Dynamic Adaptive Sliding Mode Control of Large Erecting Mechanism

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Abstract. In this paper, a dynamic adaptive sliding mode control algorithm is proposed to overcome the nonlinearities, uncertainties and circumstance disturbances that exist in the process of mechanism erecting. The backstepping methodology is applied in developing the Lyapunov-based controller. Then the dynamic surface control which using first integral filter to evaluate the derivative of virtual control item combined with sliding mode control is introduced to realize nonlinear control. Furthermore, adaptive algorithm is adopted to estimate the uncertainty parameters, and the control law and adaptive scheme are also presented. By Lyapunov stability theorem, the system is proven to be asymptotically stable. Simulation results show that compared to the PID controller and conventional sliding mode controller, the proposed control method has nicer robustness and more accurate tracking ability, and the stability of the erecting process has improved.

Introduction

Electro-hydraulic servo (EHS) systems are widely used as actuators in erecting system of large mechanism, heavy engineering equipments and armaments. Because compared with motor actuators they have many advantages such as great power capability, fast response characteristic, good control precision, and large output force. However, it is well known that the EHS system has many uncertainties, time varying and highly nonlinear characteristics due to the flow-pressure relationship, oil leakage, dead zone of valve, actuator friction, volume flow unbalance of asymmetrical cylinder, and so on. Apart from the nonlinear natures of EHS system, the large erecting mechanisms are always subjected to many kinds of disturbance, variable and heavy external load, and various working environments. These inconveniences may lead to degradation of control performance in position tracking of erecting mechanism. So the conventional linear control methods cannot guarantee robustness and tracking accuracy, and they are hard to realize fast and steady erecting.

Sliding Mode Control (SMC) has emerged as a powerful tool in the position tracking control for its robustness of parametric uncertainties and external disturbances. So it is widely used in motor control, robot control, servo system control, and so on. Recently, more and more research efforts about using SMC in hydraulic servo system control have appeared. For example, it has been successfully implemented to compensate the effects of the load variations [1], friction and internal leakage [2], and uncertain original volume [3]. But only the single SMC may lead to chattering, and it is hard to realize perfect control performance. Combining the SMC with the other control methods such as adaptive control [4], fuzzy control [5], and neural network control [6] has proved to
be a good solution. For EHS systems, Guan presented an adaptive sliding mode controller to overcome the effects of the nonlinear unknown parameters in 2008, Shiuh-Jer proposed an adaptive sliding controller with self-turning fuzzy compensation for vehicle suspension control in 2006 [7], Guo applied a cascade-control algorithm based on a sliding mode to realize the trajectory tracking control in 2008 [8]. So SMC has become a most attractive method for nonlinear system, especially for the EHS systems.

This paper is organized as follows. In section 2, the detailed nonlinear model of the erecting mechanism is established. In section 3, the proposed dynamic adaptive sliding mode controller is given, and the control law and adaptive law are presented. In section 4, the simulations are set up and results are discussed. In section 5, conclusions are exhibited.

Nonlinear Mathematic Model

The erecting mechanism is mainly composed of a hydraulic pump, an electro-hydraulic proportional valve, an asymmetrical cylinder, a large erect arm and an angle sensor as shown in Fig.1. The cylinder can rotate around the point \( O_1 \) and \( O_2 \), and the erect arm can rotate around the point \( O \). The angle of the erect arm is controlled as follows: once the reference angle \( \theta_d \) and the actual \( \theta \) are transmitted to the controller, the output current \( u \) is calculated from the control algorithm. Then, the valve spool position and direction are controlled according to the output current. Depending on the spool position, the flows as well as the direction supplied to each cylinder chamber are determined. The motion of the erect arm actuated by the cylinder is controlled by the flows.

![Figure 1. Schematic diagram of the erecting mechanism.](image)

Building the coordinates \( XOY \) and \( O \) is the origin as shown in Fig.1. Let \( OO_1=l_1 \), \( OO_2=l_2 \), \( O_1O_2=l_3 \), \( OO_3=l_4 \), \( \angle OO_2O_1=\alpha \), \( \angle XOO_3=\gamma \) where \( O_3 \) is the erect arm center of gravity, and at zero second \( \angle O_1OO_2=\theta_0 \). The cylinder of the erecting mechanism is an asymmetrical cylinder, and its dynamic characteristics are different from the traditional symmetrical cylinder. The nonlinear model is composed of three equations: force balance equation, valve flow equation, and flow continuity equation of the cylinder.

