Wavelet Autocorrelation and Denoising of Spread Spectrum Signal for Narrowband Wireless Indoor Positioning

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Abstract. Channel noise is a key factor that affects the measurement accuracy of narrowband wireless indoor positioning. To reduce noise effects, it is often required to effectively filter the received spread spectrum signals. However, current filtering methods in the time or frequency domain can only filter high frequency band noise, making the measurement accuracy low. In this paper, a low frequency band noise reduction algorithm based on the threshold segmentation of the rectangular window is proposed. The method is based on thresholding on time-frequency wavelet domain by variance analysis and signal despreading is achieved by wavelet autocorrelation at low frequency band after denoising with reference to locally generated spread spectrum signal without added noise at the same subband. Simulation results show that compared with other filtering methods, the proposed method by wavelet autocorrelation and denoising can eliminate the noise effects at the receiver, leading to improving the positioning accuracy significantly.

Introduction

The demand for indoor high precision positioning has increased rapidly in many public services. In particular, centimeter-level precision positioning technology based on microwave transmission has been sought for the past years to fit in a rising number of applications such as production process to locate robots. However, current indoor positioning systems with either short-range positioning technology based on wireless local area networks (WLAN) and ZigBee achieving meter-level accuracy, or ultra-wide band (UWB) and 60 GHz communication technology achieving high precision but with high cost required, could not meet the need of indoor wireless positioning [1,2,3,4]. Current methods based on measuring microwave transmission time to calculate the distance between the base station and the transmitter, use triangulation algorithm to determine the location of the transmitter. The accuracy of measured location is dependent on the timing measurement. Higher accuracy of timing measurement results in higher accuracy of positioning. Spread spectrum technique is often employed to achieve high precision positioning by high accuracy of timing measurement and anti-jam characteristics. However, due to strong noise effects, current indoor positioning by narrowband spread spectrum technique can only achieve low precision, where various noise and interference degrade the performance of timing measurement [5]. High precision of timing measurement is possible if spreading code is very long, but leading to very high sampling rate and high hardware cost. This can make the implementation of positioning system difficult and limit its applications [6]. As phase measurement is more sensitive to the distance of microwave transmission than timing measurement, indoor positioning system can measure coherent phase [7] by autocorrelation of received spread spectrum signals from base stations, to accurately calculate the time difference of arrival. The sampling rate required here can be low thus leading to low implementation cost. To overcome the limited accuracy of timing measurement by relatively short spreading code for indoor positioning, we propose wavelet autocorrelation phase measurement of spread spectrum signal transmitted by carrier frequency lower than 1 GHz with low cost requirement to achieve high precision.
**System Model**

The proposed positioning system is built based on the time difference of arrival (TDOA) positioning principle [8], BPSK modulation [9], and autocorrelation of spread spectrum (DSSS) technique [10], and can be implemented by the following procedure (as shown in Fig.1):

![UHF band narrowband wireless positioning system flow chart.](image)

The spreading code is denoted by \( m(t) \), let \( \cos(2\pi f_1 t) \) be the carrier signal with carrier frequency \( f_1 \) (intermediate frequency), we obtain the signal by BPSK modulation:

\[
s(t) = m(t)\cos(2\pi f_1 t + \theta)
\]  

(1)

Signals \( s(t) \) is upconverted to the transmitting carrier frequency under 1 GHz and additive white Gaussian noise (AWGN) \( n(t) \) would apply to the channel when \( s(t) \) is transmitted to the base stations. The distances between the transmitter and the base stations are normally different, so the times of arrival of transmitting signal to the base stations are different. Suppose the times of arrival of base stations 1, 2, 3 are \( t_1, t_2, t_3 \) respectively, we have:

\[
\begin{align*}
    s_1(t + t_1) &= m(t + t_1)\cos[2\pi f_1(t + t_1) + \theta] + n_1(t + t_1) \\
    s_2(t + t_2) &= m(t + t_2)\cos[2\pi f_1(t + t_2) + \theta] + n_2(t + t_2) \\
    s_3(t + t_3) &= m(t + t_3)\cos[2\pi f_1(t + t_3) + \theta] + n_3(t + t_3)
\end{align*}
\]  

(2)

