Routing Hazmat Multimodal Transportation Based on CVaR Assessment

Liping Liu, Rongxue Du, Shuxia Li, Tijun Fan and Yifan Wu

ABSTRACT

Multimodal plays a more significant role in long distances transportation of hazardous materials, while there haven’t been systemic risk evaluation methods to hazmat multimodal transportation. Considering that the risk of hazmat transportation has the characteristics of low probability and high consequence, this paper studies how CVaR applied to the routing optimization of hazmat multimodal transportation while CVaR could better compute the tail loss of risk. We build a multimodal hazmat transportation risk assessment model in this paper, and prove its effectiveness by a computational study. Results show that the proposed model can provide a CVaR optimal transportation path to hazmat multimodal transportation in realistic.

INTRODUCTION

In recent years, the demand for hazmat has a rapid growth. Apparent in 2010, two-chlorine demand reached 467248 tons, in spite of the influence of the financial crisis of 2008, compared with 257218 tons in 2005, and grew by almost half [1]. At present, the multimodal transport system has also been gradually perfected. For example, Jincheng trans logistics is one of the largest international logistics enterprises in China, has built from inland China to the surrounding countries. Under the background of the increasingly perfect transportation network, how to design the lowest risk path is what we should pay more attention to. Considering the risk of hazmat transportation has obvious fat tail (low probability and high consequences), this paper applies CVaR to risk assessment of hazmat multimodal transportation, where CVaR is seen as risk measurement tools to optimize the portfolio of financial market, can better capture the investment risk in extreme
market conditions, and compute the tail loss. In recent years it has been applied to the risk assessment in all walks of life.

**REVIEWS**

The liquid process of hazmat transportation makes it difficult to forecast the risk. Scholars have studied in many fields with objective of minimizing transportation cost or risk, considering constrains of time or transportation modes selection at the same time [2].

According to statistics, in 2007, 111 million tons of dangerous goods by the multimodal transport system in America [3]. Shuai et al. summarize literatures in recent years, points out that the railway and multimodal transport are lower risk of transport modes, and points out that establishing adaptive multimodal transport risk assessment model based on CVaR is the research direction of the future [4]. CVaR was first used in the financial sector to optimize portfolio [5]. Based on the good nature of CVaR used in a financial portfolio, Kwon et al. [6] mainly studied the application of VaR and CVaR in hazmat transport, assume the probability of risk occurrence and consequences as a function of time. The above literatures laid a solid foundation to the study of CVaR optimization in hazmat transport. Solving CVaR model is now a difficult problem, Meng et al. [7] studied Multistage dynamic CVaR problem, and equal the whole decision-making process as CVaR overla. Meng et al. [8] presented Sample mean smoothing approximation method to solve the single and mixed CVaR optimal problems, can be extended to solve other smooth function, combined with smooth method and sample mean approximation method.

**MODEL BUILDING**

**Model of Single Transportation Mode of Hazmat Based on CVaR**

Assumes the continuous random variables $\xi$, the probability density function indicated as $p(\xi)$ and the loss function indicated as $f(x, \xi)$; so, the cumulative distribution function could be described as follows when $f(x, \xi)$ less than a constant $y \in R$,

$$
\phi(x, y) = \int_{y(x, \xi) \leq y} p(\xi)d\xi \quad y \in R
$$

$$
y_{\alpha}(x) = \min\{y \in R : \phi(x, y) \geq \alpha\}
$$

$y_{\alpha}(x)$ represents a $\alpha-VaR$ value. At the same time, considering the following function,

$$
\phi_{\alpha}(x) = (1 - \alpha)^{-1} \int_{y(x, \xi) \geq y_{\alpha}} f(x, \xi)p(\xi)d\xi
$$

$\phi_{\alpha}(x)$ represents a $\alpha-CVaR$ value. For convenient calculation, consider the following equivalent problem. But in real life, the fundamental of risk measurement is each section from origin to destination. Therefore, the established model is discrete.

