Three-loop Control Strategy of High Power Harmonic Voltage Generator Based on FFT

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Abstract. Voltage feedback control of the inverter, such as dual-loop control with voltage outer loop and current inner loop, the deadbeat control, RMS control and so on, can’t eliminate the steady-state error of the harmonic. In order to solve the above problems, this paper proposes a three-loop control strategy based on FFT. On the basis of the dual-loop control system with good dynamic performance, the FFT outer loop is added, which controls the real part and the imaginary part of each harmonic respectively in spectrum to eliminate the static error. The simulation results and experimental results show that the control algorithm not only has high dynamic performance, but also can eliminate the harmonic tracking error.

Introduction

The development of voltage harmonic generation system is an indispensable condition for studying the theory of power quality theory. On the one hand, both the power quality control device and the harmonic compensation device require a harmonic voltage generation system to verify its effectiveness. On the other hand, the correctness of the power quality analysis theory and the effect of the power quality analyzer are required to test [1]. According to the PWM theory, the three-phase voltage source inverter [2] can generate arbitrary voltage waveforms within a certain spectrum range, so it is suitable as the main equipment of the harmonic voltage generation system. In the research of PWM inverters, a variety of control algorithms have been generated. The dual-loop control algorithm [3] is adding the current inner loop control based on the voltage loop control of the inverter to suppress the influence of the load disturbance quickly and timely, and therefore dynamic performance and steady-state accuracy of the system are improved to some extent. However, to track higher harmonics must increase the scale factor of the controller to obtain a large enough bandwidth, which affects the stability of the system. Deadbeat control [4] is a unique control scheme for digital control, which can correct the output voltage error caused by load disturbance within one sampling period. It has very fast dynamic response, and obtains small waveform distortion rate. However, the effect of the control depends on the accuracy of the model estimation, and the system is not robust enough. Proportional resonance control [5] can achieve infinite gain at a fixed frequency by setting its resonant frequency, thus achieving tracking of the voltage at the resonant frequency, but for this control system, it is necessary to track 2nd to 50th harmonic at the same time, which has a very high computational speed for the arithmetic processing chip and is difficult to implement. In order to meet the dynamic performance and steady-state accuracy requirements of the harmonic voltage generator, this paper adds the fast Fourier transform (FFT) voltage outer loop control to eliminate the steady state error of output harmonic voltage based on the double loop control with voltage outer loop and current inner loop.
The control system based FFT three loop proposed in this paper is shown in Fig 1. As shown in Fig 1, \( U_{(ref)re} \) and \( U_{(ref)im} \) are vectors of real part and imaginary part of the setting fundamental and 2nd to 50th harmonic voltage, respectively. \( U_{re} \) and \( U_{oim} \) are vectors of real part and imaginary part of the fundamental and 2nd to 50th harmonic output voltage, respectively. T is the period of the sampling switch, which is equal to 0.02s of a power frequency period. In the \( \alpha \beta \) two-phase stationary coordinate system, the three-phase inverter is completely decoupled and can be equivalent to two single-phase half-bridge inverters. Therefore, the three-loop control of the three-phase inverter can be equivalent to controlling the single-phase inverter in the \( \alpha \beta \) two-coordinate system. In addition, the system is a linear system, which satisfies the principle of superposition. The total response can be obtained by superposition of the responses of each harmonic acting alone. The system does not have a coupling relationship between the control of different harmonics, so that different harmonics can be controlled independently. Through the fast Fourier transform(FFT), different harmonics are converted into two DC quantities, the real part and the imaginary part, and the steady state error can be eliminated by a certain control algorithm. The reason why the real part and the imaginary part of harmonics are separately controlled without using separate control of the amplitude and phase angle of harmonics is that the former can avoid large amount of operation. The analysis and design of the following three-loop control are performed on the \( k^{th} \) harmonic control of the single-phase inverter.

**Figure 1. Schematic diagram of three-loop control system based on FFT.**

The dual-loop control system has high dynamic performance, and its dynamic response speed is much higher than the adjustment speed of the FFT outer loop by 0.02s/time. Therefore, when output voltage waveform \( U(s) \) of the FFT outer loop regulator maintains a power frequency cycle, the FFT result of the double-loop system output voltage \( U_o(s) \) during this period will be approximated by the steady-state response spectrum of the double-loop system for the excitation \( U_r(s) \). Therefore, when designing the FFT outer loop, the frequency characteristics of the dual loop system must be analyzed firstly.