Define the state variables as \( x=[x_1 \ x_2 \ x_3]=[X_p \ X_p \ X_p] \), where \( x_1, x_2, x_3 \) represent displacement, velocity, and acceleration of the piston rod, respectively. The system model can be expressed in a state-space form as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= a_1 x_2 + a_2 x_3 + a_3 g(x_1)u + a_4
\end{align*}
\] (1)

with
\[
\begin{align*}
a_1 &= \frac{B_c (1 + n^2)}{m V_0} (A_1^2 + C_i B_c) , \\
a_2 &= -\frac{B_c (1 + n^2)}{m V_0} , \\
a_3 &= \frac{m V_0}{A_1 B_c (1 + n^2) C_j w X_p} \left( \frac{2}{\sqrt{\rho (1 + n^2)}} \right) ,
\end{align*}
\]

where \(X_p\) is the displacement of the piston rod, \(A_1\) is the effective area of the head side of the piston, \(A_2\) is the effective area of the rod side of the piston, \(m\) is the equivalent mass of the cylinder, \(B_c\) is the equivalent viscous damping coefficient, \(K\) is the load spring gradient, \(P_1\) and \(P_2\) denote the supply and return pressure, respectively. And \(n = \frac{A_2}{A_1}\), \(P_s\) is the supply pressure, \(C_d\) is the discharge coefficient, \(w\) is the spool valve area gradient, \(\rho\) is the oil mass density, \(\beta_e\) is the fluid bulk modulus, and \(V_0\) is the initial volume of the chamber, \(k_p\) is the proportional coefficient.

\[
F_L = \frac{J \ddot{\theta} + G l_1 \cos(\gamma + \theta)}{l_1 \sin(\theta + \theta_0) / (l_1 + X_p)}
\] (2)

where \(J\) is the erect arm moment of inertia, \(F_L\) is the output force from the cylinder, \(\theta\) is the erect angle, and \(G\) is the gravity of the erect arm.

**Dynamic Adaptive Sliding Mode Controller**

Suppose the desired angle signal is \(\theta_d(t)\) and the displacement of the piston rod is \(X_{pd}(t)\), in the triangle \(O O_1 O_2\), they have the following relationship:

\[
X_{pd} = \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta_d + \theta_0) - l_3}
\] (3)

The objective of this paper is to design a controller to control the angle \(\theta(t)\) tracking as closely as possible to the desired signal \(\theta_d(t)\) or the displacement \(X_p(t)\) tracking the desired displacement signal \(X_{pd}(t)\). For the realistic system, the following assumption is given.

Suppose the tracking error of the piston rod displacement is \(e = x_1 - x_{1d}\), where \(x_{1d}\) is the desired displacement and \(x_{1d} = X_{pd}\). Derivative of the error \(e_1\) is \(\dot{e}_1 = x_2 - \dot{x}_{1d}\) where \(\dot{x}_{1d} = \dot{X}_{pd}(t)\). The Lyapunov function is defined as follows \(V_1 = \frac{1}{2} e_1^2 \geq 0\) the derivative of the Lyapunov function \(V_1\) is obtained \(\dot{V}_1 = e_1 \dot{e}_1 = e_1 (x_2 - \dot{x}_{1d})\). Based on the Lyapunov function steady theory, \(\dot{V}_1\) must be satisfy the inequation \(\dot{V}_1 \leq 0\), so the virtual control of \(x_2\) is chosen as \(\dot{x}_2 = -c_1 e_1 + \dot{x}_{1d}\) where \(c_1\) is a positive definite constant, and \(\dot{x}_2\) is the virtual control item. Using first integral filter \(\frac{1}{\tau_1 s + 1}\) to evaluate the derivative of the virtual control item, the following equation is obtained

\[
\tau_1 \dot{x}_{2d} + x_{2d} = \overline{x}_2, \quad x_{2d}(0) = \overline{x}_2(0)
\] (4)

where \(x_{2d}\) is the output of the first integral filter, \(\tau_1\) is a constant.