$$
F_{\alpha}(x, y) = y + \frac{1}{1 - \alpha} \int_{y_{\alpha} \leq y} \left[f(x, \xi) - y\right] p(\xi)d\xi
$$
Defined $G = (N, E)$ as transportation networks, $N$ is the set of nodes to connect traffic lines; and $E$ is the set of traffic lines. For a certain origin-destination (OD) pair, there are many connecting paths $l \in L$, each path $l$ consists of many segments $j \in \mathcal{A}$, the consequences of risk occurrence is $c_{ij}$ and the corresponding probability of risk-happening is $p_{ij}$; The conditional value at risk can be expressed as (6). $x_i$ indicates whether to choose a path for transport.

$$
F_y(x,y) = y + \frac{1}{1-\alpha} \sum [c_{iy} - y] p_{iy}
$$

$$
\min CVaR^\alpha_{yi} = \min F_y(x,y) = y + \frac{1}{1-\alpha} \sum_{j \in \mathcal{A}} P_{ij} [c_{ij} - y]^* x_{ij}
$$

S.T.

$$
\Omega = \{ x : \sum_{i \in \mathcal{O}, j \in \mathcal{D}} x_{ij} = 1, \sum_{j \in \mathcal{D}} x_{ij} = 1, \sum_{j \in \mathcal{N}} x_{ij} = \sum_{j \in \mathcal{N}} x_{ij}, x_0 \in \{0, 1\}, \forall (i,j) \in \mathcal{A} \}
$$

**Risk Assessment Model of Multimodal Transport Based on the CVaR**

**Model assumptions and parameters introduction.**

There is a pair of OD in the traffic network, there is highway or railway can be chosen between two cities, supplier will take risk as the main consideration to path choices. A multimodal transportation network $G = (N, E)$, and $N = (N_H, N_R, N_{HR})$ is the set that combine each traffic line; $N_H$ is the set nodes that both are highways, $N_R$ is the set nodes that both are railways, $N_{HR}$ is the set of transit nodes; Only transportation mode conversion occurs, there will be risk occurrence. Otherwise, even if there is a pass by of transit nodes $k \in N_{HR}$, no transfer risk will happen. $E = (E_H, E_R)$ is the set of road lines and rail lines.

**Table 1. Decerebration of the parameters.**

<table>
<thead>
<tr>
<th>variables</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Confidence, on behalf of the supplier's risk aversion level</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>The consequences of different transportation modes (person / segment)</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>The probability of risk occurrence of transportation modes</td>
</tr>
<tr>
<td>$y_0$</td>
<td>The risk threshold of Select path $l$ under the condition of CVaR*</td>
</tr>
<tr>
<td>$x_{ij}$</td>
<td>0-1 variables, whether to select the mode of transport for the road</td>
</tr>
<tr>
<td>$y_k$</td>
<td>0-1 variables, whether to select a point to transport</td>
</tr>
</tbody>
</table>

**Model building and calculation method**

Considering highway, railway transport risk and transhipment risk, build following models:

$$
\min CVaR^\alpha_{yi} = y_0 + \frac{1}{1-\alpha} \left[ \sum_{i \in \mathcal{O}, j \in \mathcal{D}} P^{H} (c_{ij} - y_0)^* x_{ij}^{H} + \sum_{i \in \mathcal{D}, j \in \mathcal{D}} P^{R} (c_{ij} - y_0)^* x_{ij}^{R} + \sum_{i \in \mathcal{R}, j \in \mathcal{R}} P^{HR} (c_{ij} - y_0)^* x_{ij}^{HR} \right]
$$

$$
\Omega = \left\{ x_{ij}^{H} + \sum_{j \in \mathcal{D}} x_{ij}^{R} = 1, \forall i \in \mathcal{O}, \sum_{j \in \mathcal{D}} x_{ij}^{H} = 1, \forall j \in \mathcal{D} \right\} \Omega_1
$$

$$
\Omega_2 = \left\{ \sum_{j \in \mathcal{D}} x_{ij}^{R} + \sum_{j \in \mathcal{D}} x_{ij}^{H} = 0, \forall i \notin \{O,D\}, x_{ij}^{H}, x_{ij}^{R} \in \{0,1\} \forall i, j \in \mathcal{N} \right\}
$$

$$
\Omega_3 = \left\{ \sum_{j \in \mathcal{D}} x_{ij}^{R} - \sum_{j \in \mathcal{D}} x_{ij}^{H} = 0, \forall i \notin \{O,D\}, x_{ij}^{R}, x_{ij}^{H} \in \{0,1\} \forall i, j \in \mathcal{N} \right\}
$$