**Figure 2. Block diagram of digital dual-loop system.**

The block diagram of the digital dual-loop system ignoring the load disturbance is shown in Figure 2. The closed-loop pulse transfer function is:

\[
\left\{ \begin{array}{l}
\phi(z) = \frac{U_o(z)}{U_i(z)} = \frac{KG_i(z)PI(z)}{KG_i(z)z + z + KG_i(z)PI(z) - G_i(z)} \\
G_i(z) = (1 - z^{-1})Z\left(\frac{1}{s(LCs^2 + RCS + 1)}\right), \quad G_2(z) = (1 - z^{-1})Z\left(\frac{C}{LCs^2 + RCS + 1}\right)
\end{array} \right.
\]

The steady-state transfer function of the digital dual-loop system can be obtained as follows:

\[
\phi(z) = \frac{U_o(z)}{U_i(z)} = \frac{KG_i(z)PI(z)}{KG_i(z)z + z + KG_i(z)PI(z) - G_i(z)}
\]

The analysis of FFT Outer Loop

\[\phi(z) = \frac{U_o(z)}{U_i(z)} = \frac{KG_i(z)PI(z)}{KG_i(z)z + z + KG_i(z)PI(z) - G_i(z)}
\]
\[ \phi(z)|_{t=0} = \text{RE}[\phi(e^{j100\pi T})] + j\text{IM}[\phi(e^{j100\pi T})] = \phi_r(k) + j\phi_i(k) \]  

(2)

Where, \( k \) is the number of harmonics, \( \phi_r(k) \) and \( \phi_i(k) \) are the real and imaginary components of the steady-state transfer of the digital dual-loop system, respectively. The expressions of \( \phi_r(k) \) and \( \phi_i(k) \) are complicated, which can be directly calculated by matlab using equations (1)~(2).

Substituting \( z \) in Eq. (1) with \( e^{j\omega T} \) and combining equation (4), the outputs of dual-loop control system can be listed as follow:

\[
\begin{align*}
U_{\text{re}}(k) &= U_{\text{re}}(k) \cdot \phi_r(k) - U_{\text{im}}(k) \cdot \phi_i(k) \\
U_{\text{im}}(k) &= U_{\text{im}}(k) \cdot \phi_r(k) + U_{\text{re}}(k) \cdot \phi_i(k)
\end{align*}
\]  

(3)

Where, \( U_{\text{re}}(k) \) and \( U_{\text{im}}(k) \) are the real and imaginary parts of the harmonic of the output voltage of the dual-loop system, respectively. \( U_{\text{re}}(k) \) and \( U_{\text{im}}(k) \) are the real and imaginary parts of the harmonic of the input voltage of the dual-loop system, respectively.

It can be seen from Eq. (3) that there is a coupling relationship between the real part and the imaginary part of the frequency characteristic of the double loop system, so it is necessary to introduce a decoupling link into the control.

The relationship between the output and the input of the dual-loop system in the frequency domain can be expressed by matrix as:

\[
\Phi(k) = \begin{bmatrix} \phi_r(k) & -\phi_i(k) \\ \phi_i(k) & \phi_r(k) \end{bmatrix}
\]  

(4)

The inverse matrix of \( \Phi(k) \) in series before the dual-loop system can achieve complete decoupling between the real and imaginary parts, and the input-to-output gain is 1. The decoupling matrix is as follows:

\[
\Gamma(k) = \Phi^{-1}(k) = \frac{1}{M^2(k)} \begin{bmatrix} \phi_r(k) & \phi_i(k) \\ -\phi_i(k) & \phi_r(k) \end{bmatrix}, \quad M(k) = \sqrt{\phi_r^2(k) + \phi_i^2(k)}
\]  

(5)

After introducing the decoupling link, the real and imaginary parts of the closed-loop control object of the FFT outer loop are symmetrical and uncoupled, so that the real and imaginary parts of the system can be independently controlled. In addition, the real part and the imaginary part of the closed loop system can be uniformly designed.

**Design of FFT Outer Loop Controller Based on Minimum Beat Discrete System**

Since the delay of calculation is generated by FFT, IFFT, digital control and so on, in order to facilitate the design of the controller, the calculating result of the digital controller is output after the next beat, that is, one power frequency period \( T \). The input waveform \( U_i(s) \) of the dual-loop system calculated in the \( n \)th beat is applied between the \( n+1 \)th beat and the \( n+2 \)th beat, and the feedback in the \( n \)th beat used for closed loop control is the FFT result of the output voltage between \( n-1 \)th beat and \( n \)th beat. According to the above analysis, the equivalent system block diagram of the FFT outer loop control system is shown in Fig.3. The closed-loop pulse transfer function and the error pulse transfer function of the system are:

\[
\begin{align*}
H(z) &= \frac{U_{\text{out}}^k(z)}{U_{\text{ref}}(z)} = \frac{D(z)z^{-1}}{1 + D(z)z^{-2}}, \\
H_e(z) &= \frac{E(z)}{U_{\text{ref}}^k(z)} = \frac{1}{1 + D(z)z^{-2}}
\end{align*}
\]  

(6)
According to Eq.(6), the relation between $H_0(z)$ and $H(z)$ and the expression of digital controller $D(z)$ can be expressed as:

$$H_0(z) = 1 - H(z) \cdot z^{-1}, \quad D(z) = \frac{(1+D(z)z^{-1})H(z)}{z^{-1}} = \frac{H(z)}{H_z(z)z^{-1}}$$  \hspace{1cm} (7)$$

The denominator of $D(z)$ contains the $z^{-1}$ factor. In order for $D(z)$ to be realized in practice, $H(z)$ should also contain factor $z^{-1}$ to cancel it. The input signal $U_{ref}^k(z)$ is the real or imaginary part of the setting $k$th reference harmonic, which is a step signal, and its amplitude is $A$. So $U_{ref}^k(z)$ and the $z$ transformation of the error signal are:

$$U_{ref}^k(z) = \frac{A}{1-z^{-1}}, \quad E(z) = H_0(z)U_{ref}^k(z) = \frac{AH_0(z)}{1-z^{-1}}$$ \hspace{1cm} (8)$$

According to the final value theorem of the $z$ transform, the steady state error of the discrete system is:

$$e(\infty) = \lim_{z^{-1} \to 1} E(z) = \lim_{z^{-1} \to 1} AH_0(z)$$ \hspace{1cm} (9)$$

It can be seen from Eq. (9) that in order to make the steady-state error of the system to be zero, the factor $(1-z^{-1})$ should be included in $H_0(z)$. Taking $H(z) = z^{-1}$, $H_0(z)$ can be expressed as:

$$H_0(z) = (1-z^{-1})(1+z^{-1})$$ \hspace{1cm} (10)$$

Equation (10) contains the factor $(1-z^{-1})$, which can make steady-state error of the system to be zero. The expression of the digital controller $D(z)$ can be obtained from equation (7):

$$D(z) = \frac{1}{1-z^{-1}}$$ \hspace{1cm} (11)$$

From the closed-loop pulse transfer function $H(z) = z^{-1}$, the system has only one pole at the origin of the $z$ coordinate, so the discrete system is stable. The transition process is ended within one beat when the load disturbance is ignored, and the minimum beat tracking of the input signal is realized.

**Experimental Verification**

To verify the effectiveness of the three-loop control strategy based on FFT, a 6kW hardware experiment platform including three-phase voltage source inverter was built and tested. The system parameters are as follows: The filter inductor $L$ is 0.1mH. The filter capacitor $C$ is 150uF. The switching frequency $f_s$ is 12.8kHz. The DC bus voltage is 650V. The load is a resistive load with the resistance is $10\Omega$ and the inductance is 1mH. In this paper, the DSP chip TMS320F28335 provided by TI is used to realize the three-loop control algorithm and PWM signal generation of the inverter. The power quality monitoring device FLUKE435 is used to measure the output waveform and record the experimental data.
The setting reference voltage parameters are as follows. The RMS fundamental voltage is 110V. The amplitude and the phase of 4th harmonic are 10V and 0°, respectively, which also means the real part and the imaginary part of 4th harmonic are 10V and 0V, respectively. The amplitude and the phase of 10th harmonic are 10V and 0°, respectively, which also means the real part and the imaginary part of 10th harmonic are 10V and 0V, respectively. The screenshot of the FLUKE435 of the experiment is shown in Fig. 4. As can be seen from Fig. 4(b), the RMS output voltage of phase A of 4th harmonic is 7.3V, which means the amplitude of 4th harmonic voltage is 10.3V, and the amplitude error rate is 3%. The phase of 4th harmonic voltage is advanced by 1°, which also means the real part error and the imaginary part error of 4th harmonic voltage is 0.3V and 0.2V, respectively. It can be seen from Fig. 4(c) that the RMS output voltage of phase A of 10th harmonic is 7.2V, which means the amplitude of 4th harmonic voltage is 10.2V, and the amplitude error rate is 2%. The phase of 10th harmonic voltage is advanced by 1°, which also means the real part error and the imaginary part error of 10th harmonic voltage is 0.2V and 0.2V, respectively. The above error is caused by the accuracy of the sampling, but the result meets the accuracy requirements on the whole.

In order to study the dynamic performance of the proposed algorithm, the load is cut off at 0.28s, and the changes of the real and imaginary parts of the 4th and 10th harmonic output voltage of phase A are observed, and shown in Fig. 4(d). It can be seen that the adjustment time of the system transition from the zero initial state to the set value is 0.12 s, that is, 6 power frequency cycles. The adjustment time of returning to the set value after 0.28 s of cutting off the load is 0.08 s, that is, 4 power frequency cycles. It can be seen from the above experimental results that the minimum beat controller of the FFT outer loop has high dynamic performance.

**Summary**

In this paper, the FFT outer loop control is added based on dual-loop control with voltage outer loop and current inner loop. The real and imaginary parts of each harmonic are controlled in the frequency domain. The steady state error is eliminated, and the decoupling strategy between the real and imaginary part, and the design scheme of the FFT outer loop controller are given. The experimental
result shows that the harmonic tracking with this control strategy has higher accuracy and higher dynamic performance.

References


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