Define the error \(e_2 = x_3 - x_{2d}\), and its derivative can be derived \(\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = x_3 - \dot{x}_{2d}\)
And the Lyapunov function is defined as \( V_2 = \frac{1}{2} e_2^2 \geq 0 \). The derivative of \( V_2 \) is

\[
\dot{V}_2 = e_2 \dot{e}_2 = e_2 (x_1 - \dot{x}_d) \]

Similarly, to make sure that \( \dot{V}_2 \leq 0 \), the virtual control item of \( x_3 \) is chosen as \( \tau_3 = -c_2 e_2 + \dot{x}_2d \) where \( c_2 \) is a positive definite constant.

Set \( x_{3d} \) is the first integral filter output of the virtual control item \( \tau_3 \), and the following equation is obtained

\[
\tau_2 \dot{x}_{3d} + x_{3d} = \tau_3, \quad x_{3d}(0) = \tau_3(0)
\]

where \( \tau_2 \) is a constant.

In this step, the SMC is added to overcome the nonlinearities. Based on the SMC theory, define the sliding surface

\[
s = k^2 e_1 + 2 ke_2 + e_3
\]

where \( k > 0 \) and \( e_3 = x_3 - x_{3d} \). And the Lyapunov function is defined as \( V_3 = \frac{1}{2} a_3 s^2 \geq 0 \)

We obtain the derivative of \( V_3 \)

\[
\dot{V}_3 = s[k^2 a_3 (x_2 - \dot{x}_2d) + 2ka_1 (x_3 - \dot{x}_2d) + (a_1 a_3 x_2 + a_2 a_3 x_3 + g(x_3)u + a_3 a_4 - a_3 \dot{x}_2d)]
\]

Let \( \xi_1 = a_1 a_3 \), \( \xi_2 = a_2 a_3 \), \( \xi_3 = a_3 a_4 \), Eq. (7) can be written as

\[
\dot{V}_3 = s[k^2 a_1 (x_2 - \dot{x}_2d) + 2ka_1 (x_3 - \dot{x}_2d) + (\xi_1 x_3 + \xi_2 x_3 + g(x_3)u + \xi_3 - a_3 \dot{x}_2d)]
\]

It is clear that the above equation does not include the nonlinear uncertain parameters, but in fact, the parameters \( \xi_1, \xi_2, \xi_3, a_3 \) are uncertain or difficult to conform. So the adaptive control method is added to compensate the parameters uncertainties. In a practical erecting mechanism, these uncertain parameters are bounded. Thus the following assumption is made.

Suppose the estimated parameters of \( \xi_1, \xi_2, \xi_3, a_3 \) are \( \hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \hat{a}_3 \), and the estimate errors are \( \hat{\xi}_i, \hat{\xi}_i, \hat{\xi}_i, \hat{a}_i \) with \( \hat{\xi}_i = \xi_i - \hat{\xi}_i, i = 1, 2, 3, \) \( \hat{a}_i = a_i - \hat{a}_i \). In order to obtain the update laws of parameters, the following Lyapunov function is defined

\[
V_4 = \frac{1}{2} (a_3 s^2 + \lambda_1 \hat{\xi}_1^2 + \lambda_2 \hat{\xi}_2^2 + \lambda_3 \hat{\xi}_3^2 + \lambda_4 \hat{a}_i^2) \geq 0
\]

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0 \).

Taking into account the Eq. (3.16), the derivative of \( V_4 \) is

\[
\dot{V}_4 = s[k^2 a_1 (x_2 - \dot{x}_2d) + 2ka_1 (x_3 - \dot{x}_2d) + (\hat{\xi}_1 x_3 + \hat{\xi}_2 x_3 + g(x_3)u + \hat{\xi}_3 - a_3 \dot{x}_2d)] + \lambda_1 \hat{\xi}_1 (\hat{\xi}_1) + \lambda_2 \hat{\xi}_2 (\hat{\xi}_2) + \lambda_3 \hat{\xi}_3 (\hat{\xi}_3) + \lambda_4 \hat{a}_i (\hat{a}_i)
\]

To make sure \( \dot{V}_4 \leq 0 \), the control law is chosen as

\[
u = \hat{\alpha}_1 \dot{x}_{3d} - k^2 \hat{\alpha}_1 (x_2 - \dot{x}_2d) - 2k \hat{\alpha}_1 (x_3 - \dot{x}_2d) - \hat{\xi}_1 x_2 - \hat{\xi}_2 x_1 - \hat{\xi}_3 - \epsilon s - \eta \text{sat}(s)/g(x_3)
\]
where η and φ are positive definite constant. And sat(s) is the saturation function which replaces the sign function to reduce the effect of chattering exists in the conventional sliding mode controller. The sat(s) expressed as

\[
sat(s) = \begin{cases} 
1 & s > \Delta \\
\frac{s}{|s|} & |s| \leq \Delta \\
-1 & s < -\Delta 
\end{cases}
\]

where Δ is thickness of the boundary layer, and Δ>0.

