Control Algorithm of Six Cable-Driven Parallel Manipulator for FAST

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Keywords: FAST, Feed cabin, Cable length, NAIPID, CPM.

Abstract. The feed cabin supporting system in a Five-hundred-meter Aperture Spherical Radio Telescope (FAST) project is a large-span cable-driven parallel manipulator (CPM). In this article, the kinematics model of the six CPM was established in the global coordinate O-XYZ. A corresponding difference of cable length between the current position and next position of the feed cabin was determined by controlling the servo motor. A nonlinear adaptive interaction PID (NAIPID) controller was proposed to improve the control performance of the control system. The simulations program of traditional PID and NAIPID were implemented in MATLAB software, respectively. A 3-meter model experiment was carried out to prove the control accuracy of the feed cabin. Both simulation and experimental results showed that the proposed control method had rapid response, and can effectively improve the control performance for the six CPM systems.

Introduction

Five-hundred-meter aperture spherical radio telescope (FAST) is the largest single dish radio telescope in the world [1, 2], sited in Guizhou province of China [3, 4]. The main reflector, a spherical dish, has a radius of 300 meter. When working, the feed cabin moves on the focus surface of the main reflector which forms the shape of paraboloid under the control of the supporting cables [5, 6].

The feed cabin supporting and positioning system in FAST is a multi-level hybrid mechanism which consists of a large-span cable-driven parallel manipulator (CPM) for the first-level mechanism, an AB rotator for the second-level mechanism, and a rigid parallel manipulator known as a Stewart manipulator for the third-level mechanism [7]. The large-span (CPM) consists of six towers built on mountainside at the diameter of 600 meters which support cables. One end of the cable is fixed on the feed cabin and the other end is linked with the manipulator. So, the feed cable is suspended in the sky by six cables. By controlling these six cables, the feed cable can be adjusted to appropriated position for collecting electromagnetic signals [8]. This paper aims to study control algorithm of six CPM to ensure the feed cable at demand position when the FAST is in its working state.

Kinematics Model of the Six CPM

The large-span (CPM) consists of six towers build on mountainside to support cables as shown in Fig. 1. Those six towers were distributed equality in a circle. A\(_i\) (i=1,...,6) are the connected points of the cables and the towers and distributed symmetrically in a circle. B\(_i\) (i=1,...,3) are the connected points of the cables and the feed cabin and distributed symmetrically in a circle [9]. The feed cable was dragged under demand trajectory on the focus surface by six cables which were driven by six servomotors, respectively.
Coordinate Frames and Kinematics Description

Global-coordinate and local-coordinate of the FAST were established, as shown in Fig. 1. The global coordinate O-XYZ is placed at the bottom of the tower, and its origin is the center of the six towers. The local-coordinate O1X1Y1Z1 was attached at the geometric center of the feed cable. The top point Ai (i=1,2…6) of those towers in the global coordinate O-XYZ can be described as:

\[
G_{A1} = \left[ \frac{1}{2} a, -\frac{\sqrt{3}}{2} a, h \right]^T, \quad G_{A2} = [a, 0, h]^T, \quad G_{A3} = \left[ \frac{1}{2} a, -\frac{\sqrt{3}}{2} a, h \right]^T, \\
G_{A4} = \left[ -\frac{1}{2} a, -\frac{\sqrt{3}}{2} a, h \right]^T, \quad G_{A5} = [-a, 0, h]^T, \quad G_{A6} = \left[ -\frac{1}{2} a, -\frac{\sqrt{3}}{2} a, h \right]^T
\]

(1)

As shown in Fig. 2, the kinematic model for the CPM based on the spatial vector chain is established. Ai is named as the coordinate of top point of the tower in the global coordinate O-XYZ. Bi is named as the coordinate of point B on the feed cabin in the local-coordinate O1X1Y1Z1. G01 is named as the coordinate of point O1 in the global coordinate O-XYZ. GBi is named as the coordinate of point Bi on the feed cabin in the global coordinate O-XYZ. By coordinate transformation, GBi can be described as:

\[
\begin{align*}
G_{Bi} &= R \cdot S_{Bi} + G_{01} \\
\left( x_{Bi}, y_{Bi}, z_{Bi} \right)^T &= R \cdot \left( x_{iBi}, y_{iBi}, z_{iBi} \right)^T + \left( x, y, z \right)^T
\end{align*}
\]

(2)

where \( R \) is a transform matrix when the feed cabin rotates around X axis, Y axis and Z axis. Cardan angle \( (\alpha, \beta, \gamma) \) is named as attitude angle of the feed cabin. The transform matrix \( R \) is described as:

\[
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & c\alpha & -s\alpha \\
0 & s\alpha & c\alpha
\end{bmatrix} \begin{bmatrix}
c\beta & 0 & -s\beta \\
0 & c\gamma & -s\gamma \\
0 & s\gamma & c\gamma
\end{bmatrix} = \begin{bmatrix}
c\beta c\gamma & -c\beta s\gamma & s\beta \\
sas\beta c\gamma + cas\gamma & -sas\beta s\gamma + cac\gamma & -sac\beta \\
-cas\beta c\gamma + sas\gamma & cas\beta s\gamma + sac\gamma & cac\beta
\end{bmatrix}
\]

(3)

where s and c symbols stand for sine and cosine operations respectively.

