination with LabVIEW to implement the signal demodulation. The results shown that the scheme is feasible.

Introduction
Quantum cryptography communication is a new technology utilizing the quantum nature of light to realize communication security [1]. As quantum key distribution(QKD) scheme, the continuous variable coherent optical communication described by an infinite dimensional Hilbert Space, have more attractive to the researcher's interest than single photon communication scheme, especially the advantages of good compatibility and high transmission rate, etc. Now, most of, continuous variables coherent optical QKD communication adopt the Gauss-type coherent light signal as the carrier wave to encode the two quadrature components: the amplitude and the phase of a polar light beam, and transmits signal with the M-Z interferometer [2]. It is utilized the two polarized parameters of coherent light to follow the Heisenberg uncertainty principle and quantum no-cloning theorem, and guarantee the security of signal transmission. This scheme needs transmitting two split light beams. One is the signal light and the other is the local oscillation light [3]. Here, we apply a new transmission method, that is the single-mode spatial optical signal transmission, in which, the linearly polarized light is modulated by rotating HWP and EOM, and the Stokes quadrature parameter is regarded as an observable, so that the coherent optical communication is carried out by single light beam. We demonstrates the feasibility of the novel scheme with theoretical derivation and take a series experimental measurements. The experimental results show that the scheme is feasible.

Theoretical analysis
The Stokes parameters are one representation of the polarization of light, and the Poincare sphere is a map of Stokes parameters, on which face, every point presents a polarization states. As shown in Fig. 1, the three Stokes parameters for any polarized light can be presented as,

\[
    S_1 = S_o \cos 2\alpha \cos 2\beta \\
    S_2 = S_o \cos 2\alpha \sin 2\beta \\
    S_3 = S_o \sin 2\alpha
\]  

(1)

where $S_o = \sqrt{S_1^2 + S_2^2 + S_3^2}$. The quantum Stokes operators are described throughout in this work, their commutators follow the uncertainty relations:
The Eq. 2 shows that the Stokes operators follow the Heisenberg uncertainty principle and quantum no-cloning theorem. Two parameters $\hat{S}_2$ and $\hat{S}_3$ are relatively easy to measure and modulate are selected as the observable base, and the signal is loaded to the quantum carrier wave via the EOM and magneto-optical modulation (MOM). When the two quadrature components $\hat{S}_2$ and $\hat{S}_3$ are encoded, it can be known that the change of $\alpha$ and $\beta$ can modulate the $\hat{S}_2$ and $\hat{S}_3$ as Eq. 1. This theory was applied to modulate $\hat{S}_2$ and $\hat{S}_3$ component by the EOM and MOM.

When analyzing the Stokes parameter, the Jones matrix is used to present the function of optical device in experiments and the light status of each stage [4]. At first, the no polarized light emitted by the laser are presented by Jones matrix as $E = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$. When passing through the polarizer and two polarized beam splitters (PBS), the light is transform to a pure horizontal polarization. The horizontal polarized light is $E = \begin{pmatrix} E_x \\ 0 \end{pmatrix}$. The EOM perform a micro-modulation to horizontal polarized light. The electro-optical crystal can be equal to a phase delay adjustable wave plate with a fixed angle. Thus the Jones matrix of the electro-optical crystal is presented by the phase delay adjustable wave plate as following:

$$E_{\text{out}} = \begin{pmatrix} e^{-i\tau} \cos \phi + e^{i2\sin 2\phi} & -i\sin \tau / 2 \sin 2\phi \\ -i\sin \tau / 2 \sin 2\phi & e^{i\tau} \sin^2 \phi + e^{i2\sin 2\phi} \end{pmatrix}$$

where $\phi$ is the angle between the crystal axis and the horizontal direction; $\tau$ is the delay phase of the crystal, which is controlled by the driving voltage of EOM. After passing the EOM, the Stokes component of signal light is: $S_2 = \hat{E}_\text{EOM} P_s E_{\text{EOM}}$ and $S_3 = \hat{E}_\text{EOM} P_s E_{\text{EOM}}$, where $P_s$, $P_t$ is the sandwich matrix [5]. So $S_2$ and $S_3$ can be described as follows:

$$S_2 = -E_s^2 \sin^2 \tau / 2 \sin 4\phi$$
$$S_1 = E_s^2 \sin 2\phi \sin \tau$$

When $\phi \approx 45^\circ$, i.e. the $S_3$ near to zero, the $S_3$ component can modulated by the EOM independently. So, we set the crystal axis of the EOM near to $45^\circ$ angle with horizontal direction, and encode the $S_3$ component as shown in Fig. 2. In this case, $E_{\text{out}} = \frac{1}{2} E_0 \begin{pmatrix} \cos(\tau / 2) & -i\sin(\tau / 2) \end{pmatrix}^T$, when $\tau$ delay is make as a very small value, the intensity of the horizontal polarized light is much larger than that of the vertical polarized light. Therefor the stronger horizontal polarized light was used as the local oscillator light, and the weaker vertical polarized light was used as the signal light, both of them
were transmitted on the same spatial transmission mode. Compared with the general two split beams transmission mode, this process has fairly stronger anti-interference performance, while the single beam transmission can guarantee the better coherence and synchronization.

When the light signal passes the EOM, the $S_y$ component is encoded by continuously rotating the HWP. along the azimuth $\phi$, the $S_x$ component of polar light will be changed such as MOM. The Stokes component of the emergent light is $S_z = \frac{1}{2} E_x^2 \cos \phi \sin 4\phi$, $S_x = \frac{1}{2} E_y^2 \sin \phi$. so that, when the phase delay $\tau$ of EOM is a fixed value, the $S_z$ component keeps invariable, and the $S_y$ is modulated by changing $\phi$.

**Experimental system**

*Optical schematic*

Based on the design conception in above section, the integrated optical schematic is shown in Fig. 3. At Alice end the laser diode emits the beam to pass the $f = 8.5$ mm convex for focusing. Then beam passes through the light isolator to eliminate the impact of reflected light. The horizontal polarized light is achieved by passed through horizontal polarizer, and its polarized status is $(E_y)$. Then the beam passes two polarized beam splitter (PBS) to further filter the vertical polarized light. The beam reaching the EOM is the purer horizontal polarized light $(E_x)$, and the encoding of $S_y$ component is completed by the EOM, and rotating the HWP to control the $S_x$ component. At the Bob end, the measurement base is firstly selected via the Q-Q-H wave plates called as SU(2) box composed of one HWP and two quarter-wave plates (QWP). Finally the emergent light is split to horizontal polarized light $(E_x)$ and vertical polarized light $(E_y)$ via the PBS. After detected by the balance homodyne detection circuit, the detected signal is captured by PCI acquisition card.

In Fig. 3, the LD is a semiconductor laser diode SDL5412 drive by LDI800; EOM is an electro-optic amplitude modulator Model 4102M drive with Model 3363B; DA/DB is the self-made homodyne detection circuit [6-7], PCI-DAQ is the data acquisition card PCI6111E.

![Figure 3. Integrated optical schematic structure.](image)

**Stokes component detection system**

At the detection end the measurement base is selected by rotating the Q-Q-H wave plate. $a_x$, $a_y$ is set as the coherent polarization status of input light, and $c_x$, $c_y$ as the coherent polarization status of output light. The Stokes component represents as the following:

\[
S_z = \langle a_x^\dagger a_x \rangle + \langle a_y^\dagger a_y \rangle \\
S_x = i(\langle a_x^\dagger a_y \rangle - \langle a_y^\dagger a_x \rangle)
\]  

(5)
The homodyne detection light density is \( I = \langle c'_x \mid c_x \rangle - \langle c'_y \mid c_y \rangle \). The output \( E_{su} \) is \( E_{su} = E_0 E_\phi E_H \), where \( E_0 \) and \( E_H \) present the Jones matrixes of QWP and HWP, respectively. If the crystal axises of three wave plates set of particular angles different angles with the horizontal direction, respectively, and the \( E_{su} \) will satisfy the following equations:

\[
I = S_2, \ E_{su} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad I = S_3, \ E_{su} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}
\]  

(6)

The angles between the crystal axis and horizontal direction of Q-Q-H plates set as \( \alpha, \beta, \phi \) respectively. According to Eq. 6, let \( \alpha = 0 \), if \( \beta = 0, \phi = -\frac{3}{8}\pi \), measure \( S_2 \) component, otherwise \( \beta = -\frac{1}{4}\pi, \phi = -\frac{1}{8}\pi \), measure \( S_3 \) component. By selecting the measurement base, the balance homodyne detection is implemented [8].

