Research on Optimal Control Method of Active Suspension Based on AMEsim Modeling

Huan XIE*, Hong-yan WANG and Qiang RUI

Department of Mechanical Engineering, Academy of Armored Forces Engineering, Beijing 100072 China
*Corresponding author

Keywords: AMEsim, Active suspension, Optimal control, Co-simulation

Abstract. On the basis of establishing the mathematical model of two degree of freedom 1/4 vehicle suspension, the suspension model is established in AMEsim. Then the optimal controller is designed, and the road input model and the optimal controller model are built in MATLAB/Simulink. Finally, the AMEsim/MATLAB simulation is performed. The simulation results show that the active suspension can improve the comfort of the vehicle compared to the passive suspension.

Introduction

Suspension system is an important part of vehicle in transmitting load and attenuating vibration. The performance of the car suspension system directly affects the overall performance of the car. The stiffness and damping characteristics of the traditional passive suspension system are fixed, so it is difficult to adapt to complex road conditions and driving conditions [2]. Therefore, it is a major trend in the development of suspension research to develop active suspension which can make active adjustment to suspension parameters.

However, most of the predecessors in the study of suspension performance mostly use MATLAB software and Adams software, which makes a particular study in a class of suspension, such as the air suspension and hydraulic suspension, more abstract and more difficult to mode.

The AMEsim is the software about the hydraulic/mechanical system modeling, simulation and dynamics analysis based on the bonding diagram. The orientation of engineering applications makes AMEsim widely used in the fields of automobile manufacturing, aerospace industry and hydraulic machinery [1]. By using the AMEsim software, we can build up the correct suspension model more quickly, and then provide a feasible way for further study of different suspension performance.

Active Suspension Modeling

The two-degree-of-freedom 1/4-car model is the most simplest and easiest model for analyzing vehicle vibration characteristics. Although the 1/4-car model cannot characterize all the parameters, it studies several basic characteristics of the practical problems, which has a good reference for the further research.

The model assumes the following assumptions: assuming that both the frame and the wheels are rigid bodies, while ignoring the tire damping characteristics, and the elasticity of the tire is simulated with a spring with constant stiffness [3].

The two-degree-of-freedom 1/4 active suspension model is shown in Figure 1.
Figure 1. The two-degree-of-freedom 1/4 active suspension model.

\[ z = z_s - z_u \]

\[ z_s = q - z_u \]

\[ z \] is the suspension dynamic deflection, and \( z_u \) is the tire dynamic deformation.

The differential equation of motion is obtained according to the mechanics analysis:

\[
\begin{align*}
\dot{m}_s z + F &= k_s z_s \\
\dot{m}_u z_u &= k_s z_s - F - k_s z
\end{align*}
\]

Design of Optimal Controller

Derivation of State Space Equation

To select the sprung mass velocity, the unsprung mass velocity, the sprung mass displacement, the unsprung mass displacement, and the road input displacement as the state variable.

\[
\mathbf{x} = \begin{bmatrix} z & \dot{z}_s & z_t & \dot{z}_u \end{bmatrix}^T
\]

The state space equation of the 1/4 car suspension is

\[
\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} + \mathbf{E} \mathbf{w}
\]

Among them:

\[
\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\
\frac{k_s}{m_s} & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
-\frac{k_s}{m_s} & \frac{k_t}{m_u} & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\
\frac{1}{m_s} \\
1 \\
-\frac{1}{m_u} \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 \\
0 \\
1 \\
0 \end{bmatrix}
\]

Determination of Output Equation

In order to improve the ride comfort and handling stability of passengers, it is necessary to minimize the body vertical acceleration and the tire dynamic deformation, and at the same time limit the suspension dynamic deflection, so as to prevent the suspension impact buffer block [4].

Select the system output vector: \( \mathbf{y} = \begin{bmatrix} \dot{z}_s & z_t & \dot{z}_u \end{bmatrix} \), Then the output equation is

\[
\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u}
\]
Among them, Output Status Matrix: 
\[
\begin{bmatrix}
k_{i}/m_{i} & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad \text{Transfer matrix: } \begin{bmatrix}
1/m_{i} \\
0 \\
0
\end{bmatrix}
\]

**Determination of Objective Function**

The optimal control objective of active suspension is to make the vehicle get good ride comfort and handling stability.

To take the integral of the function of the output variable \( y \) and the control variable \( u \) as the performance index function, and determine the following objective function.

\[
J = \int_{0}^{\infty} \left[ q_{1}\dot{z}_{1}^{2} + q_{2}\dot{z}_{2}^{2} + q_{3}\dot{z}_{3}^{2} + q_{4}F^{2} \right] dt = \int_{0}^{\infty} \left[ y^{T}Qy + u^{T}Ru \right] dt
\]

Among them, \( q_{1}, q_{2}, q_{3} \) are respectively the weighting factor of the body vertical acceleration, suspension dynamic deflection, tire dynamic displacement and actuator output force. The selection of the weighting factor determines the tendency of the designer to the performance of the suspension. And when choosing a larger body vehicle vertical acceleration weighting factor, it means that the main objective of the suspension system is to improve vehicle comfort. If you choose a larger tire motion displacement weighting factor, then the vehicle handling stability is considered to be [5]. Taking into account various factors, the weighting factor this article selected are as follows.

