Vibration Behavior Analysis for Simply Supported Beam under Multiple Moving Vibrating Systems

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Abstract. In this paper, the transverse vibration mode of simply supported beam subjected to moving mass and moving spring-mass system has been studied considering the gravity of the beam and the inertia of the moving mass. A coupled vibration model for multiple moving spring-mass systems (MSMSs) moving on the simply supported beam is established. The numerical simulations of lateral transient response of simply supported beams subjected to four different moving systems are carried out. It is found that the vibration frequency of the midpoint of the beam will be changed once the moving mass system (MMS) is changed into the MSMS. This phenomenon is more pronounced under the action of a single moving system. In addition, be different from the single MMS, the results show that the lateral displacement curves of the positions near the supports will occur obvious bimodal characteristics when a single MSMS pass through the simply supported beam.

Introduction

The elastic beam structure which is influenced by the moving system is widely used in mechanical and civil engineering applications[1-7], such as vehicle-bridge coupling system, projectile-barrel coupling system, and heavy-bridge suspension system. Therefore, the study of this problem has important practical significance, and has also been an important issue for people to study. For this sort of problem, there are three different models: the first way to solve the vibration dynamic response of the beam is to simplify the moving system as a moving load on the elastic beam. Bolotin and Kurilhara et al. studied the dynamic response of simply supported beams under moving loads[2]. The second method is using the moving mass model to simulate the actual situation. Sung studied the vibration of the simply supported beam under the action of moving mass system[3]. The third method is using moving mass-spring system. Pesterev and Biondi et al. have studied this model[4,5]. The first treatment does not take the influence of the inertia force of the moving mass on the beam into consideration, which can produce an error that unacceptable. Some researches only focus on calculating the history of displacement, the change of the beam vibration frequency effected by the compliance of the vibrating system has not been studied. The vibration frequency will endanger the safety of the structure.

In this paper, the vibration equation of simply supported beam under the action of moving mass-spring system is established. And the Euler-Bernoulli beams are discretized by finite element method, meanwhile the global mass matrix and stiffness matrix are established. In addition, the transverse vibration characteristics of simply supported beams under four loading modes are analyzed by numerical simulation. The vibration characteristics of the midpoint are compared under different loading conditions.

Vibration Analysis and Discretization of a Simply Supported Beam

As shown in Fig. 1, five mass-spring systems pass through a simply supported beam in uniform
motion. The vibration equation of the system is established in this situation. The moving system is in contact with the beam, without bouncing, during the course of the movement. Length of the beam is $L$, bending stiffness as $EI$, linear density of the beam as $\rho$, damping coefficient as $c$, the deflection at position $(x, 0)$ as $\omega(x, t)$, the speed of the moving mass-spring system as $v$, the displacement in the $y$ direction of the moving mass $i$ is $Y_i(x, t)$.

![Figure 1. Simply Supported Beam under Multiple Moving Spring-mass Systems.]

The external load of the simply supported beam is

$$f(x,t) = \sum_{i=1}^{n} \left\{ \delta(x-s_i(t)) \{ k[Y_i(t) - \omega(x,t)]_{x=s_i(t)} \} + c[Y_i - \frac{d\omega(x,t)}{dt}]_{x=s_i(t)} \right\} + \rho g \tag{1}$$

Where $\delta(x)$ is Dirac function, $x = s_i(t)$ is the displacement function in the $x$ direction of the moving system $i$.

![Figure 2. Spring-mass System.]

As shown in Fig. 2, the moving system consists of three parts: centralized mass $m_i$, spring $k_i$, and damper $c_i$.

The balance equation for the moving mass $i$ is

$$m_i \ddot{Y}_i(t) + c[Y_i - \frac{d\omega(x,t)}{dt}]_{x=s_i(t)} + k[Y_i - \omega(x,t)]_{x=s_i(t)} = m_i g \tag{2}$$

$\mathbf{w}^i = [y_1, \theta_1, y_2, \theta_2]^T$ is the degree of freedom of the beam element, where $y_i$ is the vertical displacement of the node $i$ of the element and $\theta_i$ is the rotation of the node $i$.

Shape function is

$$N_i(\xi) = [1 - 3\xi^2 + 2\xi^3, l(\xi - 2\xi^2 + \xi^3), 3\xi^2 - 2\xi^3, l(-\xi^2 + \xi^3)] \tag{3}$$

The elements not effected by moving system directly are under gravity only:

$$\{F\}^e_n = \int_0^l \rho(x)g[N]^T \, dx \tag{4}$$

The balance equation on the mass $i$ can be written as
\[ m \ddot{Y}(t) + c \dot{Y}(t) + k Y(t) = m_i g + k[N^*]_i \{w\}_i + c[N^*]_i \{\dot{w}\}_i + c v \frac{d[N]}{d s} \{w\}_i \]  

(5)

Based on the principle of virtual work, the external load of the element which is effected directly by moving system is

\[ \{F\}_n^e = \int_0^t f(x,t)[N]^T dx = k[N]^T Y(t) - k[N]^T[N] \{w\}_n + 
\]

\[ c[N]^T \ddot{Y}(t) - c[N]^T[N] \{\dot{w}\}_n - cv[N]^T \frac{d[N]}{d s} \{w\}_n + \int_0^t \rho(x)g[N]^T d x \]  