Autocorrelation of the Eq. 2, can be obtained:

\[
\begin{align*}
    R_{xx1} &= \lim_{T \to \infty} \frac{1}{T} \int_0^T s_1(t + t_1)s(t)dt \\
    R_{xx2} &= \lim_{T \to \infty} \frac{1}{T} \int_0^T s_2(t + t_2)s(t)dt \\
    R_{xx3} &= \lim_{T \to \infty} \frac{1}{T} \int_0^T s_3(t + t_3)s(t)dt
\end{align*}
\]  

(3)

The autocorrelation operation can be performed by fast Fourier transform (FFT) to obtain the complex valued autocorrelation peak. Assume that the peak location of the autocorrelation is \( k \), and the peak value is \( y \) that can be calculated from real and imaginary parts \( a_k, b_k \) as:

\[
y = \sqrt{a_k^2 + b_k^2}
\]  

(4)
Let \( \theta_{k1}, \theta_{k2}, \theta_{k3} \) be the arc tangent of the ratio of the real and imaginary parts calculated from three peaks of autocorrelation:

\[
\begin{align*}
\theta_{k1} &= \tan^{-1} \frac{a_{k1}}{b_{k1}} \\
\theta_{k2} &= \tan^{-1} \frac{a_{k2}}{b_{k2}} \\
\theta_{k3} &= \tan^{-1} \frac{a_{k3}}{b_{k3}}
\end{align*}
\]  

According to the linear relationship between the phase and time, we have:

\[
\begin{align*}
\Delta\tau_{21} &= t_2 - t_1 = \frac{\theta_{k2} - \theta_{k1}}{2\pi f_1} \\
\Delta\tau_{31} &= t_3 - t_1 = \frac{\theta_{k3} - \theta_{k1}}{2\pi f_1} \\
\Delta\tau_{32} &= t_3 - t_2 = \frac{\theta_{k3} - \theta_{k2}}{2\pi f_1}
\end{align*}
\]  

(5)

Then TDOA location method can be employed to estimate the position of the transmitter. An example of positioning by BPSK modulation and 8\( f_1 \) sampling frequency was performed with spreading code length 511 and Gaussian noise from -15 to 30dB added to the signal. The result is shown in Fig.2.

![Figure 2](image)

Figure 2. Influence of gaussian white noise on positioning accuracy at 8\( f_1 \) sampling frequency.

It can be seen that at high SNR, very high accuracy of positioning can be achieved. But at low SNR, positioning accuracy decreased rapidly, which shows that the influence of Gaussian noise on the positioning accuracy is significant. In order to improve the accuracy, effective filtering is required.

**Time-Frequency Wavelet Denoising**

Reducing noise of the received signal is a complicated task. The classic strategy is based on the theory of Fourier transform. Specifically, in order to achieve the goal of denoising, we transform the original noisy signal of time domain into frequency domain and filter the components that are thought as noise. However, in the view of frequency analysis, we map the signal from time domain to frequency domain such that several properties of signal in time domain will be ignored. In that case, we will lose the advantage of local analysis since the time domain and frequency domain cannot be observed at the same time. Many advanced methods were proposed to address this problem such as windowed
Fourier transform and wavelet analysis. Short time Fourier transform (STFT) has the ability of local analysis but unfortunately its resolution of analysis will be fixed once the window function has been chosen. This limitation will be removed when we take the wavelet analysis into consideration. The properties of signal in time domain will still be reserved after wavelet transformation and furthermore arbitrary details observed rely on the advantage of time-frequency localization property of wavelet analysis in not only time domain but also frequency domain [11]. By wavelet decomposition, multiscale analysis on the signals with the operators of shrinkage and translation and more information can be exploited efficiently [12]. After wavelet transformation on the noisy signal, the discriminants of the coefficients of noise and pure signal are obvious in different scales, which is the mechanism behind the wavelet denoising algorithm. Specifically we can cut some or even all of the noisy coefficients and retain the desired coefficients as many as possible at the same time and obtain the denoised signal in time domain after performing inverse transform on the processed wavelet coefficients.

Typical wavelet denoising methods include modulus maximum method, correlation denoising method, wavelet shrink threshold denoising method and translation invariant denoising method [13,14,15,16]. The threshold denoising method is the most popular method since it has the best performance on the suppression of Gaussian noise and the features of the pure signal can also be well reserved [17,18]. The key of wavelet threshold denoising method is the selection of appropriate threshold value. If the threshold is higher, not only the noise but also some useful information will be filtered. On the other hand, if the threshold is lower, we cannot suppress enough noise such that bad performance will be obtained. So many methods are proposed to select the threshold values in the last decades such as Sqtwolog, Maximin, Stein and Heursure [19,20,21,22,23,24]. In general, most of the methods proposed are based on the global threshold selection. However, the drawbacks are obvious: with the threshold value that calculated based on global scale, many local features of the signal cannot be captured. When the issues of over-denoising and under-denoising on the local segments come up, that may result in an unsatisfied reconstructed signal. In order to tackle this problem, Reference [25] gave an idea of wavelet denoising based on entropy theory, where different threshold values of the segments were given based on the ratio of their entropy values with the smallest one. The threshold value is computed with Sqtwolog. The method can solve the problems faced by global threshold methods, but still cannot satisfy the denoising performance of indoor high precision positioning system by UHF band narrowband spread spectrum communication. In this paper, we propose wavelet denoising algorithm by autocorrelation and thresholding at low frequency band based on threshold segmentation of rectangular window.

Wavelet Autocorrelation and Denoising

Wavelet Autocorrelation at Low Frequency Band

Wavelet coefficients at low frequency band usually contain less noise but more signal energy, thus can be taken to perform autocorrelation for despreading. In the wavelet domain, coefficients are processed as follows:

1. Wavelet decomposition at one level. At the low band, the frequency range should cover and meet the requirements of the Nyquist sampling theorem as:

\[ f_s/4 > 2f_1 \]  

(7)

where \( f_s \) is the sampling frequency, \( f_1 \) is the carrier frequency of the signal.

2. Obtain the low level wavelet coefficients \([w_0], [w_1]\) from the local signal and the received noisy spread spectrum signal respectively. So wavelet autocorrelation is performed at low frequency band as follows:

\[ R = ifft[fft[w_0] \ast \text{conj}(fft[w_1])] \]  

(8)
And the time of arrival of transmitting signal to the base station can be calculated from real and imaginary parts at the peak location \(k\) of the autocorrelation as:

\[
t = \frac{\tan^{-1} \frac{\text{real}(R_k)}{\text{imag}(R_k)}}{2\pi f_1}
\]  

(9)

(3) Following the principle of TDOA positioning method derived, the position estimation is performed by calculating the arrival time differences given in Eq. 6.

Fig. 3 shows the comparison of indoor positioning results with SNR ranging from 0 to 30 dB, where signals are denoised by wavelet heursure threshold, reference [25] wavelet segmentation threshold, and are processed by wavelet autocorrelation at low frequency band respectively.

![Figure 3. Comparison of wavelet denoising with other algorithms.](image)

It is noted that the operation of wavelet autocorrelation at low frequency band for signal despreading achieves similar results as the wavelet heursure threshold processing and wavelet segmentation threshold. However, as autocorrelation operation is performed in the wavelet domain, it is possible at the same time to further reduce noise at low frequency band with reference to locally generated spreading signal, leading to improving positioning accuracy.

**Wavelet Denoising at Low Frequency Band**

By discarding high frequency wavelet coefficients (with oversampling) and performing wavelet autocorrelation at low frequency band, noise effects are significantly reduced. To further reduce noise at low frequency band, wavelet coefficients can be decomposed to lower subbands in the wavelet domain, where the high and low frequency wavelet coefficients are thresholded by calculating the variance of the rectangular windowing coefficients. The flow chart of denoising process is shown as follows (Fig. 4).

(1) Because of the phase calculation from complex signal, biorthogonal wavelets with linear phase are used to decompose and reconstruct the wavelet coefficients at low frequency band \([w_0]\) and \([w_1]\) of the local signal and the received noisy signal respectively.
(2) Select the appropriate window length $L_w$ as well as the moving window length $L_{mw}$, and then the high-frequency wavelet coefficients $[H_{lsk}]$ and $[H_{nsk}]$ are divided into $m$ segments.

(3) Take the $k$th segment and calculate the variance of it:

$$V_{lsk} = \text{var}( [H_{lsk}] )$$
$$V_{nsk} = \text{var}( [H_{nsk}] )$$

where $[H_{lsk}]$ and $[H_{nsk}]$ are the wavelet coefficients of the $k$th segment after segmentation, $V_{lsk}$ and $V_{nsk}$ are the $k$th segment variance of high frequency wavelet coefficients from the local signal and the noisy signal respectively.

