An Analysis of Signals De-noising by Using CEEMDAN

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Abstract. Dynamical properties of mechanical systems can be obtained with the vibration signals from the systems. However, noises makes it difficult to accurately acquire the features of systems. Therefore, de-noising operation is significant step for analysis of vibration signal in the practical engineering. In order to resolve this problem, the Complementary Ensemble Empirical Mode Decomposition (CEEMDAN) method is introduced to try to eliminate noises from the analyzed signal. At first, we illustrate the theory of the method. On this base, the signal, which contains several harmonic components with white noise, is constructed, and processed by CEEMDAN. As the result shown, the method can effectively remove noise; moreover, the harmonic components can be accurately separated. And these indicate that the CEEMDAN is an effective method for the de-noising.

Introduction

Vibration signals are important data for the analysis to the dynamic features of the mechanical systems. But, the signal is easily polluted by random noises in the collection process, which can result in reducing the analysis precision and accuracy. At present, de-nosing processing is a key point of signal processing, and the scholars, at home and abroad, were working in this. At present, many transforms are proposed, such as Fourier transform, wavelet transform [1,2], short time Fourier transform, nonstationary Gabor transform [3,4] and frame transform [5] and other transform. On the basic of these transforms, a number of investigation about de-noising are conducted. However, the properties of signals were not taken into consideration in these methods; therefore, the result is not promising sometimes.

In the end of 20th century, Empirical Mode Decomposition (EMD) was proposed by Huang [6], based on the Hilbert transform, in which, intrinsic mode function (IMF) is taken as the decomposition unit. This method is the general expression form for the decomposition operation based on the inner product. As driven by the data itself, the universality of algorithm is excellent. However, for the influence of the modal aliasing, the application range is limited. The property that all frequencies are contained in white noise is masterly utilized by Wu, and the Ensemble Empirical Mode Decomposition was proposed in 2008 [7]. The method can reduce modal aliasing, which expand the application scope of the EMD, and improve the de-noising effect. Torres, etc. proposed complete Ensemble Empirical Mode decomposition with adaptive noise (CEEMDAN), which can reduce number of IMFs, decreasing computation amount [8]. Therefore, we introduce CEEMDAN to do a try for de-noising operation for signal with white noises.

The Principle of the CEEMDAN

In CEEMDAN, decomposition modes are denoted as $d_l$ and are proposed to calculate a unique first residue as:

$$r_l(t) = f(t) - d_l(t)$$

(1)

Then, the first mode are computed over an ensemble of $r_l$ plus different realizations of a given noise obtaining $d_l$ by an averaging calculation.
The operator \( E_j(\cdot) \) is defined as generating the \( j \)th mode obtained by EMD. \( \omega' \) is denoted as the white noise with \( N(0,1) \). If \( f(t) \) is the analyzed signal, the method can be described by the following steps [8]:

1. \( I \) realizations \( f(t) + \epsilon_i \omega'(t) \) are decomposed by EMD to obtain their first modes by

\[
\hat{a}_i(t) = \frac{1}{I} \sum_{i=1}^{I} d_i(t) = \overline{d}_i(t)
\]  

(2)

2. At the first stage \( (k = 1) \), the first residue is calculated as in Eq. (3):

\[
r_1(t) = f(t) - \hat{a}_i(t)
\]  

(3)

3. Decomposition of realizations \( r_1(t) + \epsilon_i E_i(\omega'(t)), i = 1, \ldots, I \) is done, until their first EMD mode. The second mode is defined as:

\[
\hat{a}_2(t) = \frac{1}{I} \sum_{i=1}^{I} E_i(r_1(t) + \epsilon_i E_i(\omega'(t)))
\]  

(4)

4. The \( k \)th residue is calculated by \( (k = 2, \ldots, K) \):

\[
r_k(t) = r_{k-1}(t) - \hat{a}(t)
\]  

(5)

5. Decompose realizations \( r_k(t) + \epsilon_i E_k(\omega'(t)), i = 1, \ldots, I \), until their first EMD mode and define the \((k + 1)\)th mode as

\[
\hat{a}_{k+1}(t) = \frac{1}{I} \sum_{i=1}^{I} E_i(r_k(t) + \epsilon_i E_k(\omega'(t)))
\]  

(6)

6. Go to step 4 for next \( k \).

Steps 4 to 6 are conducted until the obtained residue is no longer feasible to be decomposed (the residue does not have at least two extrema). The final residue satisfies:

\[
R(t) = f(t) - \sum_{k=1}^{K} \hat{a}_k(t)
\]  

(7)

With \( K \) is the total number of modes. Therefore, the analyzed signal can be expressed as:

\[
f(t) = \sum_{i=1}^{K} \hat{a}_i(t) + R(t)
\]  

(8)

Eq. (8) makes the proposed decomposition complete and provides an exact reconstruction of the original signal.

The Test of Simulate Signal

In order to verify the effectiveness of the CEEMDAN, the simulate signal is made, and the formula is as followed

\[
x = \sin(15 \cdot 2\pi t) + \sin(30 \cdot 2\pi t) + \sin(50 \cdot 2\pi t)
\]  

(9)

The white noise with certain degree energy is added to the signal. There sampling time is 8s, and the sample frequency 1 kHz. The time series figure of the signal and the corresponding Fourier spectrum are shown in Figs. 1 and 2, respectively. As can be seen Figs. 1 and 2, the noise results in burrs on the sample signal in time domain and the corresponding Fourier spectrum.
The Application of CEEMDAN

The sample signal is processed by using CEEMDAN, and the EMD decomposition is conducted for 200 times. The mirror method is used to extend the sample signal to reduce the effect of the boundary effect. 4 IMFs are obtained in the decomposition. As shown in Fig. 3, after CEEMDAN decomposition operation, the noise is separated, and the harmonic component at 15, 30 and 40 Hz are also separated.

The Contrast between the Processed Signal and the Original Signal

To obtain the de-noising signal, we add IMFs 2-4 together, and the result is shown in Fig. 4. Comparing Fig.4 and Fig.1, it can be known that the signal become smooth, after de-noising operation.
Summary

In order to eliminate noise from the vibration signal, CEEMDAN is introduced in this paper. At first, the principle of the algorithm is illustrated, and the specific decomposition process is elaborated. The method is applied to a simulate signal, which contain harmonic components and white noise. It can be concluded that the result shows that the noise can be effectively eliminated by using this method. However, as shown in Fig.3, a decomposition error that part of harmonic is separated can be found in the right endpoint, which results from the endpoint effect. And this problem should be further studied.

References


