Study on the Energy-Efficient Driving Strategy for Trains Running in the Steep Downhill Section

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Abstract. Energy-efficient train driving is one of the effective methods to reduce the traction energy consumption of trains, which is consequently being paid more attention at present. In the steep downhill section, the speed will continue to rise when the trains do not apply traction due to the ramp force. In order to make better use of the steep downhill section and reduce the energy consumption of the train, this paper presents an approach for studying the energy-efficient driving strategy of trains running in the steep downhill section. For a given line condition, the steep downhill section in the line is identified firstly, and a method of calculating the driving strategy of this section is proposed without changing the running time of this section. Furthermore, the gradient descent method is integrated to solve the optimal cruising speed corresponding to the minimum traction energy consumption of the train. Finally, the effectiveness of the algorithm is verified by comparing the energy-saving effect with the classical energy-efficient driving strategy of “Maximum traction-Cruising-Coasting-Maximum braking”.

Introduction

With the increasing scale of the rail transit operation and the shrinking of the running headway, the energy consumption of the system has increased dramatically. Therefore, how to reduce the total energy consumption has become an important problem to be solved urgently in the sustainable development of the rail transit system. Taking the urban rail transit system for example, the traction energy consumption of trains accounts for about 53% of the total energy consumption of the system [1]. So the total energy consumption of the system can be effectively decreased by cutting down the train traction energy consumption to reduce the operational cost and carbon emission.

Studying the energy-efficient train driving strategy is an effective way to reduce the traction energy consumption. In 1980, Milroy [2] formulated a speed profile optimization model based on continuous train control, which lays the foundation of the optimal train control theory. Asnis [3] analyzed the problem with constant gradients and applied the Pontryagin maximum principle to find the necessary conditions on an optimal speed profile. Howlett [4,5] formulated the energy-efficient driving problem as a finite dimensional constrained optimization model and used the maximum principle to get the basic method of solving the energy-efficient driving conditions and the switching points. Khmelnitsky [6] formulated a continuous energy-efficient driving model with kinetic energy as state variable, in which varying gradients and speed limits were considered. Liu [7] proposed a complete calculation approach to finding the switching points among different phases by using the Pontryagin maximum principle. Considering varying gradients, speed limits and variable traction efficiency, Su [8] presented a numerical algorithm based on energy consumption allocation, and the algorithm is extended to the calculation of energy-efficient driving strategy among multi-stations, so as to optimize the distribution of the multi-station running time.

From the studies above, it is concluded that the classical energy-efficient driving strategy consists of maximum traction, cruising, coasting, and maximum braking as well as their corresponding switching points (see Figure 1).
For an individual steep uphill section, the Howlett team [9] found that the cruising phase was interrupted. The maximum traction was applied at the point $x=c$ before the steep uphill, then the cruising was restored at the point $x=d$ beyond the steep section (see Figure 2).

![Figure 2. Optimal speed profile on a steep uphill section.](image)

In the steep downhill section, the train speed will increase even if the traction and braking force are not applied by the train due to the ramp force. Trains can run through the section of steep downhill with no energy consumption and the same running time. Therefore, in order to make better use of the steep downhill, a calculation method, on the basis of the classical energy-efficient driving strategy, is proposed to solve the energy-efficient control problem for trains running in the steep downhill section.

**Problem Description**

**Model Formulation**

The train driving strategy, which is intuitively reflected by a speed profile, is the combination of an operating control sequence and switching points among different control phases. According to literature [10], the objective function is presented in equation (1):

$$
\min E = \int_0^s u_f(x) F(v(x)) dx.
$$

(1)

where $u_f$ is a proportional coefficient of the output traction force, $F$ is the maximum traction force, $v$ is the train speed, $x$ is the position of the train. For a point mass train the equation of motion can be written as

$$
\begin{align*}
\frac{dv}{dt} &= \frac{u_f F(v) - u_b B(v) - r(v) - G(x)}{mv}, \\
\frac{dx}{dt} &= \frac{1}{v}.
\end{align*}
$$

(2)
where \( B \) is the maximum braking force, \( u_b \) is a proportional coefficient of the output braking force, \( r(v) \) is the frictional resistance when the speed is \( v \), \( G(x) \) is the grade and curve resistance, \( m \) is the mass of train, \( t \) is the running time. The train speed, the traction force and the braking force should satisfy the following constraints

\[
u_f \in [0,1], u_b \in [0,1] \text{ and } u_f \neq u_b = 0,
\]

(3)

\[
v(x) \leq V(x).
\]

(4)

And the boundary conditions of the train movement are

\[
v(0) = 0, v(S) = 0, t(0) = 0, t(S) = T
\]

(5)

**Definition of Steep Downhill**

The track is steep downhill if the speed \( v_c \) the train running in the section \([b,c]\) cannot be maintained or decreased without braking \([9]\) (see Figure 3). Thus,

\[
G(x) - r(v_c) > 0
\]

(6)

for each \( x \in [b,c] \). Where \( r(v_c) \) is calculated by equation (7):

\[
r(v_c) = a_c v_c^2 + b_c v_c + c_c.
\]

(7)

![Figure 3. Speed increases on a steep downhill section without braking.](image)

**Solution**

The overall framework of the algorithm proposed in this paper is shown in Figure 4, in which the gradient descent method is one of the most classical methods to solve the optimization problem \([11]\), so it is not described in detail here. The following two sub-sections highlight the 2nd and 3rd steps of the algorithm.