Substituting controller u into Eq. 10 the following equation is derived:

\[
\dot{V}_s = -\varepsilon s^2 - \eta s \cdot sat(s) + \xi_1 (x_1 - \lambda_1 \dot{\xi}_1) + \xi_2 (x_2) - \lambda_2 \dot{\xi}_2 + \tilde{a}_1 [k^2 (x_1 - \dot{x}_d) - 2ks(x_1 - \dot{x}_d) - s\dot{x}_d] \\
+ 2ks(x_1 - \dot{x}_d) - s\dot{x}_d - \lambda_3 \dot{\tilde{a}}_1
\]

To make sure \( \dot{V}_s \leq 0 \), the adaptive law is chosen as

\[
\dot{\xi}_1 = \frac{1}{\lambda_1} s x_2, \quad \dot{\xi}_2 = \frac{1}{\lambda_2} s x_1, \quad \dot{\xi}_3 = \frac{1}{\lambda_3} s, \\
\dot{\tilde{a}}_1 = \frac{1}{\lambda_4} [k^2 (x_1 - \dot{x}_d) + 2ks(x_1 - \dot{x}_d) - s\dot{x}_d]
\]

Simulation

To test the effectiveness of the proposed dynamic adaptive sliding mode controller, the compared simulations are presented in this section.

Tab.1 has presents the original parameters of the system, in practice, some parameters have uncertainties or hard to confirm because of the external disturbances and environment changes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply pressure</td>
<td>( P_s )</td>
<td>18</td>
<td>MPa</td>
</tr>
<tr>
<td>Discharge coefficient of the spool valve</td>
<td>( C_d )</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Circumference gradient of the spool valve</td>
<td>( w )</td>
<td>2.51×10^3</td>
<td>m</td>
</tr>
<tr>
<td>Bulk modulus of the oil</td>
<td>( \rho )</td>
<td>7.5×10^2</td>
<td>Pa</td>
</tr>
<tr>
<td>Length of the piston rod</td>
<td>( l )</td>
<td>1.5915</td>
<td>m</td>
</tr>
<tr>
<td>Equivalent viscous damping coefficient</td>
<td>( B_0 )</td>
<td>800</td>
<td>N/m/s</td>
</tr>
<tr>
<td>Mass density of the oil</td>
<td>( \rho )</td>
<td>868</td>
<td>Kg/m^3</td>
</tr>
<tr>
<td>Equivalent mass of the cylinder</td>
<td>( m )</td>
<td>7.834</td>
<td>Kg</td>
</tr>
<tr>
<td>Length of the piston rod</td>
<td>( l )</td>
<td>1.831</td>
<td>m</td>
</tr>
<tr>
<td>Nominal value of the cylinder</td>
<td>( C_n )</td>
<td>2.4×10^-9</td>
<td>m^3/Pa</td>
</tr>
<tr>
<td>Outwards leakage coefficient of the cylinder</td>
<td>( C_{oa} )</td>
<td>7.1×10^-11</td>
<td>m^3/Pa</td>
</tr>
<tr>
<td>Equivalent mass of the erect arm</td>
<td>( M_e )</td>
<td>1155.98</td>
<td>Kg</td>
</tr>
<tr>
<td>Moment of inertia of the erect arm</td>
<td>( J )</td>
<td>10023</td>
<td>Kg/m^2</td>
</tr>
<tr>
<td>Volume of the piston chamber</td>
<td>( V_e )</td>
<td>1.5×10^3</td>
<td>m</td>
</tr>
<tr>
<td>Length of O_3O_2/O_2O_2/O_2</td>
<td>( l_1/l_2/l_3 )</td>
<td>1.132/1.62/1.032/3.5</td>
<td>m</td>
</tr>
<tr>
<td>Angle of ( \phi )</td>
<td>( \phi )</td>
<td>0.6816/0.1047</td>
<td>rad</td>
</tr>
</tbody>
</table>