Difference of Cable Length (DCL)

When an astronomical observation trajectory is given, the path of the feed cabin should be determined firstly. And, in theory, the CPM alone is sufficient to perform the required motion of the receivers. However, there are many disturbances in the motion of CPM, such as temperature variation, wind load, elastic deformation of the cables, friction and so on, which make the feed cabin deviate from its normal path.

In order to improve the path accuracy, the desired cable length is ignored. But the difference of cable length between the current position and next position is required. When the FAST is in its working state, the current position and attitude angle of the feed cabin are detected by total station device. The next position and attitude angle of the feed cabin are deduced according to observation
target. Then the DCL can be determined by desired motions of CPM. In controlling period of CPM, the disturbances are too little to be considered when DCL is minuteness.

The calculation of DCL is described as follows: firstly, the current position and attitude angle of the feed cabin are detected by total station device. Secondly, the difference between current position and attitude angle of the feed cabin and next position and attitude angle of the feed cabin are judged, respectively. If the difference between current position of the feed cabin and next position of the feed cabin is greater than $10^{-8}$, the DCL is calculated according to vector decomposition, else the DCL is zero. In the same way, the DCL is calculated when the difference between current attitude angle of the feed cabin and next attitude angle of the feed cabin is greater than $10^{-8}$. Lastly, the total DCL is output.

**Nonlinear Adaptive Interaction PID Control**

**Control System of Feed Cabin**

The feed cabin support system is closed loop control system which consists of detection equipment, feedback control and executive mechanism. In the motion of feed cabin, the real trajectory of the feed cabin is measured by three non-contact laser total stations in time. Then the difference between real trajectory and theoretical trajectory is analyzed and compensated in next motion. The flow diagram of the control system of feed cabin is shown in Fig. 3.

![Figure 2. Vector of the cable i.](image1)

![Figure 3. Control system flaw of feed cabin.](image2)

**Self-Adaptive Interaction PID Controller**

PID controllers are the basic component in modern multi-level hierarchy of control [12]. The multi-level hybrid mechanism can, however, pose a great challenge to the PID controller design. The adaptive interaction algorithm is an effective method for variable parameter PID controlling [13, 14, 15]. This method can be used in PID controller for parameter optimization. According to adaptive interaction algorithm, a complex control system can be decomposed into a series of integral subsystems. The input and output of each subsystem are $x_i$ and $y_i$, respectively, which can be described as:

$$y_i(t) = (F_i \circ x_i)(t) = F_i\left[ x_i(t) \right] \quad i=1, 2, \ldots, N$$  \hspace{1cm} (4)

where $x_i(t)$ is the input of the $i$th subsystem, $y_i(t)$ is the output of the $i$th subsystem, $\circ$ is the function of $F_i$. It is assumed that $F_i$ is Frechet differentiable, and each subsystem is single input single output. The interaction between the subsystems is the function of other subsystem and external input as shown in Fig.4. The interaction is defined as $c$, and the interaction set is defined as $C$.

It is assumed that the interaction between subsystems is linear. The input of each subsystem can be described as

$$x_i(t) = u_i(t) + \sum_{c \in I} \alpha_i \cdot y_i(t)$$

where $u_i(t)$ is external input of the $i$th subsystem, $\alpha_i$ is the
interaction weight between subsystems, \( y_i \) is the input of the \( i \)th subsystem. According to the linear interaction, the dynamic interaction of the system is described as:

\[
y_i(t) = F_i \left[ u_i(t) + \sum_{c \in I_i} \alpha_c y_i(t) \right] \quad i=1, \ldots, N
\]

The purpose of adaptive interaction is to minimize the performance function \( E(y_1, y_2, \ldots, y_i, u_1, u_2, \ldots, u_i) \) by self-adaptive controlling the interaction weight \( \alpha_c \) between subsystems. If the interaction weight \( \alpha_c \) is changing as Eq.6, the Eq.7 is unique solution of Eq.5 [15]. The performance function \( E \) is decreasing monotonically.

\[
\dot{\alpha}_c = \sum_{c \in \Omega_c} \alpha_c \phi \left( \frac{dE}{dy_{\alpha c}} \circ F'_{\alpha c} \circ y_{\alpha c} - \gamma \frac{\partial E}{\partial y_{\alpha c}} \right) \circ F'_{\alpha c} \circ y_{\alpha c}
\]

(6)

\[
\dot{\alpha}_c = -\gamma \frac{dE}{d\alpha}
\]

(7)

where \( y_{\alpha c} \) is the output of subsystem with interaction \( c \), \( \Omega_{\alpha c} \) is the output set of interaction \( c \), \( \gamma \) is the self-adaptive coefficient.

PID controlling has shown better performance in process control, which consists of proportional gain \( K_P \), the integral gain \( K_I \) and derivative gain \( K_D \). According to self-adaptive interaction, PID controller can be divided into four subsystems, such as proportional part, integral part, derivative part and controlled plant. For PID controller, the function of system error is denoted as the performance function \( E = e^2 = (u - y)^2 \), where interaction weight \( \alpha_c = (K_P, K_I, K_D) \), \( O_4 = y_4 \). Then the Eq.7 can be equivalent to as follows:

\[
\begin{align*}
\dot{K}_P &= 2 \gamma y F'_{4} \circ y_1, \\
\dot{K}_I &= 2 \gamma y F'_{4} \circ y_2, \\
\dot{K}_D &= 2 \gamma y F'_{4} \circ y_3
\end{align*}
\]

(8)

In most cases, \( F'_{4} \circ y_1, F'_{4} \circ y_2 \) and \( F'_{4} \circ y_3 \) can be linear approximation [16]. The Eq.8 can be simplified into Eq.9.

\[
\begin{align*}
\dot{K}_P &= 2 \gamma y_1, \\
\dot{K}_I &= 2 \gamma y_2, \\
\dot{K}_D &= 2 \gamma y_3
\end{align*}
\]

(9)

PID parameters are obtained by solving the least of performance function \( E = e^2 = (u - y)^2 \). The self-adaptive interaction PID controller is obtained by combining the simplified self-adaptive algorithm of PID parameter with normal PID controller as shown in Fig.5.

![Figure 4. The interaction between the subsystems.](image_url)

![Figure 5. The self-adaptive interaction PID controller.](image_url)
Nonlinear Self-Adaptive Interaction PID Controller

The output of normal PID controller is linear combination PID parameters, which will cause conflict between quick response and low overshoot. In order to improve the performance of control system, nonlinear PID controller was obtained by combining nonlinear function with PID controller. The nonlinear function is constructed by the system error in this paper. When nonlinear function and self-adaptive interaction PID are stringed together, the nonlinear adaptive interaction PID (NAIPID) controller was built. The nonlinear function can be described as:

\[ y(e) = ae^b + 1 \]  \hspace{0.5cm} (10)

\[ y(e) = \frac{\exp(ce) + \exp(-ce)}{2} \]  \hspace{0.5cm} (11)

where \( a, b \) and \( c \) are constant,

\[ e = \begin{cases} 
  e, & e \leq e_{\text{max}} \\
  e_{\text{max}} \text{sgn}(e), & e > e_{\text{max}}
\end{cases} \]  \hspace{0.5cm} (12)

The range of the nonlinear function is from 1 to \(+\infty\). The range is within a limited range to avoid system oscillations.

Simulations and Experiments

Simulations of NAIPID

In order to verify the control performance of NAIPID, the simulation program is implemented in MATLAB software. The same random disturbance was added to the simulation program of traditional PID and NAIPID, respectively. Take tower No.1, for instance. The performance of traditional PID and NAIPID were shown in Fig.6. From Fig.6, we can see that the maximum error of DCL is 0.0422m using traditional PID controller, and the maximum error of DCL is 0.0099m using NAIPID controller. The NAIPID controller shows better performance than using traditional PID controller. When the six CPM are disturbed by wind, temperature changes and other matter that can block the cable moving freely, the NAIPID controller has rapid response and low overshoot, and can effectively improve the tracking control performance.

Scale Model Experiments

A 3-meter model (the cable tower is uniformly distributed in a 3 meter diameter circle) of the six CPM is built to verify NAIPID performance. The same trajectory of feed cabin was carried our using traditional PID and NAIPID controller, respectively. Since there is no wind in the laboratory room, the random disturbances were added into the control program at the 15th step to simulate the reality. The position error of feed cabin was recorded with time variation, as shown in Fig.7.

Figure 6. Simulation result of NAIPID and PID.

Figure 7. Experimental result of NAIPID and PID.
From Fig. 7, we can see that the max position error of feed cabin using NAIPID controller is smaller than using traditional PID controller without the random disturbances. When the random disturbances were added into the control program at the 15th step, the NAIPID controller has rapider response than traditional PID controller. Therefore, the NAIPID controller has fast convergence speed, high accuracy and reliable stability for the six CPM systems.

Conclusion

In this paper, Kinematics model of the six CPM for FAST was established in the global coordinate O-XYZ. In order to improve the path accuracy of the feed cabin, the difference of cable length between the current position and next position was determined by controlling the servo motor. The NAIPID controller was proposed to improve the performance of control system. The simulation result showed the NAIPID controller had fast convergence speed, high accuracy and reliable stability. The control experiments were carried out on the scale model. The experiment result showed that the proposed control method had rapid response and low overshoot, and can effectively improve the control performance for the six CPM systems.

References