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\[ x_{ij}^H + x_{ij}^R \leq 1 + Y_j, \quad j \in (E_H, E_R), j \in N_{ij}, i \neq j \] (11)
\[ x_{ij}^H + x_{ij}^R > Y_j, \quad j \in (E_H, E_R), j \in N_{ij}, i \neq j \] (12)

\[ Y_i = \begin{cases} 1 & \text{distribution of traffic mode, } k \in \frac{1}{2}N_{ij} \\ 0 & \text{otherwise} \end{cases} \] (13)

(8) is objective for path \( l \) at a certain confidence level for a certain OD pair; and \( [x]^\alpha = \max(x, 0) \), where(9)(10) is the constraint of decision variables, ensure the continuity of transport; (11)(12) ensure that only in the event of transport mode changes, there will be transhipment risk. (13) is the choice decision variable of the transit point, equal to 1 when be chosen as a transit centre, otherwise 0. In the objective function, because the value \( y_0 \) is uncertain, the objective function is a stochastic nonlinear function, which is more difficult to solve. And because the optimality meet \( y_0 \in (C'^0, C' \alpha) \), and arrange all segments of the path \( l \) in descending order, which are defined as \( C_0, C_1, C_2, \ldots, C_n \), \( C_i \) is the \( k \) th minimum value among \( (C_i^0, C_i^\alpha) \), \( C_0 = 0 \). \( y_0 \in (C'^0, C' \alpha) \) is the stable values set, the objective function becomes a linear function, the optimal CVaR value for path \( l \) just as follows,

\[
\min_{y_0} \min \left\{ CVaR_{l} \left| y_0 + \frac{1}{1 - \alpha} \left( \sum_{\left(i,j \in E_H \right)} P_{ij}^H [C_{ij} - y_0] \cdot x_{ij}^H + \sum_{\left(i,j \in E_R \right)} P_{ij}^R [C_{ij} - y_0] \cdot x_{ij}^R + \sum_{k \in N_{ij}} P_{ij}^k [C_{ij} - y_0] \cdot Y_i \right) \right. \right\} \ (14)
\]

The optimal CVaR value for each OD pair is as (14), the objective function is a nonlinear function and cannot be solved by existing software. After the known range of \( y_0 \) values, it can be transformed into a linear function.

\[
CVaR'_{\alpha} = \min \{ CVaR'_{\alpha} \mid \forall l \in L \} \ (15)
\]

**COMPUTATION ANALYSIS**

**Numerical Example Analysis**

As shown in Figure 1, with nine points, twelve lines. Assuming an OD pair exists. Each segment has two data, the length and the consequences of the risk. The length refers to the actual distance between cities, \( l_{ij}^H \sim (40, 90), l_{ij}^R \sim (150, 200) \). The risk value is calculated as: \( C_{ij}^H = C_{ij}^R = \rho_{ij} \cdot \text{Len}_{ij} \); \( C_{ij}^{HR} = \pi \rho_{ij} \). The probability of risk-happen for road and rail are \( P_{ij}^H = 0.62 \times 10^{-4} \) and \( P_{ij}^R = 0.19 \times 10^{-4} \); and \( P_{ij}^{HR} = P_{ij}^H = 0.62 \times 10^{-4} \). Through cplex12.0 calculation, it will give the optimal path directly from the data.

![Figure 1. distribution network.](image-url)
Table 2. The length of each segment and the value of risk.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Segment length (Miles)</th>
<th>Average density (person / km²)</th>
<th>Risk Value (person / segment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>82</td>
<td>150</td>
<td>12300</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
<td>215</td>
<td>14625</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>225</td>
<td>14760</td>
</tr>
<tr>
<td>4 (rail)</td>
<td>180</td>
<td>45</td>
<td>8100</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>220</td>
<td>12320</td>
</tr>
<tr>
<td>6 (rail)</td>
<td>165</td>
<td>30</td>
<td>4950</td>
</tr>
<tr>
<td>7 (rail)</td>
<td>190</td>
<td>35</td>
<td>6650</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
<td>190</td>
<td>13300</td>
</tr>
<tr>
<td>9 (rail)</td>
<td>170</td>
<td>40</td>
<td>6800</td>
</tr>
<tr>
<td>10</td>
<td>49</td>
<td>225</td>
<td>11025</td>
</tr>
<tr>
<td>11</td>
<td>78</td>
<td>140</td>
<td>10920</td>
</tr>
<tr>
<td>12</td>
<td>63</td>
<td>235</td>
<td>14805</td>
</tr>
</tbody>
</table>

Table 3. Risk values for each transit yard.