**Calculation of Classical Energy-Efficient Driving Strategy**

On track with a steep downhill, the optimal coast starting point \( s \) of the train is solved by applying the dichotomy according to the range of a given trip time \( T \), and then the classical energy-efficient driving strategy of the train is obtained. The process can be divided into three steps:

- For an individual given cruising speed, the steep downhill interval \([b,c]\) in the line is identified by equation (6);
- Four critical states of the driving strategy, as shown in Figure 5, and the corresponding running time \( t_1, t_2, t_3 \) and \( t_4 \) are solved respectively;
- According to \( t_1, t_2, t_3, t_4 \) and the given trip time, the optimal starting point of the coasting phase is found by using the dichotomy, and then a classical energy-efficient driving strategy is obtained.
Set a trip time $T$

Initialize a cruising speed $v_c$

Solve classical e-e driving strategy

If cruise on $(b,c)$

Optimize local strategy on $(b,c)$

Calculate energy consumption $E$

$|E(v_c)| < e$

Output the optimal strategy with $E_{\text{min}}$

Identify steep section $(b,c)$

Load vehicle data and line data

Solution to classical e-e driving strategy

Local optimization on $(b,c)$

Figure 4. Flow chart of the proposed approach.

By the analysis, the length and position of the steep slope differ in the different steep slope lines, which leads to five relationships between the running time $t_1$, $t_2$, $t_3$ and $t_4$. According to the range of $T$, the distribution of the optimal starting point of the coasting phase is shown in Table 1:

**Table 1. Five Relationships between $t_1$, $t_2$, $t_3$ and $t_4$ & Distribution of $s$.**

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Figure 5. Driving strategies for the train respectively coasting from $S_1$, $S_2$, $S_3$, $S_4$. 

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Optimization of Driving Strategy on Steep Downhill

As shown in Figure 6, \([b,c]\) is the steep downhill. If the train adopts the cruising regime at the interval \([a,d]\) in the original classical driving strategy, the speed profile is shown as the horizontal dotted line. Specifically, the train will apply partial power at the interval \([a,b]\) to maintain the cruising speed \(v_c\). The traction energy consumption at such interval is \(E_{(a,b)}\). The train applies partial braking at the interval \([b,c]\) to maintain the cruising speed. The traction energy consumption at this interval is \(E_{(b,c)} = 0\), and the traction energy consumption at interval \([c,d]\) is \(E_{(c,d)}\).

On the premise of meeting the constant trip time, the optimization of the local driving strategy in the steep downhill is to switch the regime from cruising to coasting at point \(x=a\) before the steep section and from coasting to cruising at point \(x=d\) after passing by the steep section. In this way, the ramp force of the steep downhill section can be fully used. Substituting the cruising with the coasting without consuming the traction energy and reducing the local traction energy consumption of train at the interval \([a,d]\) so as to optimize the energy-efficient driving strategy of the overall trip and minimize the energy consumption of the train.

Figure 6. Train operation speed profile at the steep downhill section.

Simulation Results

In order to illustrate the effectiveness of the proposed approach, two simulations are conducted based on the infrastructure data between two successive stations (Jinghai - Ciqu) of Beijing Yizhuang rail transit line.

The distance between the two stations is 2086 meters and the speed limit of the whole section is 80 km per hour. The gradient information is shown in Figure 7. According to equation (6), the interval from 690 to 1389 is the steep downhill section.

- When the given trip time \(T=162\) s, the classical energy-efficient driving strategy is calculated with the traditional algorithm (see Figure 7(a)). The cruising speed of the train is 68 km/h and the traction energy consumption is 14.14 kW.h.

Figure 7. Energy-efficient driving strategy with the classical method and the proposed method.
For the same given trip time, the optimized energy-efficient driving strategy solved by the proposed approach (see Figure 7(b)). The optimal cruising speed of the train is 70 km/h and the traction energy consumption is reduced to 13.84 kW.h.

As shown in Figure 7(b), the optimized energy-efficient driving strategy consists of maximum traction regime in \([x_1, x_2]\), cruising regime in \([x_2, x_3]\), coasting regime in \([x_3, x_4]\) and maximum braking in \([x_4, x_5]\).

**Conclusion**

The paper studies the energy-efficient driving strategy for trains running in the line including single steep downhill section. It uses the actual inter-station line information of Yizhuang Line for simulations. On the same station section and the same trip time, the simulation results show that the proposed approach has a good performance on energy-saving, which can reduce the traction energy consumption by 2.12% than the classical energy-efficient driving strategy. Thus, the energy-efficient algorithm described in this paper is proved to be effective.

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**References**