According to Eq. 4, \( S_3 \) parameter is a sine function with \( \tau \), and \( \tau = \frac{\pi}{V_x} V \), here \( V \) is the modulation voltage of EOM. The result is that a sine relationship between \( S_3 \) and \( V \). The modulation voltage \( V \) of the electro-optical crystal is a 1.6 MHz sine wave with a \( V_0 \) DC bias voltage. It can be written as \( V = V_0 + V_x \sin \omega t \), where \( \omega \) is the driving frequency. \( V_x \) is the input drive voltage of EOM. We obtain \( S_3 = \frac{E_x}{4} \sin \frac{\pi}{V_x} (V_0 + V_x \sin \omega t) \). Because the change of \( V_x \) is fairly small, \( S_3 \approx \frac{k}{4} E_x \frac{\pi}{V_x} (V_0 + V_x \sin \omega t) \), and let \( M = \frac{k}{4} E_x \frac{\pi}{V_x} \), the \( S_3 \) is simplified as follows relationship:

\[
S_3 \approx M(V_0 + V_x \sin \omega t) = MV_0 + MV_x \sin \omega t
\]  

(7)

when the detection signal passes through the bandpass filter at the end of the detection circuit, the DC signal is filtered, the measurement value \( S_3 = MV_x \sin \omega t \). In fact, the captured signal is the reserved \( MV_x \sin \omega t \) parameter. The signal is acquired via PCI data acquisition card and cascade to PC, then processed via LabVIEW. Through the above analysis, the \( V_x \) and \( S_3 \) is a nonlinear relationship. For the convenience of encode and decode of the acquired data, it should be at first scanned by output modulation voltage during signal encoding, then the firstly acquired data is split with equal interval. Each interval point is distributed by test value of Gaussian number to guarantee the relative linearity and make the demodulation process convenient.

**Result and discussion**

When the random selection measurement base is \( S_2 \), the horizontal azimuth \( \phi \) of HWP is changed and the \( S_2 \) component value is measured. In Theory, when the delay \( \tau \) of electro-optical crystal is a fixed value, let \( A = \frac{1}{4} E_x^2 \cos \tau \), \( S_2 = A \sin 4\phi \). The measured data is shown in Fig. 4. that \( S_2 \) component is a sine relationship with \( \phi \), which is agreed with the theory. When the modulation \( \phi \) varies in a small range, \( S_2 \) and \( \phi \) can also follow the linear modulation relationship.
When $S_i$ is selected as the measurement base, the EOM performs Gaussian number encoding. The acquired voltage of $S_i$ component after scanned by the initial drive voltage, which is split to 16 equal interval points as the value taking of Gaussian number. Since the EOM creates a 1.6 MHz baseband signal, and thus the max encoding rate in this system is 1.6 MHz. The signal acquired by PCI acquisition card is sent to PC, and the sampled data is demodulated and analyzed via LabVIEW programming. The sampled signal is performed by linear demodulation. After random encoding of EO drive voltage with $10^5$ sample points. The signal is demodulated to Gaussian distribution, which is in conformity with the test assumption and requirement. Fig. 6 show the receiving and sending of $S_2$.

The error code analysis of demodulation signal adopts the array indexing. The sending data is compared with the demodulation data one by one and once the difference is found, the error code number increases by one. $10^5$ samples are analyzed, and the error code number approaches to zero.

**Summary**

We mainly adopts the HWP, EMO and PBS optical devices to build up setup the single mode spatial coherent light communication system and constitute the Stokes parameter transmission system. The results show that the modulation and encoding for $S_2$ or $S_3$ component is feasible. At the detection end the SU(2) box randomly selected the measurement base. Subjecting to the Heisenberg uncertainty principle in the quantum status when $S_x$ and $S_y$ component are measured at the same time, the phenomenon of quantum broadening shows that a potential eavesdropper cannot at the same time accurately measure the $S_x$ and $S_y$ value, which guarantee the security of the communication.
Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 61177072).

Reference


