The Dynamic Properties of Planetary Transmission of Wind Power Growth Gearbox Considering Installation Constraints

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Abstract. The natural mode shape of planetary gear train is closely related to vibration and noise. It is valuable to study installation constraints of the planetary transmission, which is very important in the theoretical and engineering. A new method was proposed to study the influence of the planet carrier's different constraints on the planetary gear train's inherent vibration mode. The supporting stiffness of the planetary gear and the center member were considered respectively in the dynamic model of planetary gear train. The planet carrier constraint way simulation was realized through the support stiffness of different combination. A dynamic model with considering the planet carrier constraints was built to study the influence of different installation methods of the carrier on the natural vibration mode. The research indicates that the different installation way has a significant effect on the inherent vibration mode of planetary gear train.

Introduction

Compared with ordinary gear transmission, planetary gear transmission has many obvious advantages [1], which is widely used in wind power generation, ship engineering, mining machinery and other fields. It is of great engineering significance to study the inherent characteristics of planetary gear transmission. Many scholars at home and abroad have done a lot of useful research on the inherent characteristics of planetary gear dynamics. Pure torsion inherent characteristics of planetary gear system dynamics using lumped parameter model [2-6] and [7-8] literature. The literatures adopt torsion on the intrinsic characteristics of the planetary gear system dynamic lumped parameter model of translation. Although these studies consider such factors as the floating of components and the time-varying meshing stiffness of gears, the influence of planetary mounting constraints on the natural characteristics of the system is neglected.

It is of great theoretical significance and practical value to study the influence of the constraint of planetary frame installation on the inherent characteristics of planetary gear trains. In order to study the planet carrier different restraint on natural vibration mode of gear planetary influence, we propose a new method to realize the simulation of planetary gear constraint method, this method is the dynamic model in the planetary gear train, considering the planetary gear planetary gear train center member and the support stiffness of each component, through the support stiffness of different combinations to achieve the simulation of planetary gear constraint mode. In order to analyze the influence of the constraint mode of planetary carrier on the inherent characteristics of planetary gear train, the 2K-H type straight planetary gear train of wind power increasing box is taken as the research object, and the coupling dynamic model of planetary gear is established by the lumped parameter method. According to the principle of mechanics, the dynamic equations of the model are listed, and the eigenvalues of vibration differential equations and the corresponding vibration modes vectors are solved. The mode shapes of the system are compared and analyzed.
Acceleration Analysis of Component Centroid and Vibration Model of Planetary Gear

The coordinate system of the planetary frame of the key component of planetary gear transmission is shown in figure 1.

The OXY shown in the diagram for the absolute coordinate system, Oxy coordinates in planetary gear system, the angular velocity to know $\omega_c$, Oxy relative to the X axis through the angle of $\phi = \omega_c t$, t is the time, $r_i$ is a member of the planetary transmission displacement vector centroid, the projection of the X axis and Y axis are $x_i$ and $y_i$, and the unit vector set selection for the x axis x axis respectively. Then the displacement vector can be expressed in the Oxy coordinate system:

\[
\mathbf{r}_i = x_i\mu + y_i\nu
\]

The selection $\mu$ and $\nu$ can be expressed in absolute coordinate system:

\[
\mu = \left|\mu\right|e^{i\phi}
\]

\[
\nu = \left|\nu\right|e^{i(\theta_i + \pi/2)}
\]

The formulas (2) and (3) are available:

\[
\dot{\mu} = \omega_c \nu
\]

\[
\dot{\nu} = -\omega_c \mu
\]

The formula (1) obtains the two derivative of the time t, and combines the formula (4) and the formula (5) to obtain the acceleration of the mass center of the component:

\[
\ddot{r}_i = (\ddot{x}_i - 2\omega_c \dot{y}_i - \omega_c^2 x_i)\mu + (\ddot{y}_i + 2\omega_c \dot{x}_i - \omega_c^2 y_i)\nu
\]

Fig. 2 is the coupling dynamic model of the 2K-H type planetary gear train used in the wind power increasing box. The model considers the support stiffness of the planetary gear and the center component of the planetary gear train respectively.

Where $k_{pn}$ support stiffness ($n=1,2,\ldots, N$), $N$ is the total number of planetary gear; $\psi_n = 2\pi(n-1)/N$, $k_{ij}$ is the center member support stiffness ($i = c, r, s; j = x, y, u$), $x_i$, $y_i$, and $u_i$ respectively ($i = c, r, s, 1, 2, \ldots, N$), $u_i = r_i\theta_i$; $u_i$ and $\theta_i$ are the components of the torsion line displacement and torsional displacement; $r_i$ is the radius of rotation of each member; $k_{rn}$ and $k_{sn}$ respectively in meshing and the outer meshing stiffness ($n=1,2,\ldots, N$), $\alpha_s$ and $\alpha_r$ are the internal and external meshing angle.
Differential Equation of Planetary Transmission

