Plastic Modifying of Dynamic Stress Intensity Factor under Impact Load

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Abstract. The aim of this paper is to analyze the feasibility of calculating Dynamic Stress Intensity Factor (DSIF) considering crack propagation by plastic zone. At first, the plastic zone size is calculated and the equivalent crack can be obtained. Then based on which, the DSIF subjected to the impact load is calculated. The results showed that the DSIF after plastic modifying is obviously lower than the one calculated based on linear elastic material model. Furthermore, there is a maximum value in the time history curve of DSIF. Then, the equivalent crack propagation velocity is analyzed. The result shows that when the impact load reaches the peak value, crack propagation velocity also reaches the maximum value, and so does the DSIF.

Introduction

The development of dynamic fracture theory often be simplified into two types of problems: The crack is stable but the external force changes rapidly with time, such as vibration, impact and wave (explosion wave or seismic wave etc.); another type is having stable external force but the crack spread and extend rapidly \cite{1}. For dynamic loads, the researchers have already done extensive researches on the measuring techniques of material’s dynamic fractures under various experimental conditions, such as drop hammer \cite{2-4}, Split Hopkinson Pressure Bar (SHBP) \cite{5-8} and explosive load \cite{9}. In the aspect of structural impact, Chen et at. \cite{10} researched a typical submarine structure with surface cracks; Hu \cite{11} discussed the crack’s dynamic responses of rock material under impact load. Through analyzing the time history of stress field and strain field, the corresponding DSIF could be achieved. As a key indicator of dynamic fracture, DSIF is often been used to study the initiation, propagation and arrest of cracks, however, since DSIF adopts constitutive model with linear elasticity, it is apparently not applicable on metal and alloy material. Wu et al. \cite{12} raised a theoretical model to calculate plastic zone under dynamic loads, which provided a thinking of plastic analysis of crack tip under dynamic loads. Gong Nengping \cite{13} analyzed the dynamic fracture properties of plastic material with dynamic J-integral. Considering the DSIF has the characteristics of convenient measurement and simple calculation, therefore for some high-strength steel and alloy, the DSIF after plastic modifying could be used to represent material’s fracture property. Based on that, this paper proceeded modifying on DSIF through analyzing the size of plastic zone of crack tip and achieved plastically corrected DSIF, than the propagation rate of equivalent crack was analyzed.

Dynamic Stress Intensity Factor under Impact Load

Consider a three-point bending beam model shown in Figure. 1(a), a represents the length of crack, S represents the span of beam; B and W respectively represent the width and height of beam. The DSIF was analyzed under the action of impact load P(t). Bacon et al. \cite{14} pointed out that under the experimental condition of three-point bending, DSIF could be obtained by the displacement response of loading point. Since the displacement of loading point is only consideration, it can be equivalent to a spring-mass system shown in Figure 1(b). The equivalent mass could be solved by the following equation \cite{15}. 

\[ m = \frac{P(t)}{k} \]
m_e = \frac{17 \rho B W S}{35} \quad (1)

The equivalent stiffness could be obtained by the midpoint deformation of beam with cracks under static load[16],

\[ k(a) = \frac{P}{u_{nc} + u_c} = \frac{48EI}{L} \left[ 1 + 2.85 \left( \frac{W}{L} \right)^2 - 0.84 \left( \frac{W}{L} \right)^3 + 6 \left( \frac{W}{L} \right)^{4} \right] \left[ V \left( \frac{a}{W} \right) \right]^{-1} \quad (2) \]

where \( P \) represents the static load, \( u_{nc} \) and \( u_c \) respectively represent the displacement of beam without cracks under Load \( P \) and caused by cracks. The \( V \left( \frac{a}{W} \right) \) appeared on the equation above can be represented as:

\[ V \left( \frac{a}{W} \right) = \left[ \frac{a}{W} \right] \left[ 5.58 - 19.57 \left( \frac{a}{W} \right) + 36.82 \left( \frac{a}{W} \right)^2 - 34.94 \left( \frac{a}{W} \right)^3 + 12.77 \left( \frac{a}{W} \right)^4 \right] \quad (3) \]

After equivalence, the mass block has the same dynamic displacement response as the loading point of simply-supported beam, the solution is

\[ u(t) = \int_0^t P(\tau) h(t - \tau) d\tau \quad (4) \]

where, \( h(t - \tau) = \begin{cases} 1/\omega_m t \quad & t \geq \tau \quad \omega_m t \sin[\omega_m (t - \tau)], \quad t < \tau \end{cases} \quad (5) \]

direct proportion to \( u(t) \) [16]:

\[ K_{kl} (t) = C u(t) \]

where the coefficient \( C \) can be obtained by static fracture theory,

\[ C = \frac{35 \sqrt{a}}{2BW^2} Y \left( \frac{a}{W} \right) k(a) \quad (6) \]

When the sample satisfying \( S/W \approx 4 \), there is

\[ Y \left( \frac{a}{W} \right) = \frac{1.99 - \frac{a}{W} \left( 1 - \frac{a}{W} \right) \left[ 2.15 - 3.93 \frac{a}{W} + 2.7 \left( \frac{a}{W} \right)^3 \right] \left( 1 + 2 \frac{a}{W} \right)^{3/2}}{\left( 1 + 2 \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right)^{3/2}} \quad (7) \]

Finally, substitute equation (6) and (4) into equation (5) to obtain DSIF. Here the DSIF did not consider the plastic modifying of crack tip, therefore, for the majority of metal and alloy materials,
there will always exist calculation errors of DSIF caused by certain yield. The next section will consider the plastic zone of crack tip and correct the DSIF.

**Plastic Modifying of Dynamic Stress Intensity Factor**

Consider the material as ideal elastic-plastic model. It is considered that when reaching yield stress, such material’s strain will increase even without increased stress. At this time, the material cannot bear any load, calculate the yield at the crack tip and correct the original crack to correct the DSIF. Select the Mises yield criterion,

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{ys}^2
\]

where, \(\sigma_{ys}\) is called effective yield stress, its relationship with material’s yield stress should satisfy:

\[
\sigma_{ys} = \begin{cases} 
\sigma_s \quad \text{(pl an st r ess)} \\
\sigma_s/(1-2\mu) \quad \text{(pl an st r ai n)}
\end{cases}
\]

If the crack’s actual length is \(a\), considering the existence of plastic zone, the length of crack could be equal to effective crack length:

\[
\bar{a} = a + r_y
\]

At this circumstance, the plastic zone does not have to be considered, so the DSIF with linear elasticity could still be adopted. In the equation, \(r_y\) represents the equivalent crack propagation that considered plastic zone and stress relaxation, in the static fracture mechanics, the expression is [17]:

\[
r_y = \begin{cases} 
1/\pi \left( \frac{K_1}{\sigma_s} \right)^2 \quad \text{(pl an st r ess)} \\
1/(4\sqrt{2}\pi) \left( \frac{K_1}{\sigma_s} \right)^2 \quad \text{(pl an st r ai n)}
\end{cases}
\]

Under impact loads, substitute \(K_1\) with \(K_{id}\), then the plastic zone at crack tip could be solved. Since the material is ideal elastic-plastic model, the crack’s propagation is approximately considered as \(r_y\), and from which the crack’s new length \(\bar{a}\) could be obtained. Substitute the corrected equivalent crack length into equation (2), (6) and (5), then the corrected DSIF is achieved.

**Numerical Example**

Consider the three-point bending sample shown in Figure.1, the physical dimensions are: \(B=W=10\text{mm}, S=55\text{mm}\). The material constants are: \(\rho=7800\text{kg/m}^3\), \(E=210\text{GPa}\), \(\mu=0.3\), \(\sigma_s =1685\text{MPa}\). Consider a Hopkinson pressure bar with impact load, then select the typical loads shown in Figure.2 as excitatory input (81 spots in total). Taking plane stress as example, the influence characteristics of plastic modifying on DSIF was analyzed.

Figure 3(a) provided the result of contrast between DSIF with linear elasticity and DSIF with plastic modifying under the condition of plane stress. Apparently, the DSIF with linear elasticity and DSIF with plastic modifying were almost identical before \(12\mu s\). However, the difference was getting larger after \(12\mu s\). That is because after plastic modifying, the material is no longer considered having bearing capacity after yielded, however, it was considered as the propagation value of cracks and used as new cracks to calculate the DSIF at the next moment. This result is basically in accordance with the result in reference [16]. As shown in Figure. 3(b), the author believes that the initiation of cracks caused the separation between FEM result and formula result.
In Figure 3(a), it can be noticed that the DSIF with plastic modifying reached its peak at about 17.71 $\mu$s, after that, the DSIF experienced a rapid decrease. Comparing it to the result of reference [16] shown in Figure 3(b), it is apparent that the DSIF obtained by FEM method also achieved its maximum at 17.71 $\mu$s, which is just the same time as the impact load reached its peak value. That is because the stress at the crack tip decreased after passing the peak load, then the DSIF was decreased correspondingly.

**Conclusion**

This paper analyzed the size of plastic zone on crack tip under impact loads and applied it as the modifying of cracks to solve the equivalent crack length. Through the method of simultaneously correcting crack length and calculating, the DSIF under impact loads could be obtained. It is found that the plastically corrected DSIF under plane stress condition is apparently lower than the DSIF calculated solely by linear elastic theory, and on the time history curve of plastically corrected DSIF, there is a maximum value. When the impact load reached the peak, the DSIF achieved its maximum value, and the propagation rate of cracks at this time is also the fastest.

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References