\[
q_{1} = 5 \times 10^{6}, q_{2} = 5 \times 10^{11}, q_{3} = 5 \times 10^{13}, q_{4} = 1
\]

Putting \( y = Cx + Du \) into (5) will gains:

\[
J = \int_{0}^{\infty} \left[ x^{T}Q_{d}x + x^{T}N_{d}u + u^{T}R_{d}u \right] dt
\]

Among them: \( Q_{d} = C^{T}QC \), \( N_{d} = 2C^{T}QD \), \( R_{d} = D^{T}QD + R \), and \( Q_{d} \) is the state variable weighting matrix, \( N_{d} \) is the control variable weighting matrix, and \( R_{d} \) is the weighted correlation matrix.

**Control Law**

To select the feedback control force as the linear combination of state variables, there are:

\[
u = \{ F \} = -Kx = -\left( K_{i}z_{i} + K_{2}\dot{z}_{2} + K_{3}\dot{z}_{3} + K_{4}\dot{z}_{4} \right)
\]

Among them, Optimal feedback control matrix: \( K = R_{d}^{-1} \left( N_{d}^{T} + BL \right) \), and \( L \) can be obtained from Riccati equation.

\[
A^{T}L + AL + Q_{d} - LBR_{d}^{-1}B^{T}L = 0
\]

Finally the optimal feedback control matrix is obtained

\[
K = [K1,K2,K3,K4] = [-12913.3,1668.0,49014.1,759.8]
\]
AMEsim / Simulink Joint Simulation Calculation

The Establishment of Simulation Road Model

For different road grades, the roughness is usually expressed by the road roughness coefficient. This paper studies the road roughness of vehicle vibration input, and mainly uses the road power spectral density to describe its statistical characteristics [6].

The time-domain expression of the road roughness for filtering white noise:

\[ \dot{z}_g(t) = -2\pi f_0 z_g(t) + 2\pi \sqrt{G_0} \omega(t) \quad (9) \]

\[ f_0 = n_0 \mu \quad (10) \]

The road roughness coefficient is related to the road grade, and this paper chooses B-class pavement for simulation experiment.

In this paper, the road roughness coefficient is \( G_0 = 64 \times 10^{-6} \text{m}^3 \), the vehicle speed is 40 m/s, and space frequency is 0.011. \( \omega(t) \) is a Gaussian white noise function. The road simulation input model established in simulink is shown in Figure 2.

![Figure 2. The road simulation input model.](image)

Establishment of Simulink Model for Optimal Controller

Simulink Model for Optimal Controller is shown in Figure 3.

![Figure 3. Simulink Model for Optimal Controller.](image)
AMEsim/Simulink Co-simulation

Table 1. The parameters of a car suspension.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Physical meaning</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{wb}$</td>
<td>Wheelbase</td>
<td>2800/2=1400</td>
<td>mm</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Distance from the front axle to the body center of gravity G in X direction</td>
<td>1120</td>
<td>mm</td>
</tr>
<tr>
<td>$m_{s1/2}$</td>
<td>1/2 car on the sprung mass</td>
<td>1800/2=900</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{sd}$</td>
<td>Distribution of front suspension</td>
<td>$m_{s1/2}(l_{WB}-l_f)/l_{wb}=540$</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{swf}$</td>
<td>The mass of front axle</td>
<td>27/2=13.5</td>
<td>kg</td>
</tr>
<tr>
<td>$m_{uf}$</td>
<td>The mass of front axle</td>
<td>27</td>
<td>kg</td>
</tr>
<tr>
<td>$k_{sf}$</td>
<td>The stiffness of front suspension spring</td>
<td>30000</td>
<td>N/m</td>
</tr>
<tr>
<td>$c_{sf}$</td>
<td>The damping of front suspension</td>
<td>3000</td>
<td>N/(m/s)</td>
</tr>
<tr>
<td>$k_{tf}$</td>
<td>The vertical stiffness of front tire</td>
<td>270000</td>
<td>N/m</td>
</tr>
<tr>
<td>$c_{tf}$</td>
<td>The vertical damping of front tire</td>
<td>200</td>
<td>N/(m/s)</td>
</tr>
</tbody>
</table>

So you can build two degrees of freedom AMEsim suspension model, and the simulink controller is embedded into the AMEsim model through the data interface provided by AMEsim. And then achieves AMEsim/MATLAB joint simulation.

Figure 4. AMEsim/Simulink Simulation Model.

The simulation data are shown in Figure 5.
Conclusion

Compared with the passive suspension, the active suspension under the optimal control of LQG has different degrees of improvement in the vehicle body vertical acceleration, suspension dynamic deflection and tire dynamic deformation. It shows that the optimal control of the active suspension can better improve the vehicle ride comfort and handling stability.

It is feasible to use AMEsim to establish the active suspension model and establish the controller model by MATLAB. The AMEsim model is accurate and reliable, which provides experience for the further study of complex and specific suspension system.

References