Uncertain Systems Synchronization Control Based on Robust Adaptive

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Keywords: Nonlinear, Robust control, Adaptive control, Uncertainty.

Abstract. The uncertain systems with robust adaptive control algorithm is proposed in this paper. The design strategy of the algorithm lies in using the boundedness of chaotic systems. The unknown information of driven system and response system is given uncertainty range by the hypothesis presence bounded function. In order to simplify the design of adaptive PID composite synchronous rule and use the robust control method to deal with uncertainty, the robust adaptive controller is designed to complete synchronization. It can be seen from the simulation that chaotic systems can achieve synchronization and eliminate the vibration phenomenon at the same time.

Introduction

Robust control is a strong measure to deal with uncertain problems, and the measure has been sufficiently developed. Robust control and self-adaptive control is used by many systems at once and they are interdependent. Good results can not be acquired if robust control or self-adaptive control is used alone. Based on Lyapunov function method and model reference adaptive control theory, Hou Yan-ze put forward and analyzed a robust adaptive switching control program that a controller has dead zone nonlinearity for uncertain switching systems whose switching sign is dependent on independent decision variable, and the simulation results indicate that the system can track the expected track quickly. Based on the passivity method, Ke Hai-sen put forward a saturated robust adaptive controller for uncertain and non-holonomic mobile robots that satisfy matched condition, and the controller need not know the upper bound of uncertain interference. Guan Xin-ping researched the robust adaptive synchronization problem of two systems that exist interference. The method can effectively overcome the damage that is caused by uncertainty, and the method has a good effect to synchronization. A numerical calculation is conducted of Lorenz system, and the result of the numerical calculation shows the effectiveness of the method. Wang Hong-wei use Lyapunov stability theory to confirm weight updating rules of orthogonal neural network controller, the error of weight and track is guaranteed bounded [1-3]. Based on Chebyshev orthogonal neural network, an uncertain chaotic system robust adaptive synchronized method is come up. He method can effectively overcome the damage that is caused by uncertainty, and the method has a good effect to synchronization.

Through the research on adaptive PID synchronous control law, in view of the complexity that the uncertainty hypothesis brings to control law design, the design of the adaptive PID composite synchronous rule is simplified further. Considering to abstract simplify model uncertainty, in order to use the robust control method to deal with uncertainty. So, in view of the model uncertainty, the assumption of relative error state first-order bounded is put forward. And because of the boundedness of chaotic systems, the assumption is met easily by general chaos system [4-7].

Problem Description

Considering the following driver system and response system, where parameters of response system is known and there exists unknown parameters and uncertain nonlinear function in driven system.

The driven system can be written as
\[ \dot{x} = f_x(x) + F_x(x)\theta + \Delta(x,t) \]  

(1)

The response system can be written as

\[ \dot{y} = f_y(y) + bu \]  

(2)

Take a three dimension system as an example, the main driven system can be written as

\[ \dot{x}_1 = f_{x1}(x_1, \ldots, x_4) + \sum_{j=1}^{p_1} F_{x1j}(x_1, \ldots, x_4)\theta_{x1j} + \sum_{j=1}^{q_1} \Delta_{x1j}(x,t) \]  

(3)

\[ \dot{x}_2 = f_{x2}(x_1, \ldots, x_4) + \sum_{j=1}^{p_2} F_{x2j}(x_1, \ldots, x_4)\theta_{x2j} + \sum_{j=1}^{q_2} \Delta_{x2j}(x,t) \]  

(4)

\[ \dot{x}_3 = f_{x3}(x_1, \ldots, x_4) + \sum_{j=1}^{p_3} F_{x3j}(x_1, \ldots, x_4)\theta_{x3j} + \sum_{j=1}^{q_3} \Delta_{x3j}(x,t) \]  

(5)

And the slave response system can be written as

\[ \dot{y}_1 = f_{y1}(y_1, \ldots, y_4) + b_1u_1 \]  

(6)

\[ \dot{y}_2 = f_{y2}(y_1, \ldots, y_4) + b_2u_2 \]  

(7)

\[ \dot{y}_3 = f_{y3}(y_1, \ldots, y_4) + b_3u_3 \]  

(8)

When \( \theta \) is unknown parameter, and the number of unknown parameter is \( \sum_{i=1}^{n} p_i \), and the number of uncertain nonlinear function is \( \sum_{i=1}^{n} q_i \), \( b_i \) is a known constant [8-10].

So the robust adaptive control target for chaotic system with unknown parameter and uncertain nonlinear function is too complex, so the design of robust adaptive synchronization need benefit the boundedness of chaotic system. And use a bounded function to describe the uncertain region of the action is to design the control \( u = u(x, y, \hat{r}_y), \hat{r}_y = f(z_1, z_2, z_3) \) such that the state of slave system can trace state of main system, such as \( y \rightarrow x \).

