The Calculation and Simulation of a Current Loop’s Magnetic Field

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Abstract. Basic electromagnetic theories and elliptic integral method are adopted to derive analytic expressions of the magnetic field due to elliptical loops carrying stable currents, and the corresponding spatial distributions are calculated numerically and simulated in detail. The discussions on the distribution rules are given according to the above results.

Introduction

The calculation of the magnetic field due to a symmetrical current loop is a classical mathematical physics problem[1,2], and also one of the important parts of electromagnetic theories. It has great practical significance, e.g., stimulated field of the current loops can be directly applied to seek water by NMR on the ground. The questions mentioned above have been studied in many works with various methods[3,4], such as superposition principle, Legendre polynomial method. However, only the magnetic fields along the central vertical axis of circular or elliptical loops have been discussed dominantly in the papers above, while the general spatial distributions are rarely studied up to now.

In this paper, we adopt the elliptic integral methods as well as basic electromagnetic theories to derive the theoretical expressions of spatial distributions for elliptical loops in Section 2, then the detailed three dimensions (3D) image simulations base on numerical calculations and the relevant discussions on the distribution rules are illustrated in Section 3. Finally, Section 4 is a brief summary.

Theoretical Analysis of the Magnetic Field Due to an Elliptical Current Loop

According to the basic theories of magnetic field and the Biot-Savart Law, the magnetic field formed by stable current is given by

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{r^3}$$

(1)

![Figure 1. Elliptical current loop geometry in coordinate system.](image)

For an elliptical loop shown as figure 1, the geometrical parametric equation is $x' = a \cos \phi$, $y' = b \sin \phi$, where $a/b$ represents semi-major/semi-minor axis, so the integral element $Id\vec{l}$ in Eq. 1
located at \( Q(x', y', 0) \) on the current loop is \( \mathbf{I}d\mathbf{l}' = \mathbf{I}d\left(l_x \cdot \mathbf{i} + l_y \cdot \mathbf{j}\right) = \mathbf{I}d\left(-a \sin \phi \mathbf{i} + b \cos \phi \mathbf{j}\right) \). The relative position vector from the source point Q to arbitrary field point \( P \) is:

\[
\mathbf{r} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}d\mathbf{l} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \left[ b_z \cos \phi \mathbf{i} + a_z \sin \phi \mathbf{j} + (ab - bx \cos \phi - ay \sin \phi) \mathbf{k} \right] \frac{1}{\left[(x - a \cos \phi)^2 + (y - b \sin \phi)^2 + z^2\right]^{3/2}} d\phi
\]

(2)

In the following, we shall give the simplified expressions of the three components of magnetic field for several special cases for the field point \( P \) at \((0, 0, 0)\), on one of three axes, and in one of the coordinate planes. The simplest case is the field point \( P \) at \((0, 0, 0)\), with Eq. 2, one can get \( B_x = B_y = 0 \). While the \( z \) component is

\[
B_z = \frac{\mu_0 I b}{a^2 \pi} \frac{E(k)}{k^2} = \frac{\mu_0 I b}{\pi b} \frac{E(k)}{k^2} = \frac{\mu_0 I b}{\pi a} \frac{E(k)}{k^2}
\]

(3)

as is known that \( E(k) = \frac{1}{\sqrt{1 - t^2}} \frac{k^2 dt}{1 - k^2 t^2} \), \( t = \sin \phi \) is the standard format belonging to elliptic integral of the second kind[5]. The case of \( P \) on \( z \)-axis is relatively complex, and we find \( B_z \) is the only non-vanishing component of the magnetic field:

\[
B_z = \frac{\mu_0 I b}{\pi (a^2 + z^2)^{3/2}} \frac{E(k)}{k^2} = \frac{\mu_0 I b}{\pi a} \frac{E(k)}{k^2}
\]

(4)

For \( P \) in one of the coordinate planes, the three components are much more sophisticated which are hard to be simplified visibly.

**Numerical Analysis and Discussion**

If there is an elliptical current loop carrying 1 Amp lying in \( z=0 \) plane when the origin point is just located at its center. The semi-major axis is 2.0cm and semi-minor axis is 1.0cm. The distributions of magnetic field on one of three axes are shown as follows (here Matlab is used[6]):

![Figure 2](image1.png)

Figure 2. \( B=B_z \) for \( P \) on one of three axes: (a) for \( x \)-axis, (b) for \( y \)-axis and (c) for \( z \)-axis. The vertical axis refers to the specific values of \( B_z \), while the horizontal axis refers to the distance(cm) from the origin point.

For simplification, here we take \( \mu_0 \) as 1, so the unit of vertical axis is \( T/\mu_0 \).

Fig. 3 shows the distribution of the three components of magnetic field \( B \) for \( P \) in one of the coordinate planes.
Figure 3. (a), (b) and (c) are $B_x$, $B_y$ and $B_z$ respectively for $P$ in x=2.1cm plane; (d), (e) and (f) are $B_x$, $B_y$ and $B_z$ respectively for $P$ in y=2.1cm plane; (g), (h) and (i) are $B_x$, $B_y$ and $B_z$ respectively for $P$ in z=2.1cm plane. Here the vertical axis refers to the specific values of $B_x/B_y/B_z$ for $P$ in different coordinate planes, while the horizontal axis refers to the distance(cm) of $P$ from the origin point respectively.