According to figure 2, the relative displacement and force condition of each component in the system are analyzed. According to Newton's second law and formula (6), the dynamic equation of the system is established:

\[
\begin{align*}
& m_i (\ddot{x}_i - 2\omega \dot{y}_i - \omega^2 x_i) - \sum_n \kappa_{i,n} \Delta_{i,n} \sin (\varphi_{i,n}) + k_i x_i = 0 \\
& m_i (\ddot{y}_i + 2\omega \dot{x}_i - \omega^2 y_i) + \sum_n \kappa_{i,n} \Delta_{i,n} \cos (\varphi_{i,n}) + k_i y_i = 0 \\
& (1/f_i^2) \alpha_i \dot{x}_i + \sum_n \kappa_{i,n} \Delta_{i,n} + k_i u_i = -T_i/\omega_i \\
& m_i (\ddot{x}_i - 2\omega \dot{y}_i - \omega^2 x_i) + k_i \Delta_{i,\infty} \sin (\varphi_{i,\infty}) + k_i \Delta_{i,\infty} = 0 \\
& m_i (\ddot{y}_i + 2\omega \dot{x}_i - \omega^2 y_i) - k_i \Delta_{i,\infty} \cos (\varphi_{i,\infty}) - k_i \Delta_{i,\infty} = 0 \\
& (1/f_i^2) \alpha_i \dot{x}_i - k_i \Delta_{i,\infty} = 0 \\
& m_i (\ddot{x}_i - 2\omega \dot{y}_i - \omega^2 x_i) + \sum_n \kappa_{i,n} \Delta_{i,n} + k_i x_i = 0 \\
& m_i (\ddot{y}_i + 2\omega \dot{x}_i - \omega^2 y_i) + \sum_n \kappa_{i,n} \Delta_{i,n} + k_i y_i = 0 \\
& (1/f_i^2) \alpha_i \dot{x}_i + \sum_n \kappa_{i,n} \Delta_{i,n} + k_i u_i - T_i/\omega_i \\
\end{align*}
\]

(7)

Where \( \Delta_{i,n} \), \( \Delta_{i,\infty} \), \( \Delta_{i,m} \), \( \Delta_{p,m} \), \( \Delta_{p,m} \) is the relative displacements of each component of planetary gear trains respectively, \( \psi_{i,n} = \psi_{i,n} - \alpha_i \), \( \psi_{i,m} = \psi_{i,m} + \alpha_i \), \( T_c \) and \( T_S \) are input torsion and load torsion, respectively; \( m_i \), \( M \), \( m_i \) and \( m_n \) are the mass of each component of planetary gear train; \( I_c, I_s \), and \( I_n \) are the rotational inertia of each component of planetary gear train, respectively.

The differential equations of the system can be obtained by assembling the above equations and representing them in matrix form:

\[
M \ddot{q} + \omega \dot{G} \dot{q} + (K_b + K_m - \omega^2 K_m)q = T
\]

(8)

Where \( q = [x_c, y_c, x_r, y_r, x_s, y_s, x_u, y_u, x_1, y_1, x_2, y_2, x_3, y_3, \ldots, x_N, y_N, u_N] \) coordinate vector of the system, \( T \) excitation matrix, \( M \) mass matrix, \( K_m \) centripetal stiffness matrix, \( G \) gyroscopic matrix, \( K_b \) support stiffness matrix, \( K_m \) mesh stiffness matrix.

Influence of Planetary Cage Restraint Mode on Inherent Characteristics of Planetary Gear Train

The undamped free vibration equation (9) of the system can be obtained by the formula (8)

\[
\omega^2 M \psi_i = [K_b + K_m - \omega^2 K_m] \psi_i
\]

(9)

Where \( \omega_i \) natural frequencies of order \( i \), \( \psi_i \) vibration type of order \( I \), and

\[
\psi_i = [\psi_{i,1}, \psi_{i,2}, \psi_{i,3}, \ldots, \psi_{i,N}]
\]

(10)

\[
\psi_{i,j} = \left[ \psi_{i,j}^1, \psi_{i,j}^2, \psi_{i,j}^3, \ldots, \psi_{i,j}^N \right]
\]

(i=1,2,3,...,N+3; j=c,r,s,1,2,3, ..., N).