Considering the above adaptive e unknown information of driven system and response system, then design a robust adaptive controller to synchronize two systems[11-14].

Assumption

Two assumptions are built for the above system to simplify the analysis.

Assumption 1: the driven system and response system have the same structure, it means that it has the same dimension.

Assumption 2: the nonlinear function satisfies the below conditions, for \( 1 \leq i \leq n, 1 \leq j \leq p_2 \), there exists a unknown positive constant \( r_j \leq d_j \) such that

\[ f_y(y_1, \ldots, y_4) - f_y(x_1, \ldots, x_4) - \sum_{j=1}^{p_2} F_{yj}(x_1, \ldots, x_4)\theta_{yj} - \sum_{j=1}^{q_2} \Delta_{yj}(x,t) \leq r_i|z_i| + r_0|z_0| + r_0|z_0| \]  

(9)

where \( d_j \) is a known constant. Because the chaotic system is bounded, so it is easy to be satisfied for many chaotic systems[15-17].
Robust Adaptive Control Law Design

Define the error variable as \( z_i = y_i - x_i \), where the error system can be written as

\[
\dot{z}_i = f_{yi}(y_1, \cdots, y_4) - f_{xi}(x_1, \cdots, x_4) - \sum_{j=1}^{p_i} F_{yj}(x_1, \cdots, x_4)\theta_{yj} - \sum_{j=1}^{p_i} \Delta_{yj}(x, t) + b_i u_i
\]  

(10)

Then it also has

\[
z_i \dot{z}_i \leq r_1 \|z_i\| + r_2 \|\dot{z}_i\| + r_3 \|z_i\| + z_i b_i u_i
\]  

(11)

Use the adaptive method to design the control \( u_i \) as

\[
u_i = b_i^{-1} [\hat{r}_1 \|z_i\| + \hat{r}_2 \|\dot{z}_i\| + \hat{r}_3 \|z_i\|] \text{sgn}(z_i)
\]  

(12)

The error of assumption is defined as

\[
\hat{r}_{ij} = r_{ij} - \hat{r}_{ij}
\]  

(13)

Solve its derivative as

\[
\dot{\hat{r}}_{ij} = -\dot{\hat{r}}_{ij}
\]  

(14)

Where the parameter adaptive law is designed as

\[
\dot{\hat{r}}_{ij} = -\|z_i z_j\|
\]  

(15)

Choose the Lyapunov function as

\[
V_i = \sum_{i=1}^{n} z_i^2 + \sum_{j=1}^{p_i} \hat{r}_{ij}
\]  

(16)

It is easy to get

\[
\dot{V}_i \leq 0
\]  

(17)

So the system is stable and synchronization is fulfilled [18-21].

Numerical Simulation

Also use a three dimension system as an example

\[
\dot{x}_1 = a(x_2 - x_1) + k_{ib} x_3 \cos x_2
\]  

(18)

\[
\dot{x}_2 = bx_1 + cx_2 - x_1 x_3 + k_{ib} x_3 \cos x_2
\]  

(19)

\[
\dot{x}_3 = x_2^2 - hx_3 + k_{ib} (1 + \sin(x_2 x_3)) x_2
\]  

(20)

If \( a = 20, b = 14, c = 10.6, h = 2.8, k_{ib} = 0 \), the system has an attractor. Set \( a, b, c, h \) as unknown parameters as system and \( k_{ib} \) as coefficient of nonlinear function. The structure of response system is known as follows:

\[
\dot{y}_1 = a_y (y_2 - y_1) + u_1
\]  

(21)
\[ \dot{y}_2 = b_y y_1 - k_y y_1 y_3 + u_2 \]  \quad (22)

\[ \dot{y}_3 = -c_y y_3 + h_y y_1^2 + u_3 \]  \quad (23)

Choose parameter as \((a_y, b_y, c_y, k_y, h_y) = (10, 40, 2.5, 1, 4)\), and the initial state of response system is \((y_1, y_2, y_3) = (1, -1, 2)\), use above robust adaptive strategy, the simulation result without considering the nonlinear functions is as follows:

So we can make a conclusion that the synchronization can be fulfilled and oscillation can be reduced. And the disadvantage is that the synchronization error is not easy to eliminate. And the advantage is that the structure of system is not necessary to know accurately. And the synchronization of the system can be realized no matter there are nonlinear function or not.

**Conclusion**

The uncertain chaotic systems with robust adaptive synchronization algorithm is proposed in this paper. The design strategy of the algorithm lies in using the boundedness of chaotic systems. The unknown information of driven system and response system is given uncertainty range by the hypothesis presence bounded function. The robust adaptive controller is designed to complete synchronization. At last, it can be seen from the simulation that chaotic systems can achieve synchronization and eliminate the vibration phenomenon at the same time under the condition that control system need not be known precisely. But the precision of synchronization is limited.
References