<table>
<thead>
<tr>
<th>Transit yard</th>
<th>Average density (person / km²)</th>
<th>Risk Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>75</td>
<td>235.5</td>
</tr>
<tr>
<td>D</td>
<td>95</td>
<td>298.3</td>
</tr>
<tr>
<td>F</td>
<td>115</td>
<td>361.1</td>
</tr>
<tr>
<td>H</td>
<td>100</td>
<td>314</td>
</tr>
</tbody>
</table>

Under $\alpha = 0.9999$, solved with cplex12.0, we can get $CVaR^* = 400.12$; and the optimal path is, $A(O) \rightarrow 1 \rightarrow B \rightarrow 4 \rightarrow E \rightarrow 7 \rightarrow F \rightarrow 10 \rightarrow I(D)$. From this small scale numerical example, results show the feasibility of CVaR to the application in intermodal transportation problem, we can also get the corresponding optimal transportation path.

**Numerical Example Analysis Based on China's Middle-eastern Transportation network**

As is shown in Figure 2, 20 administrative centres can be connected by highway, rail is not surely connected. Select nine transportation centres with larger traffic flow as transit yards. Choose different OD pairs, the risk consequences of the traffic lines is computed as $c_{ij} = \rho_i \times len_{ij}$; The risk consequence of transit yard choose to the population in two kilometres $c_k = 4\pi \times \rho_k$; and the population density refer to the average population density in every province; The probability of risk occurrence could be described as $p_i^H = 0.64 \times 10^{-6} \times len_{ij}^H$; $p_i^R = 0.16 \times 10^{-6} \times len_{ij}^R$; $p_i = 0.64 \times 10^{-6} \times 4\pi$;

![Figure 2. Transport network diagram of Middle-Eastern China.](image)
Table 4. Traffic node list.

<table>
<thead>
<tr>
<th>Administrative center</th>
<th>Whether transit</th>
<th>Administrative center</th>
<th>Whether transit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>Yes</td>
<td>Fuzhou</td>
<td>No (destination)</td>
</tr>
<tr>
<td>Tianjin</td>
<td>No</td>
<td>Nanchang</td>
<td>No</td>
</tr>
<tr>
<td>Shijiazhuang</td>
<td>No</td>
<td>Jinan</td>
<td>Yes</td>
</tr>
<tr>
<td>Taiyuan</td>
<td>No</td>
<td>Zhengzhou</td>
<td>Yes</td>
</tr>
<tr>
<td>Shenyang</td>
<td>Yes</td>
<td>Wuhan</td>
<td>Yes</td>
</tr>
<tr>
<td>Changchun</td>
<td>No</td>
<td>Changsha</td>
<td>No</td>
</tr>
<tr>
<td>Shanghai</td>
<td>No</td>
<td>Chongqing</td>
<td>Yes</td>
</tr>
<tr>
<td>Nanjing</td>
<td>No</td>
<td>Guiyang</td>
<td>No</td>
</tr>
<tr>
<td>Hangzhou</td>
<td>No</td>
<td>Xian</td>
<td>Yes</td>
</tr>
<tr>
<td>Hefei</td>
<td>Yes</td>
<td>Yinchuan</td>
<td>No (origin)</td>
</tr>
</tbody>
</table>

Make an example of Yinchuan in Ningxia province, Fuzhou in Fujian province as an OD pair, after solving by cplex12.0, we can get CVaR*=5241.452, and the computation time is no more than 15 seconds; finally, the route obtained is as follows:

Yinchuan → Xian → Taiyuan → Jinan → Nanjing → Shanghai → Fuzhou

CONCLUSIONS

Multimodal transport is the effective ways to increase social transport efficiency, reduce society transportation cost, and integrate various mode of transportation advantage. Especially for the transport of hazardous materials, both the endurances of transportation time and risk factor are important. This article applied CVaR to establish risk assessment model, in the future research, we will pay attention to the time connection and provide a more effective and comprehensive reference for the suppliers.

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REFERENCE