Taking the system parameters as an example, the formula (9) is solved to obtain the inherent characteristics of the system. When the number of planetary wheels is 3, 4 and 5, the natural frequencies of each order of the system are found.
The vibration modes corresponding to the natural frequencies of the system can be obtained by calculation. As the inherent vibration mode analysis of planetary gear train, with the number of planetary gear was 3, 4 and 5 of the planetary gear train as an example, through the analysis of modal coordinates corresponding to the natural frequency of the system and, by comparison and conclusion system center member torsional vibration mode, the center member radial translational mode and planetary gear vibration model. Analysis of the data in Table 2 shows that there is a mapping relation between the natural frequency and the root system of M, when the root number m=1 and m=2, the natural frequency of the corresponding number is 6; when the weight of root number m=N-3 (N > 3), the natural frequency of the number corresponding to the 3. With the increase of the number of planet gears, the natural frequencies of the corresponding number of m=1 numerical root weight was increasing, and the natural frequency of root of the corresponding number of m=2 in the first 3 order natural frequency becomes small, after the 3 order natural frequency increases, the number of root weight m=N-3 (N > 3) value of the natural the frequency corresponds with the number of planetary gear change.

Taking the number of planetary wheels 5 as an example, the vibration modes of 3 typical vibration modes of planetary gear trains are plotted, as shown in figure 3 to figure 5. Torsional vibration mode of the center member. The multiplicity of the 6 frequencies corresponding to the rotation mode is 1; the corresponding natural frequencies are shown in Table 2, and the corresponding vibration modes are shown in figure 3. The number of the 6 frequencies corresponding to the radial translation mode of the central component in the radial translation mode of the central component is 2; the corresponding natural frequencies are shown in Table 2, and the corresponding vibration modes are shown in figure 4. Planetary gear vibration mode. The number of the corresponding 3 frequencies in the planetary gear vibration mode is N-3 (N>3). When the number of planetary wheels is less than or equal to 3, the planetary gear vibration mode does not exist. The corresponding natural frequencies are shown in Table 2, and the corresponding vibration patterns are shown in Figure 5.

![Figure 3](593.1Hz)  ![Figure 4](262.2Hz)  ![Figure 5](731.3Hz)

Figure 3. Sketch of rotational mode. Figure 4. Sketch of comparison of translational mode. Figure 5. Sketch of comparison of planet mode.

**Influence of Planetary Cage Restraint Mode on Inherent Characteristics of Planetary Gear Train**

In the dynamic model of planetary gear train, the support stiffness of planetary gear and central component of planetary gear train is taken into account respectively, and the constraint mode of planetary carrier is simulated by different combinations of support stiffness of each component.

It is assumed that the radial support stiffness of the solar wheel, inner ring gear and planetary gear is $1.0 \times 10^8$ N/m, and the meshing stiffness of the internal and external gear pair is $5 \times 10^9$ N/m. The planetary frame fully floating constraint, each component of the support stiffness combination: $k_{cx}=k_{cy}=k_{cu}=0$; planetary gear circumferential floating constraints, each component of the support stiffness combination: $k_{cx}=k_{y}=1.0 \times 10^8$ N/m, $k_{cu}=0$; planetary gear radial floating constraints, each component of the support stiffness combination: $k_{cx}=k_{cy}=0$, $k_{cu}=1.0 \times 10^9$ N/m; planetary gear completely fixed constraints each component, support stiffness combination: $k_{cx}=k_{cy}=1.0 \times 10^8$ N/m,
According to the formula (9), the natural frequencies corresponding to the planetary mounting system under various mounting modes are solved, as shown in Table 2.

### Table 2. Natural frequencies of the planetary gear set under different conditions (Hz.)

<table>
<thead>
<tr>
<th>m</th>
<th>k_cx=k_cy=k_cu=0</th>
<th>k_cx=k_cy=10^8 N/m, k_cu=0</th>
<th>k_cx=k_cy=0, k_cu=10^9 N/m</th>
<th>k_cx=k_cy=10^9 N/m, k_cu=10^7 N/m</th>
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<td>0</td>
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<td>1163.8</td>
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</tr>
<tr>
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<td>1890.9</td>
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<td>2653.3</td>
<td>2653.3</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>14252.9</td>
<td>14252.9</td>
<td></td>
</tr>
<tr>
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<td>512.3</td>
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<tr>
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<td>5963.8</td>
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<tr>
<td></td>
<td>6981.7</td>
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### Conclusion

The close relationship between the planetary gear and its natural vibration mode of the vibration and noise of the planetary gear train with key members of the planetary frame restraint influence on the inherent characteristics of the planetary gear train has important theoretical significance and practical value. The main conclusions are as follows. The planetary frame constraint mode simulation method is proposed, the method considering the planet gear and the planetary gear center member support stiffness, support stiffness of each component through different combinations to achieve the simulation of planetary gear constraint mode. The coupling dynamic model of 2K-H type planetary gear system with planetary frame constraints is established, and the dynamic equations of the model are derived. The number of planetary wheels does not change the total number of natural frequencies of the system and the corresponding frequency values in the planetary vibration mode, but it will change the frequency corresponding to the torsional vibration mode and the radial translation vibration mode. The inherent characteristics of planetary gears are closely related to the constraint modes of planetary gears, and different installation modes have a significant influence on the vibration modes of planetary gears.

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### References