Data in Table 1 show the concrete values of $B$ and exact maxima and minima positions when $P$ is in various planes, which is corresponding to the different row in Fig. 3. In the figures of (b) and (d), there are double maximum values and double minimum values, and the corresponding positions are symmetrical around the origin. So we just give two positions here. For the last column(c, f, i) in Fig. 3, there is just one maximum (or minimum) value position, so we only give one value in Table 1. Obviously it’s hard to derive maxima and minima positions with the theoretical equations like Eq. 1-4. Furthermore, although the nine images above are in the case of an elliptical loop lying in z=0 plane, the corresponding results are similar and can be obtained just by the realignment between the three lines in Fig. 3 when the loop is set on the x=0/y=0 plane.

Table 1. Maxima and minima positions of $B_x/B_y/B_z$

<table>
<thead>
<tr>
<th></th>
<th>$B_x$</th>
<th>$B_y$</th>
<th>$B_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x=2.1cm plane</td>
<td>max (2.1, 0.0816, 0.0816)</td>
<td>(2.1, 0.2449, 0.0816)</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>min (2.1, 0.0816, -0.0816)</td>
<td>(2.1, 0.2449, -0.0816)</td>
<td>(2.1, 0, 0)</td>
</tr>
<tr>
<td>y=2.1cm plane</td>
<td>max (1.1200, 2.1, 1.6000)</td>
<td>(0.0800, 2.1, 0)</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>min (-1.1200, 2.1, 1.6000)</td>
<td>(-0.0800, 2.1, 0)</td>
<td>(0, 2.1, 0)</td>
</tr>
<tr>
<td>z=2.1cm plane</td>
<td>max (0, 1.9200, 2.1)</td>
<td>(1.2800, 0, 2.1)</td>
<td>(0, 0, 2.1)</td>
</tr>
<tr>
<td></td>
<td>min (0, -1.9200, 2.1)</td>
<td>(-1.2800, 0, 2.1)</td>
<td>/</td>
</tr>
</tbody>
</table>

For x=0 plane and y=0 plane, distribution patterns are similar because of the symmetry, so we simply care about the magnetic field in x=0 (or y=0) plane and z=0 plane. There is a circular current loop carrying 1 Amp lying in z=0 plane, which the radius is 1.0cm. The distributions of magnetic field $B$ for $P$ on one of three axes are shown as follows:
Figure 4. Simulation of magnetic field in the z=0 plane. (a), (b) and (c) show the general distributions; (d), (e) and (f) present symmetric characteristic of $B_x$, $B_y$ and $B_z$.

According to the simulations in Fig. 4, it can be seen that both radial and axial components of the magnetic field reach their maximum when $P$ approaching to the $z=0$ plane and the $x=1$cm plane, but they decay pretty quickly when spreading to peripheral region. The components of $B$ as well as their gradients in coordinate system are small and keep constant in other areas.

Table 2. Inside and outside of the circular current loop

<table>
<thead>
<tr>
<th>x(cm)</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.4</th>
<th>0.6</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_z$ (T/µ₀)</td>
<td>0.0861</td>
<td>0.1227</td>
<td>0.2403</td>
<td>0.3322</td>
<td>0.6177</td>
<td>0.8196</td>
<td>1.225</td>
</tr>
<tr>
<td>$B_x$ (T/µ₀)</td>
<td>1.1</td>
<td>1.5</td>
<td>1.7</td>
<td>1.8</td>
<td>2.0</td>
<td>8.3</td>
<td>8.4</td>
</tr>
</tbody>
</table>

From Table 2, it is obvious that directions of the magnetic field inside and outside are reverse. Magnetic field located at $x=1$cm reaches the maximum and tends to be infinitive with $x$ increasing. Inside the loop, magnetic field keeps stable when $x$ belongs to $(0.4, 0.8)$, while decaying pretty quickly when $x$ belongs to $(0, 0.4)$. When approaching to the center, the decrease speed gradually shows a sign of slowing down. Outside the loop, the magnetic field decays rapidly when it belongs to $(1.7, 2.0)$. However, when $x > 8.3$, the spatial derivative becomes small far from the loop.

If there is a circular loop with radius 2.0cm or an elliptical loop with semi-major axis 2.0cm and semi-minor axis 0.2cm, which is lying in the $z=0$ plane, then the simulations for the magnetic field far from the loop are in the following:

Figure 5. Magnetic field far from the loop in $x=0$ plane. (a), (b) and (c) refer to the $B_z$ distributions of a circular loop; (d), (e) and (f) refer to the $B$ distributions of an elliptical loop.
As shown in figure 5, here the eccentricity of the elliptical loop in figure 1 is equal to 0.995, in this case there is almost no distinct difference between the elliptical loop and the circular loop with eccentricity equal to 0. Therefore, an obvious conclusion can be drawn that the eccentricity or ratio of long to short half axis has little impact on distributions of the elliptical loops under the circumstance of the far field.

**Summary**

In this work, the general spatial distributions of the magnetic field due to an elliptical loop are deduced theoretically, and then the relatively complete simulations are presented. Some important rules of the magnetic field distribution, such as the peak positions, the cases of near and far from the loop, etc., are analyzed and discussed, which rarely appeared in other relevant papers. However, there are still many questions worth to be studied further, e.g., the loop with more complex geometrical shape, or even the loop with alternating currents.

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**References**