(6)

Combine Eqs. (5) and (6), and the dynamic equation of the system is established as

\[ [M] \{\ddot{p}\} + [C] \{\dot{p}\} + [K] \{p\} = \{F\} \]  

(7)

Where \( \{p\} = \{x \ w \ y_1 \ldots y_n\}^T \), coefficient vector of the Eq.(7) is defined as follows

\[ [M] = \begin{pmatrix} [M_B] & 0 \\ 0 & [\hat{M}] \end{pmatrix} , \]

\[ [C] = \begin{pmatrix} [C_B] + [\hat{C}] \\ [c_1] & [\hat{c}] \end{pmatrix} , \]

\[ [K] = \begin{pmatrix} [K_B] + [\hat{K}] \\ [k_1] & [\hat{k}] \end{pmatrix} \]

(8)

Where \([\hat{C}]\) and \([\hat{K}]\) are the additional damping matrix of the beam element and the additional stiffness matrix, respectively. The beam elements that are not directly affected by the moving spring-mass system, the additional damping matrix and the additional stiffness matrix are zero matrix. The additional stiffness matrix and additional damping matrix of the beam element directly affected by the moving system are as follows:

\[ [\hat{C}]^T = [\hat{c}][N]^T[N] \]  

(9)

\[ [\hat{K}]^T = [\hat{k}][N]^T[N] + [\hat{c}]v[N]^T \frac{d[N]}{d s} \]

\[ [c_1]^T = [c_2]^T = -[\hat{c}][N]^T \]  

(10)

\[ [k_1]^T = -[\hat{k}][N] - [\hat{c}]v \frac{d[N]^T}{d s} \]

\[ [k_2]^T = -[\hat{k}][N]^T \]

In addition, load vector is \( \{F\} = \{F_B\}^T, [M]g \) , where \( \{F_B\} \) is the load vector of the weight of the beam, and the \( [M]g \) is gravity of the moving mass.

**Numerical Simulations**

In this numerical simulation, the finite element calculation program is compiled by APDL language, and four different moving systems are used to stimulate the simple beam. Firstly, the single movement mass uniform motion excitation. Secondly, multiple moving mass uniform motion
motions. Thirdly, single moving spring-mass system constant speed motions. Fourthly, multiple moving springs-mass system uniform motion excitation. The length of the simply supported beam is 10m, the sectional area $M = 0.18 \text{mm}^2$, the moment of inertia $I = 0.0054 \text{mm}^4$, the elastic modulus of the material $E = 2.1 \times 10^{11} \text{Pa}$, the beam material density $\rho = 7800 \text{kg/m}^3$. The weight of the moving mass is $P = 1000 \text{N}$, the elastic modulus of the spring is $k = 1 \times 10^7 \text{N/mm}$, and the gravitational acceleration $g = 9.8 \text{kg.m/s}^2$.

As shown in Fig. 3, three positions with different positions on the simply supported beam are observed, M and N are the nodes near the two ends of the beam, respectively.

![Figure 3. Three Positions on the Beam.](image)

**Figure 3. Three Positions on the Beam.**

In the first aspect, the lateral displacement data of the midpoint under the a single moving mass and a single moving spring-mass system are shown in Fig. 4. It can be found that the displacement values of the midpoint of the beam are not very different in the case of the moving mass and the moving vibration system acting on the simply supported beam, but the vibration frequency is quite different. The lateral vibration frequency of the midpoint under the moving spring-mass system is higher than that under the action of the moving mass. This difference is particularly pronounced under the action of a single moving system, that is, this kind of difference in vibration frequency is more prominent in Fig. 4.

![Figure 4. Lateral Vibration of Midpoint.](image)

**Figure 4. Lateral Vibration of Midpoint.**
In the second aspect, the lateral vibration data of the positions M and N can be seen in Fig. 5. When the beam is subjected to the moving spring-mass system, the vibration amplitude of the lateral displacement exhibits a bimodal characteristic for both of positions M and N. And the middle node does not have this vibration characteristic no matter which model of the moving system is used. This characteristic is particularly significant under the action of a single moving spring-mass. In addition, the vibration amplitude of the positions M and N on the beam does not show bimodal characteristics, whether it is a single MMS or multiple MMS as shown in Fig. 6.

**Conclusions**

The dynamic model of the simply supported beam under the action of multiple moving spring-mass system is established. And the element stiffness matrix and the unit mass matrix of the simply supported beam are obtained by the finite element method. The dynamic equations of the simply supported beams are derived. In the numerical analysis, four different moving systems are used to act on the simply supported beam, and the lateral vibration data of three positions on the beam is extracted. By analyzing and comparing, two conclusions are drawn. Firstly, the lateral vibration frequency of the midpoint of the beam is different under the action of different moving systems. The vibration frequency of the midpoint under the MSMS is obviously higher than under the MMS. Secondly, the transverse vibration curves of the nodes near the ends of the beam have a bimodal characteristic, which characteristic is only exist when the beam is subjected to a MSMS. And this feature is particularly evident when a single MSMS pass through the beam.
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