An Automatic Fuzzy Image Segmentation Method Based on Wavelet Energy Histograms and Markov Random Field Models

Guo-ying LIU, Xu SONG and Dan HONG
Department of Computer and Information Engineering, Anyang Normal University,
Anyang China 455002
*Corresponding author

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Abstract. Fuzzy C-Means (FCM) algorithm has been widely used in remote sensed image segmentation. However, it has two main defects: (1) its sensitiveness to noise outliers, and other imaging artifacts; (2) the number of clusters needed to be set previously. In order to overcome these problems, in this paper, we incorporate the wavelet energy histograms (WEHs) and Markov random field models (MRFs) into the fuzzy clustering procedure and present a novel image segmentation method. WEHs serve the determination of cluster centers and MRFs play a role of modelling spatial information. First of all, the peaks of wavelet histogram are exploited to find the initial cluster centers. Then, a fuzzy clustering procedure with MRFs is performed on each band separately. Finally, the fused label of the clustering result from each band is used as the final segmentation result. The superiority of the proposed method is demonstrated by comparing it with the some well-known methods of FCM, FLICM, HMRF-FCM, and AHFCM.

Introduction

Image segmentation is one of the key techniques in the field of remote sensing and computer vision. It aims at dividing an image into a series of non-overlapping image regions with similar characteristics such as spectral, texture, etc. Although many different kinds of methods have been proposed to solve this problem [1-4], it still keeps challenging because of noisy perturbation and low contrast of images.

In the past decades, fuzzy clustering algorithms have been widely used in image segmentation. Among them, fuzzy c-means (FCM) is one of the most popular algorithms [5], because it has robust characteristics for ambiguity and is able to retain more original image information than hard or crisp segmentation methods [6]. The conventional FCM performs well on most noise-free images; but it fails to segment images with noise, outliers and imaging artifacts, due to its ignorance of spatial information. Besides, it requires experts with adequate domain knowledge to determinate the number of clusters before clustering, which greatly limits its application.

In order to solve the first problem, recently, many improved FCM algorithms have been proposed to incorporate spatial information. S. Krinidis and V. Chatzis [6] proposed a novel robust fuzzy local c-means algorithm (FLICM), where a fuzzy factor was introduced into its objective function to guarantee noise insensitiveness and image detail preservation. This method was lately improved by by M.Gong from introducing a kernel distance measure into its objective function [7]. S. P. Chatzis and T. A. Varvarigou [8] explored the spatial coherency modeling capacities of the hidden Markov random field (HMRF) model in the fuzzy clustering procedure and proposed the HMRF-FCM algorithm. We extended this method into a multiscale version by capturing and utilizing the multiscale spatial constrains [9] and enhanced its accuracy of selecting local information by incorporating region-level information into the fuzzy clustering [10]. Besides, Zhang et al. [11] utilized the mean templates of distance function and membership function to consider spatial information. All of these methods mentioned above have shown their superiority to FCM in their realms.
Besides the improvement of the traditional FCM by incorporating spatial information, some methods were proposed to automatically determine the number of clusters. EA Zanaty introduced a new validity index method based on multi-degree entropy algorithm for determining the number of clusters automatically [12]. Chaabane et al. integrated the automatic thresholding algorithm into the FCM algorithm to segment color images [13]. More recently, Saman Ghaffarian and Salar Ghaffarian proposed an automatic histogram-based FCM method (AHFCM) to find the number of clusters [14].

In this study, we combine the merits HMRF-FCM and AHFCM to present a novel automatic fuzzy clustering method. There are two main contributions of this work. First, we refine the histogram-based method of AHFCM by extending it to the wavelet domain. Unlike AHFCM, we calculate the histogram of each band on the quantized wavelet energy instead of intensities. Second, we independently perform the gray-level HMRF-FCM on each band and simply fuse the results to form the final segmentation. It is different from AHFCM because MRF model is used to consider spatial information. It is also different from HMRF-FCM which explores multivariate distributions to deal with multispectral images.

The reminder of this paper is organized as follows: in Section 2, we provide a brief review of the AHFCM algorithm and the HMRF-FCM algorithm. In Section 3, we introduce our algorithm based on HMRF-FCM and AHFCM. The experimental results of the proposed approach are given in Section 4. Finally, Conclusions are drawn in Section 5.

Mathematical Background

The HMRF-FCM Algorithm

Let us consider a B-bands multispectral image Y be composed of N points, each pixel has a given spectral vector \( y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,B}]^T \). Let us suppose that this image has to be segmented into K classes. The objective function of HMRF-FCM is given by

\[
J = \sum_{i=1}^{N} \sum_{k=1}^{K} r_{ik} d_{ik} + \lambda \sum_{i=1}^{N} \sum_{k=1}^{K} r_{ik} \log \left( \frac{r_{ik}}{\pi_{ik}} \right),
\]

where \( r_{ik} \) is the membership of \( y_i \) in the \( k \)th clusters, \( \lambda \) is the model’s degree of fuzziness, \( \pi_{ik} \) is the prior probability of the \( j \)th cluster, \( d_{ik} \) is a distance measure between point \( y_i \) and the \( k \)th cluster. Let \( \mu_k \) and \( \Sigma_k \) denote the mean value and covariance matrix, respectively, the distance measure \( d_{ik} \) is defined as

\[
d_{ik} = -\log(p(y_i | \mu_k, \Sigma_k)),
\]

where

\[
p(y_i | \mu_k, \Sigma_k) = \frac{1}{(2\pi)^{B/2} |\Sigma_k|^{1/2}} \exp\left\{-\frac{1}{2} (y_i - \mu_k)^T \Sigma_k^{-1} (y_i - \mu_k)\right\}.
\]

The prior probability \( \pi_{ik} \) is defined as the mean-field–like approximation of MRF model. In [8], the authors adopt a second-order neighborhood system and the prior probability is defined as

\[
\pi_{ik} = \frac{\exp(\beta \sum_{j \in n_i} \delta(k, x_j))}{\sum_{k=1}^{K} \exp(\beta \sum_{j \in n_i} \delta(k, x_j))},
\]

where \( \beta \) is the inverse temperature parameter, and \( \delta(\cdot) \) stands for the Kronecker’s function

\[
\delta(k, x_j) = \begin{cases} 1, & \text{if } x_j = k \\ 0, & \text{otherwise} \end{cases}.
\]
The AHFCM Algorithm

The AHFCM algorithm [14] has two main steps:

Step 1: automatic histogram-based FCM clustering of a single band image.

In this step, the authors calculate the local slopes of the histograms to get the initial cluster centers for the conventional FCM algorithm. The slope equations are defined as below:

$$M_{i,i-1} = \frac{h_i-h_{i-1}}{g_i-g_{i-1}}, \quad M_{i,i+1} = \frac{h_{i+1}-h_i}{g_{i+1}-g_i}, \quad M_{ai} = M_{i,i-1} \times M_{i,i+1},$$

(4)

where $h_{i-1}$, $h_i$ and $h_{i+1}$ are the frequency values of the previous point $i-1$, the current point $i$ and the next point $i+1$ in the histogram, $g_{i-1}$, $g_i$ and $g_{i+1}$ are the brightness value of points $i-1$, $i$ and $i+1$, respectively. Therefore, equation (4) gives us two slopes and their multiplication. By using the following rules, the initial candidate centers can be determined:

If $M_{ai} > 0$, $p$ is not a cluster center.
If $M_{ai} < 0$
If $M_{i,i-1} \leq 0$, $i$ is not a cluster center.
If $M_{i,i+1} > 0$, $i$ can be a cluster center.
If $M_{ai} = 0$
If $M_{i,i-1} \leq 0$ or $M_{i,i+1} < 0$, $i$ is not a cluster center.
If $M_{i,i-1} = 0$ and $M_{i,i+1} = 0$, $i$ is not a cluster center.

A candidate center will be removed from the candidate set if it is too near to others, e.g., the distance is smaller than a predefined threshold.

Step 2: Another FCM clustering step is performed on the multispectral image with the initialization of the fused labeled band images.

In this step, the authors firstly fused the results of FCM on single bands to form an initial segmentation result, which is followed by another conventional FCM on the primary multispectral image.

Proposed Method

Proposed Model

In this study, we propose a fuzzy image segmentation method by combining the merits of AHFCM and HMRF-FCM. To demonstrate our algorithm, we first generate a new object function

$$J = \sum_{i=1}^{N} \left[ \sum_{b=1}^{B} \sum_{k_b=1}^{K_b} r_{i,b,k_b} d_{b,i,k_b} + \sum_{b=1}^{B} \sum_{k_b=1}^{K_b} r_{b,k_b} \log \left( \frac{r_{b,k_b}}{\pi_{b,k_b}} \right) \right],$$

(5)

where $k_b$, $r_{b,i,k_b}$, $d_{b,r,k_b}$ and $\pi_{b,i,k_b}$ are the cluster label, the membership function, the distance measure and the prior probability on the $b$th band, respectively. It must be mentioned that two gray values $y_{i,b_1}$ and $y_{i,b_2}$ will be classified into different clusters with membership $r_{b_1,i,k_{b_1}}$ and $r_{b_2,i,k_{b_2}}$ respectively although they are located at the same position $i$. In this case, the distance measure is defined as

$$d_{b,i,k_b} = -\log(p(y_{i,b} | \mu_{k_b}, \sigma_{k_b})),$$

(6)

where

$$p(y_{i,b} | \mu_{k_b}, \sigma_{k_b}) = \frac{1}{\sigma_{k_b} \sqrt{2\pi}} \exp \left[ -\frac{(y_{i,b}-\mu_{k_b})^2}{2(\sigma_{k_b})^2} \right],$$

(7)

and $\pi_{b,i,k_b}$ has the same form as in (3).

The minimization of (5) can be obtained by the minimization on each band independently and the fusion of the band-wise results.
Parameter Estimation

Parameters involved in the proposed method include $r_{b,i,k_b}$, $\mu_{k_b}$, $\sigma_{k_b}$, $K_b$ and $\beta$. The inverse temperature parameter $\beta$ is estimated by using the same method as in [8], and the number of clusters $K_b$ on the $b^{th}$ band is estimated by using the same method as in AHFCM based on the wavelet energy histogram, which will be described below. The remaining parameters can be obtained by minimizing (5). Let us first consider the derivation of the fuzzy membership function values. This can be attained by minimizing (5) over $r_{i,k_b}$ under the constraints $\sum_{k_b=1}^{K_b} r_{i,k_b} = 1$:

$$r_{p,i,k_b} = \frac{\pi_{b,i,k_b} \exp(-d_{b,i,k_b})}{\sum_{k_b=1}^{K_b} [\pi_{b,i,k_b} \exp(-d_{b,i,k_b})]}.$$  \hspace{1cm} (8)

To obtain the means $\mu_{k_b}$ and the standard variance $\sigma_{k_b}$, we need to conduct the minimization of (5) over them respectively. Then we have

$$\mu_{k_b} = \frac{\sum_{l=1}^{N} r_{b,i,k_b} y_{i,b}}{\sum_{l=1}^{N} r_{b,i,k_b}},$$  \hspace{1cm} (9)

$$\sigma_{k_b} = \frac{\sum_{l=1}^{N} r_{b,i,k_b} (y_{i,b} - \mu_{k_b})^2}{\sum_{l=1}^{N} r_{b,i,k_b}}.$$  \hspace{1cm} (10)

Histogram Based on Wavelet Energy

Wavelet transformation has shown a good performance to represent the non-stationary of images. Therefore, we employ wavelet energy to form a histogram that is more robust to noise and image non-stationaries. Let a certain wavelet band of the wavelet transformed image be $w = [w_{i,j}]$, where $w_{i,j}$ denotes the wavelet coefficient on position $(i, j)$. The wavelet energy can be defined as:

$$e_{i,j} = \frac{1}{|\eta_{i,j}|} \sum_{(i',j') \in \eta_{i,j}} W_{i,j}^2,$$  \hspace{1cm} (11)

where $\eta_{i,j}$ is a position set that is surrounding position $(i, j)$, $|\eta_{i,j}|$ denotes the number of elements in $\eta_{i,j}$. For the sake of simplicity, in this study, we employ a squared window centered at $(i, j)$ to serve as $\eta_{i,j}$.

In order to get the histogram of wavelet energy, in this study, we quantify $e_{i,j}$ into the range $[0, 255]$. Therefore, the wavelet band image becomes a piece of gray image and its histogram has 256 bins, each of which describes the frequency of certain quantified wavelet energy. Based on histogram on each band.

The Proposed Algorithm

Consequently, the proposed method can be summarized as follows.

1. For each spectral band $b$, do
   a) Decompose the $b^{th}$ band of the input image $y$ into wavelet domain, and only the low frequency wavelet subband is considered to compute the histogram of wavelet energy.
   b) Calculate the wavelet energy for each position by (11) and quantify it into $[0, 255]$.
   c) Calculate the frequency of quantified wavelet energy to form the histogram, which is further smoothed by a mean filter with the window size of 3.
   d) Use (4) to calculate slopes in each histogram and determine initial number of cluster centers $K_b$ and the initial parameters of each center by the rules described in Section 2.B. In this step, we firstly set $\mu_{b,k} = \tilde{g}_i'$ ($\tilde{g}_i'$ is the value of the quantified wavelet energy which corresponding to the $k^{th}$ peak of the histogram. Then those in $\{\mu_{b,k}\}$ with a distance smaller than the predefined threshold $T$ are merged. After this, we assign a cluster label $k_b$ for each pixel $i$ if $|y_{i,b} - \mu_{b,k}|$ has the minimum value. Finally, pixels with the same label are used to recalculate the $\mu_{b,k}$ by $\mu_{k_b} = (\sum_{y \in L_k} y_{i,b})/|L_k|$ and estimate $\sigma_{k_b}$ by $\sigma_{k_b} = (\sum_{y \in L_k} (y_{i,b} - \mu_{k_b})^2)/|L_k|$, where $L_k$ is the set of pixels labeled $k$ and
\(|L_k|\) is the pixel number in \(L_k\).

e) Based on the initial labels and parameters, perform the single-band HMRF-FCM on the wavelet energy image to get the clustering result \(X_{1b} = [x_{1b,1}, x_{1b,2}, \cdots, x_{1b,N}]^T\) on this band.

**Step 2. Perform the single-band HMRF-FCM on the primary image data.** For each spectral band \(b\), do

a) Based on \(X_{1b}\) obtained in Step 1, estimate the parameters \(\mu_{b,k}\) by \(\mu_{b,k} = (\sum_{j \in L_k} y_{i,b})/|L_k|\) and estimate \(\sigma_{b,k}\) by \(\sigma_{b,k} = (\sum_{j \in L_k} (y_{i,b} - \mu_{b,k})^2)/|L_k|\).

b) Perform the single-band HMRF-FCM on the \(b^{th}\) band of image \(y\) to obtain the clustering result \(X_{2b}\).

**Step 3. Fuse the clustering results on all bands to get the final labeled image \(X\) by exploiting the same fusion method as AHFCM.**

**Experimental Results**

In this section, in order to experimentally evaluate our proposed algorithm, we describe the experimental results on a piece of noisy image and a piece of remote sensing image. In addition, we test and compare the accuracy of the proposed method with FLICM [6], HMRF-FCM [8], and AHFCM [14]. Because FLICM and HMRF-FCM need predefined number of clusters, we set their class number \(K\) as the same as the output of our proposed method. In this study, we use the overall accuracy \(oa\) and Kappa coefficient \(\kappa\) to indicate the performance of all algorithms. Because the AHFCM algorithm may have different number of clusters in the final result, we make a simple summation to compute these two indicators: *if a label area in the segmentation result does not maximum overlap one of the the label areas in the groundtruth image, it will be considered as a misclassification area.*

Let \(P_{ij}\) be the proportion of subjects that were assigned to the \(i^{th}\) class of the segmentation result and \(j^{th}\) class of the ground truth. Denote \(P_t = \sum_{i=1}^{K} P_{ij}\) and \(P_j = \sum_{i=1}^{K} P_{ij}\), then

\[
\begin{align*}
\kappa &= \frac{\sum_{i=1}^{K} P_{ii} - \sum_{i=1}^{K} P_{i}P_{i}}{1 - \sum_{i=1}^{K} P_{i}P_{i}}.
\end{align*}
\]

\[
\begin{align*}
oa &= \sum_{i=1}^{K} P_{ii}.
\end{align*}
\]

(12)

![Figure 1. Segmentation results on a noisy image: (a) Synthetic noisy image; (b) ground truth image; (c) the result of FLICM; (d) the result of HMRF-FCM; (e) the result of AHFCM; (f) the result of the proposed method.](image)

Our first experiment applies these algorithms on a piece of noisy image to demonstrate the robustness of the proposed method to noise. Fig. 1 (a) is a piece of 128 × 128 synthetic image which is obtained from the ground truth image Fig. 1(b) corrupted by the Gaussian noise (zero mean and
0.15 variance). The segmentation results by FLICM, HMRF-FCM, AHFCM and the proposed method are shown in Fig.1 (c)-(f). Fig. 1(f) only has 3 clusters in the final result, while the proposed method has 5 clusters, which equals to the cluster number in the ground truth image. It is obvious that the proposed method obtains the best result. The comparison between Fig. 1(d) and Fig. 1(f) implies that the proposed method has better initial clusters by the analysis of the wavelet energy histogram, which highly improves the segmentation performance. Besides, the comparison between Fig. 1(e) and Fig. 1(f) clearly verifies that the proposed method is more robust than AHFCM because of the spatial information incorporated by the HMRF model. Both $\alpha$ and $\kappa$ recorded in Table 1 shows the same result.

Table 1. Comparison between different algorithms.

<table>
<thead>
<tr>
<th>Image</th>
<th>FLICM $\alpha$</th>
<th>FLICM $\kappa$</th>
<th>HMRF-FCM $\alpha$</th>
<th>HMRF-FCM $\kappa$</th>
<th>AHFCM $\alpha$</th>
<th>AHFCM $\kappa$</th>
<th>Our method $\alpha$</th>
<th>Our method $\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1(a)</td>
<td>0.5523</td>
<td>0.4405</td>
<td>0.7333</td>
<td>0.6667</td>
<td>0.5114</td>
<td>0.4127</td>
<td>0.9658</td>
<td>0.9572</td>
</tr>
<tr>
<td>Fig. 2(a)</td>
<td>0.5321</td>
<td>0.4534</td>
<td>0.6733</td>
<td>0.6167</td>
<td>0.505</td>
<td>0.4571</td>
<td>0.7624</td>
<td>0.6573</td>
</tr>
</tbody>
</table>

Figure 2. Segmentation results on a remote sensing image: (a) Synthetic noisy image; (b) ground truth image; (c) the result of FLICM; (d) the result of HMRF-FCM; (e) the result of AHFCM; (f) the result of the proposed method.

The second experiment is conducted on a piece of remote sensing image shown in Fig. 2(a) with the size of $910 \times 512$, which was acquired by an unmanned aerial vehicle of Phantom 4 Advanced on Nov. 15, 2016. It includes three different land cover types, namely wheat area, farmland ridges, and road. In this image, wheat area is rich of textures, while farmland ridge has very similar spectral response to the road, which highly increases the difficulty of clustering. Fig. 2(b) is the ground truth image, and Fig. 2(c)-(f) are the segmentation results FLICM, HMRF-FCM, AHFCM, and the proposed method, respectively. Fig. 2(f) has 3 clusters. Hence, $K = 3$ is set for FLICM and HMRF-FCM. While AHFCM has 8 clusters in Fig. 2(e), which includes some very tiny ones. It is clear that FLICM, HMRF-FCM and AHFCM have very serious misclassifications, while the proposed method is relatively better. It is mainly because of two reasons. Firstly, by making use of
spatial information, the proposed method can produce segmentation result with more homogeneous regions than AHFCM. Besides, by the determination of cluster number on each band independently, spectral bands with weaker distinguishability will be eliminated from the final clustering procedure. From Table 1, we can draw a same conclusion.

All algorithms are executed on Intel i7-3770, 3.4GHZ in Matlab. According to the test images, the proposed method used more time than FLICM and AHFCM. However, it is very near to HMRF-FCM. Comparison of the average running time of the algorithms is listed in Table 2.

Table 2. Comparison of the average running time of the algorithms in seconds.

<table>
<thead>
<tr>
<th></th>
<th>FLICM</th>
<th>HMRF-FCM</th>
<th>AHFCM</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1(a)</td>
<td>3.1205</td>
<td>22.7819</td>
<td>2.1026</td>
<td>9.0273</td>
</tr>
<tr>
<td>Fig. 2(a)</td>
<td>30.7216</td>
<td>425.2809</td>
<td>14.5153</td>
<td>437.7864</td>
</tr>
<tr>
<td>Average</td>
<td>16.9211</td>
<td>224.0314</td>
<td>8.3089</td>
<td>223.4069</td>
</tr>
</tbody>
</table>

Conclusion

In this study, we proposed a novel automatic fuzzy image clustering algorithm, which combines the merits of HMRF-FCM and AHFCM. It exploits the wavelet energy histogram to estimate the initial number of clusters and the initial centers. Thus our proposed method has more robust initial values than the competitors. Besides, the band-wise fuzzy clustering in the framework of HMRF-FCM considers more spatial information than AHFCM as well as making full use of the different distinguishability of different spectral values. Experimental results on both synthetic and remote sensing images have demonstrated the superiority of the proposed method.

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