Calculation of Thick-walled Shell Taking into Account Nonlinearity and Temperature Inhomogeneity of the Concrete at Subzero Temperatures

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ABSTRACT

Thick-walled shells found in many engineering structures, particularly in the construction of thermal power. Another example of practical application is the design of storage facilities for cryogenic liquids. We consider the problem calculating finite cylinder which is in an axisymmetric temperature field, the material of which has a non-linear stress-strain diagram. Chart options depend on the temperature.

INTRODUCTION

In [1-5] considered the one-dimensional problems of the nonlinear theory of elasticity and theory of plasticity for bodies with a one-dimensional (radial) inhomogeneity of the mechanical characteristics. In solving problems for bodies with a two-dimensional inhomogeneity including axisymmetric problem of calculation of finite-length cylinder, as well as similar problems in the theory of elasticity, one has to resort to direct numerical methods [6 etc.]. Similar problems are encountered in many engineering structures, particularly in the heat-power construction. Another example of practical application is the design of storage facilities for cryogenic liquids. Cryogenic liquids, in particular liquefied gas, stored at temperatures up to -165°C. As indicated in [7], at such low temperatures the strength of the concrete almost 4 times and the modulus of elasticity in a 2-fold increase compared with the corresponding values at normal temperature. Considering that on the outer surface of the structure temperature is close to normal, due to a significant temperature gradient occurs a significant change of deformation and strength characteristics of concrete.

STATE OF THE PROBLEM

The paper considers the problem of calculating the finite-length cylinder being in an axisymmetric temperature field, a material of which has a non-linear stress-strain diagram [8, 9] (Fig. 1):

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\[ \sigma_i = f(\varepsilon_i) = E\varepsilon_i - A\varepsilon_i^\alpha, \]  \hspace{1cm} (1) 

where

\[ \alpha = \frac{E\varepsilon_{\max}}{E\varepsilon_{\max} - \sigma_{\max}}; \]  \hspace{1cm} (2a) 

\[ A = \frac{E\varepsilon_{\max} - \sigma_{\max}}{\varepsilon_{\max}^\alpha}. \]  \hspace{1cm} (2b) 

Here \( E \) – Young modulus, \( \sigma_{\max} \) – breaking stress, \( \varepsilon_{\max} \) – breaking strain, \( E_{s,\max} \) – secant modulus. Parameters (2) are functions of two coordinates – \( r \) and \( z \) due to their dependence on the two-dimensional temperature field \( T(r, z) \). Geometric parameters correspond to the gas storage facility in Fos-Sur-Mer [7].

Below you will solve two problems: for operational mode of gas storage and for the emergency mode (fire).

**OPERATIONAL MODE**

Heat equation in axially symmetric problem in cylindrical coordinates has the form [10-Лыков]:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \lambda_T(T) \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial z} \left[ \lambda_T(T) \frac{\partial T}{\partial z} \right] = 0, \]  \hspace{1cm} (3)

where \( \lambda_T(T) \) - coefficient of thermal conductivity.

On the lateral sides we have boundary conditions of the second kind:

\[ \lambda_T(T) \frac{\partial T}{\partial n} = \alpha_n (T - T_{\text{amb}}). \]  \hspace{1cm} (4)

Here \( n \) – the normal to surface; \( \alpha_n \) – heat-transfer coefficient; \( T_{\text{amb}} \) – ambient temperature.

At the end faces will use the boundary conditions of the first kind:

\[ z = 0, \quad T = T_0(r); \]  \hspace{1cm} (5a) 

\[ z = H, \quad T = T_H(r); \]  \hspace{1cm} (5b)
where $T_0(r)$ and $T_H(r)$ – known functions.

The solution of boundary value the problem was solved used variation-difference method.

As an example, we give some results of calculation of the shell in the operating mode. Fig. 3 shows graphs of temperature and the relative changes of the limit strength and limit deformation, as well as the modulus of elasticity of concrete across the thickness of the cylinder at low temperatures (per unit were taken values at 0 ° C). Since the thickness of the shell is much less than its sizes chart $T(r)$ is close to linear.

Significant dependences of mechanical properties on the temperature, which are the parameters of the diagram $\sigma_i - \varepsilon_i$, we need to account in calculations the inhomogeneity of elastic-plastic material properties. Based on the results shown in Fig. 3 according to (1) and (2) we can construct diagrams $\sigma - \varepsilon$ for different layers of the concrete shell (Fig. 4).

The linearization of elastoplastic problem was carried out by the method of elastic solutions [11]. For the numerical solution of the elastic problem at each step of the iterative process used variation-difference method [12], the system of linear equations formed in the optimal structure and solved by the Gauss method.

The boundary conditions in the problem plasticity corresponded rigid attachment on the bottom of the cylinder. On the side surfaces and on the top surface are no power loads. Fig. 5a shows the stress distribution in the middle section of the cylinder with the dimensions: $a = 28m$, $b = 29m$, $H = 40m$. The calculation took the following values of the mechanical and thermal characteristics: $E = 2 \cdot 10^4$ MPa; $\nu = 0.2$; $\sigma_{\text{max}} = 30$ MPa; $\varepsilon_{\text{max}} = 0.01$; $\lambda_T = 1.5$ W/cm$^2$; $\alpha_T = 1 \cdot 10^{-5}$/$^\circ$C.

Figure 3. Relative change of concrete’s mechanical properties at negative temperature.

Figure 4. Stress – strain diagrams of concrete for different layers of shell.
Since the temperature field in the problem such that the temperature on the outer surface of the cylinder is higher than on the inner surface, it is natural that stress in the internal region – stretching and in the outer region – compressing.

Analysis of the temperature field in the two-dimensional problem corresponds to the operation mode shows that the temperature along the inner wall of the shell does not change, and along the outer wall varies slightly. This indicates that the two-dimensional solution is insignificantly different from the one-dimensional. This is also confirmed by nature of the stress distribution along the height of the cylinder (Fig. 5.b).

**EMERGENCY MODE (FIRE)**

In the fire mode distribution of the ambient temperature near the outer of the shell wall is axially symmetric and varies in height according to the law:

\[
T_b(z) = \left(1.1 - \frac{z^3}{H^3}\right)250^\circ C.
\] (6)

The gas temperature is equal to $-164^\circ C$. It is believed that increasing the temperature to $-100^\circ C$ occurs on exposure of the insulating layer of air.
Figure 6 shows the temperature curve along the radius on two layers of the shell for the emergency operation. It is seen that in this case the problem is essentially two-dimensional.

![Temperature curve along the radius on two layers of the shell](image)

**Figure 7.** Diagrams of stresses $\sigma_0$

1 – linear homogeneous material; 2 – nonlinear homogeneous material; 3 – linear inhomogeneous material; 4 – nonlinear inhomogeneous material

**CONCLUSIONS**

From the presented results it is clear that the inhomogeneity of concrete at low temperatures leading to an increase in the stiffness leads to an increase in stresses. In its turn, the account of nonlinearity leads to a reduction of stresses (up to 50%). These results indicate the need to consider two factors: non-linearity and inhomogeneity.

Taking into account the work of the material as reflected in deformation diagrams at different temperatures allow to reduce preliminary efforts and the wall thickness of the containment while maintaining its operational characteristics.

**ACKNOWLEDGEMENT**

Scientific work is realize under support of Ministry education and science Russian Federation (grant of President Russian Federation No14.Z57.14.6545-SS)

**REFERENCES**