In order to show the influence of the uncertain parameters, and to test the performance of the proposed control scheme. In this paper, we suppose the damping coefficient \( B_0 \), bulk modulus \( \beta_o \), mass of the erect arm \( M \) and the external disturbance \( \Delta F \) have the following expression: \( B_0 = B_0 + 0.04B_0 \sin(0.1\pi t) \), \( \beta_o = \beta_o + 0.01\beta_o \sin(0.1\pi t) \), \( M = M_o + 0.09M_o \sin(0.1\pi t) \), \( \Delta F = 4000\sin(0.1\pi t) \). The proposed controller parameters are designed as

\( k=100; \varepsilon=1200; \eta=0.5; \tau_1=\tau_2=0.02; \Delta=0.3; \lambda_1=1×10^{-9}; \lambda_2=5×10^{-6}; \lambda_3=1.2; \lambda_4=20. \)
Here, two control methods that PID control and conventional SMC are used and compared. The control law of the PID control can be expressed as 
\[ u(t) = k_p e(t) + k_i \int e(t) dt + k_d \dot{e}(t) \]
where \( k_p \) is the proportional coefficient, \( k_i \) is the integral coefficient, \( k_d \) is derivative coefficient, and \( k_p = 8 \), \( k_i = 4 \), and \( k_d = 0.005 \), respectively.

The control law of the conventional SMC can be expressed as 
\[ u(t) = [k_1 e_1 + k_2 e_2 + \tilde{x}_d - a_1 x_2 - a_2 x_3 - a_4 + k_s + \varepsilon \text{sgn}(s)] / (a_0 g(x)) \]
where \( \varepsilon = 4200 \), \( k = 2.2 \), \( k_1 = 1 \times 10^6 \), \( k_2 = 2000 \),
\[ s = k_1 e_1 + k_2 e_2 + e_3, \quad e_1 = \tilde{x}_d - x_1, \quad e_2 = \dot{x}_d - x_2, \quad e_3 = \ddot{x}_d - x_3. \]

The simulation model is established in the software Matlab/Simulink environment. Applying the above three controllers to track the desired angle curve in the condition that the system has parameters uncertainties and external disturbance. The simulation results are shown in Fig.3-7.

Fig.2 exhibits the position tracking angle curves of the three controllers and Fig.3 presents their position tracking error. It can be seen that the proposed dynamic adaptive sliding mode controller has the best tracking precision, and shows steadier and more accurate performance compare to the other two controllers. The max tracking error of the proposed controller is 0.1607 degree at about 0.2 second, but 1.1313 degree for the PID control method, and 0.4565 degree for the conventional SMC. Moreover, after about 0.3 second, the tracking error is decreased to almost zero for the proposed controller and until 50 second the 0.0413 degree error has appeared. The other two controllers have distinct vibration effected from the parameters uncertainties and external disturbance. And PID controller has about 1.0491 degree error at last second, and this cannot satisfy our request.

Fig.4 shows the control signal of the three controllers. As seen, the curve of the proposed controller is smoother than that of the other two controllers, since the adaption laws can compensate the parameters uncertainties. However, the other two curves have vibration under the influence of the parameters uncertainties and external disturbances. Furthermore, in order to realize good performance, it must be increased the value of the
parameter $\varepsilon$ for the conventional SMC. This leads to the control signal chattering with high frequency and make the erecting process unstable. Fig.5 and Fig.6 present the parameters estimations, we can see that all the parameters are bounded, and it is their updating online during the control process that makes the tracking performance improves.

![Parameter estimations](image)

**Figure 6. Parameter $\xi_3$ and $a_3$ estimations.**

**Summary**

The erecting mechanism is a complicated system which is composed of mechanism, electrical equipments and hydraulic actuator, so it has strong nonlinearities and uncertainties. The highly nonlinear dynamics prevent the use of conventional control methods. By combining the sliding mode control and adaptive control, this paper proposed a dynamic adaptive sliding mode controller using backstepping methodology. The proposed controller can compensate the parameters uncertainties and external disturbance. The compared simulation results show that the proposed control method and adaptive schemes can obtain excellent position tracking performance and realize nonlinear and robust control.

**References**


Reference to a book:


