2D-DOA Estimation Based on MSCS of Multi-FH Signal
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Keywords: Frequency-Hopping signal, 2D-DOA, Network sorting, Half-spectrum searching.

Abstract. A two dimensional direction of arrival (2D-DOA) estimation algorithm based on MSCS is proposed by using space-time-frequency analyzing method. Firstly, the spatial time-frequency (TF) matrix of each hop is constructed by using the time-frequency domain features of FH signals; Then the dimension of the noise subspace is descended by the singular value decomposition on the intersection of noise subspace and its conjugate one; Finally efficient DOA estimation is realized through half-spectrum searching. Theoretical analysis and simulation results show that this algorithm has good effectiveness and estimation performance.

Introduction

Frequency-hopping signals have been widely used in aerospace communications field because of their characteristics of good security, low probability of interception and strong networking capability.

The existing studies mainly focus on the detection of FH signal and the estimation of relevant parameters, which has little research on the spatial information of FH signal. In [1], [2], a space-time model was proposed to estimate the DOA of frequency hopping signals, which requires a known number of sources; In [3], [4], root-music approach is proposed to replace the MUSIC algorithm, which reduce the complexity of the MUSIC algorithm, but the algorithm has high demand for the array and is not suitable for engineering applications; The algorithm proposed in [1-4] only concerned with one-dimensional DOA information, without considering the 2D-DOA problem. A spatial-polarimetric distributions and ESPRIT algorithm was proposed in[5],[6] to estimate 2D-DOA of FH signals, which has high precision in the array of parallel lines and “L” type linear array to estimate 2-DOA of multiple frequency hopping signals, but this algorithm requires high SNR, and the ESPRIT algorithm requires parameter matching, which increases the computational complexity and reduces the estimated performance;

A 2D-DOA estimation algorithm based on MSCS is proposed by using space-time-frequency analyzing method. The idea of conjugate subspace is introduced into the traditional MUSIC algorithm. Only the peak search in the half spectrum is needed to realize the estimation of the frequency hopping signal. The proposed algorithm can reduce the complexity by 50% while guaranteeing the estimation performance.

Snapshots Model of FH Signal

Assume that N FH signals impinge instantaneously onto the array shown in Figure 1. The element number of the uniform linear array ULA₁ and ULA₂ is M, the array element spacing is d₁, the array spacing is d₂.
FH signal is a wideband signal with random carrier frequency hopping, but it can be simplified as narrowband signal when studying a certain hop. Assume that the narrowband plane wave \( S \) inject on the array with the azimuth angle \( \theta \in [0, 2\pi] \) and the pitch angle \( \phi \in [0, \pi/2] \). The steering vector \( \mathbf{a}_S(\theta, \phi) \) of the subarray ULA\(_1\) to incident wave \( S \) can be expressed as:

\[
\mathbf{a}_S(\theta, \phi) = [1, e^{-j2\pi d_1 \sin \theta \cos \phi / \lambda}, \ldots, e^{-j2\pi (M-1)d_1 \sin \theta \cos \phi / \lambda}]^T
\]

where \( \lambda \) denotes the wavelength. Suppose that the steering vector of the subarray ULA\(_2\) to incident wave \( S \) is \( \mathbf{a}_{S_2}(\theta, \phi) \), so \( \mathbf{a}_{S_2}(\theta, \phi) \) can be defined as:

\[
\mathbf{a}_{S_2}(\theta, \phi) = [1, e^{-j2\pi (d_1 \sin \theta \cos \phi + d_2 \sin \theta \sin \phi) / \lambda}, \ldots, e^{-j2\pi (M-1)d_1 \sin \theta \cos \phi + d_2 \sin \theta \sin \phi) / \lambda}]^T
\]

The steering vector \( \mathbf{a}(\theta, \phi) \) of array can be expressed as :

\[
\mathbf{a}(\theta, \phi) = [\mathbf{a}_S(\theta, \phi), \mathbf{a}_{S_2}(\theta, \phi)]^T
\]

The array flow pattern matrix can be formulated as:

\[
\mathbf{A} = [\mathbf{a}_1(\theta, \phi), \mathbf{a}_2(\theta, \phi), \ldots, \mathbf{a}_N(\theta, \phi)]
\]

The snapshot vector model for array can be defined as:

\[
\mathbf{X}(t) = \mathbf{Y}(t) + \mathbf{N}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t)
\]

**Construction of Space-Time-Frequency Matrix of FH Signal**

The discrete cross-time-frequency distribution of signal \( x_i(t) \) and \( x_j(t) \) can be expressed as:

\[
\mathbf{D}_{x_i x_j}(t, f) = \sum_{\tau=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \phi(l, \tau) x_i(t + l + \tau) x_j^*(t + l - \tau) e^{-j4\pi f \tau}
\]

So the spatial-time-frequency distribution of signal \( x(t) \) is defined as:

\[
\mathbf{D}_{xx}(t, f) = \sum_{\tau=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \phi(l, \tau) X(t + l + \tau) X^H(t + l - \tau) e^{-j4\pi f \tau}
\]

where \( \phi(l, \tau) \) denotes the TF distribution between the output signals of each array. According to (5) and (7), the covariance matrix of TF domain can be written as:

\[
\mathbb{E}[\mathbf{D}_{xx}(t, f)] = \mathbb{E}[\mathbf{D}_{yy}(t, f)] + \mathbb{E}[\mathbf{D}_{NN}(t, f)] = \mathbf{A}\mathbf{D}_{S}(t, f)\mathbf{A}^H + \mathbb{E}[\mathbf{D}_{NN}(t, f)]
\]

\( \mathbb{E}[\mathbf{D}_{xx}(t, f)] \) has the similar structure with the uni-polar array time covariance matrix, and it has the same subspace characteristics as flow pattern matrix.
D-DOA Estimation Based on MSCS

Suppose that the 2D-DOA of incident wave S in the time-frequency domain is $(\hat{\theta}, \hat{\phi})$, we can get (9) from the orthogonal subspace principle:

$$a^H(\hat{\theta}, \hat{\phi})U_N = 0$$

(9)

From (9), $a^*(\hat{\theta}, \hat{\phi})$ is expressed as:

$$a^*(\hat{\theta}, \hat{\phi}) = a(\hat{\theta}, \hat{\phi} \pm \pi)$$

(10)

According to the relationship, we can obtain:

$$[a^H(\hat{\theta}, \hat{\phi})]^*U_N^* = [a(\hat{\theta}, \hat{\phi} \pm \pi)]^HU_N^* = 0$$

(11)

where $U_N^*$ denotes the conjugate of noise subspace $U_N$. From (11) we can see that there is a mirror source $S'$ in the time-frequency domain whose steering vector is $a^*(\hat{\theta}, \hat{\phi})$. If $\hat{\phi} \in [0, \pi]$, the 2D-DOA of $S'$ is $(\hat{\theta}, \hat{\phi} + \pi)$, otherwise the 2D-DOA of $S'$ is $(\hat{\theta}, \hat{\phi} - \pi)$. Therefore, the spatial spectrum function $P_{MSCS}(\theta, \phi)$ of MSCS algorithm can be defined as:

$$P_{MSCS}(\theta, \phi) = \frac{1}{a^H(\theta, \phi)U_N^*U_N^*U_N^*a(\theta, \phi)}$$

(12)

Let $U_N = [U_{N_1}, U_{N_2}, \cdots, U_{N_{M-L}}]$, we have:

$$P_{SCMUSIC}^{-1}(\theta, \phi) = \sum_{i=1}^{M-L} \sum_{j=1}^{M-L} [a^H(\theta, \phi)U_{N_i}^*[U_{N_j}^*U_{N_j}^*]U_{N_i}^*a(\theta, \phi)]$$

(13)

From (12) and (13), we have:

$$P_{MSCS}^{-1}(\theta, \phi) = 0 \quad ((\theta, \phi) = (\hat{\theta}, \hat{\phi}) \lor (\hat{\theta}, \hat{\phi} \pm \pi))$$

(14)

Therefore, the spatial spectrum function $P_{MSCS}(\theta, \phi)$ of the MSCS algorithm takes the extreme at $(\hat{\theta}, \hat{\phi})$ and $(\hat{\theta}, \hat{\phi} \pm \pi)$, so $P_{MSCS}(\theta, \phi)$ is the symmetric spatial spectrum.

According to (12), the half-spectral peak search is carried out in the $(\theta, \phi)$ domain to obtain the $(\hat{\theta}, \hat{\phi})$ or $(\hat{\theta}, \hat{\phi} \pm \pi)$, which makes the extreme of $P_{MSCS}(\theta)$.

Simulation and Analysis

Suppose that the array spacing $d_1$ of the two uniform linear arrays $ULA_1$, $ULA_2$ is 2.5m, and the array spacing $d_2$ is 3m; the 2D-DOA parameters of the four far-field FH signals ($FH1$–$FH4$) are $(20^\circ, 30^\circ)$, $(40^\circ, 50^\circ)$, $(60^\circ, 70^\circ)$, $(70^\circ, 60^\circ)$, respectively.

100 Monte Carlo experiments were performed to verify the performance of the proposed algorithm, the root mean square error of 2D-DOA, and estimated success rate were used as the performance criterion. The root mean square error (RMSE) of 2D-DOA is defined as:

$$RMSE = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (\hat{\theta}_i - \theta_i)^2}$$

(15)

where $L$ denotes the source number, $\hat{\theta}_i$ and $\theta_i$ denote the estimated and true azimuth angle of i-th source respectively and the RMSE of the pitch angle is calculated in the same way.
Assume that the number of the snapshots of each hop is 2000 and the number of array element ULA$_1$ and ULA$_2$ is 4. Figure 2 and Figure 3 show the performance comparison of 2D-DOA estimation in MSCS algorithm and MUSIC algorithm when SNR increase from -10dB to 30dB.

![Figure 2. Estimated success rate in experiment 2.](image1)

![Figure 3. RMSE of parameters in experiment 2.](image2)

It can be seen from Experiment 2 that with the increase of SNR, the estimated success rate of azimuth angle and pitch angle of increases gradually and the RMSE decreases gradually in both MSCS algorithm and MUSIC algorithm; When SNR reaches about 10dB, the estimated success rate of the two algorithms is 100%; In general, the estimated success rate of MSCS algorithm is slightly higher than the MUSIC algorithm.

The time required for the 2D-DOA estimation of the two algorithms when SNR increase from -8dB to 20dB is shown in Table 1.

<table>
<thead>
<tr>
<th>Algorithm type</th>
<th>-8dB</th>
<th>-4dB</th>
<th>0dB</th>
<th>4dB</th>
<th>8dB</th>
<th>12dB</th>
<th>16dB</th>
<th>20dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUSIC</td>
<td>20.463</td>
<td>0.582</td>
<td>20.393</td>
<td>20.460</td>
<td>20.498</td>
<td>20.359</td>
<td>20.642</td>
<td>20.463</td>
</tr>
</tbody>
</table>

It can be seen from Table 1 that the time required for the 2D-DOA estimation of the MSCS algorithm is about 10.4s, while the MUSIC needs 20.4s. Therefore, the complexity of the MUSIC algorithm can be reduced to half of that.

Conclusions

The spatial information of the frequency hopping signal can effectively assist the multi-frequency hopping signal network sorting, and the 2D-DOA estimation of the multi-frequency hopping signal has the important significance. An efficient 2D-DOA estimation algorithm for multi-FH signals based on space-time-frequency analysis and MSCS is deduced and elaborated in this paper. Theoretical analysis and simulation results show that the proposed algorithm has good effectiveness and estimation performance.

Acknowledgement

This research was financially supported by the National Science Foundation of China under Grant 64601500.

References