(4) The threshold $\lambda_k$ for the $k$th segment is obtained from the comparison of the two variances as follows:

$$\lambda_k = \begin{cases} 0, & V_{lsk} \geq V_{nsk} \\ \sigma \sqrt{2 \log L_w} \cdot \frac{V_{lsk}}{V_{nsk}}, & V_{lsk} < V_{nsk} \end{cases}$$

where $L_w$ is the length of the rectangular window, $\sigma$ is the noise variance and is given by:

$$\sigma = \frac{\text{median}([H_{ns}])}{0.6745}$$

where $\text{median}( [H_{ns}] )$ is the median of high frequency wavelet coefficients.
(5) The kth segment is processed using a soft threshold based on the resulting threshold. The soft thresholding is given by:

\[
\delta^s_\lambda = \begin{cases} 
0, & |[H_{nsk}]| \leq \lambda_k \\
\text{sign}([H_{nsk}])(|[H_{nsk}]| - \lambda_k), & |[H_{nsk}]| > \lambda_k
\end{cases}
\]  

(14)

(6) After thresholding, compare the two variances again from step (3) and see if the threshold \( \lambda_k \) needs to be updated from Eq. 12. If yes, then repeat steps (4), (5); otherwise, go to step (7).

(7) The processed kth segment is stored with \([NH_{ns}]\). Compare the value of \( k \) with \( m \), if less than \( m \), then \( k = k + 1 \) and repeat steps (3), (4), (5), (6); otherwise, go to step (8).

(8) Obtain the new high frequency coefficients \([NH_{ns}]\) after processing, and then the new low frequency wavelet coefficients \([NL_{ns}]\) in the same way.

(9) The thresholded high and low frequency coefficients are reconstructed back to low frequency wavelet coefficients.

From experiments conducted, it is noted that the window length has a great influence on the threshold value. The experimental results shown in Table 1 indicate that the shorter the window length, the higher the positioning accuracy. But when window length is very short such as 8 and 4, the impact on the positioning accuracy is no longer noticeable. As the smaller the window length, the greater the computation, the window length 8 is used for experiments. Test results also illustrate that the moving window length can be chosen slightly shorter than the length of the rectangular window. The moving window length chosen for the experiments is two units smaller than the window length.

| Window length | 63   | 31   | 15   | 7    | 3
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<tr>
<td>Average positioning error(cm)</td>
<td>40.42</td>
<td>38.79</td>
<td>37.24</td>
<td>35.66</td>
<td>35.62</td>
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**Simulation Results and Discussions**

Theoretical analysis shows that the derived wavelet autocorrelation combined with denoising at low frequency band can achieve better performance for wireless indoor positioning. To test the proposed method for achieving higher positioning accuracy, four methods are simulated by the wireless channel with SNR in the range of -15dB to 30dB, and the influence of different methods on the positioning accuracy and computing time consumption is compared. The four methods are no filtering, wavelet heursure threshold filtering, [25] wavelet segmentation threshold filtering and the proposed wavelet autocorrelation and denoising. Simulation results are shown and compared in Fig.5 and Fig.6. It can be seen that when the SNR is greater than 5 dB, all filtering methods can improve positioning accuracy. However, with the decrease of SNR (stronger noise), the traditional wavelet threshold filtering and the reference [25] wavelet threshold method cannot achieve significant improvement of positioning accuracy. However the proposed method can still improve the positioning accuracy significantly at very low SNR.
To evaluate the computational complexity, the average computing time of four methods is recorded in Table 2. It includes general signal processing for positioning and noise reduction processing. Computing time consumption shows that the proposed method consumes a little more time than the other methods, mainly because of the window and moving window processing for thresholding at the low frequency band. However, the proposed method outperforms other methods by improving the positioning accuracy significantly.
Conclusions

This paper proposes a new algorithm based on the threshold filtering of the rectangular window in the wavelet domain for narrowband wireless indoor positioning by spread spectrum communication. The proposed denoising method is based on thresholding on time-frequency domain by variance analysis and signal despreadening is achieved by wavelet autocorrelation at low frequency band after denoising with reference to locally generated spread spectrum signal without added noise at the same subband. Simulation results show that compared with other filtering methods, the derived wavelet autocorrelation combined with denoising at low frequency band can eliminate the noise effects at the receiver, leading to improving the positioning accuracy significantly.

References


