Kalman Filtering for Discrete Systems with Measurement Delay

Qiuping Wang, Miao Zhang, Lei Han, Linyuan Che
Institute of Automation Engineering, Northeast Dianli University, Jilin, China

ABSTRACT: For the discrete systems with measurement delay, traditional Kalman filtering algorithm cannot be directly applied, the key problem is that the discrete measurement equation does not comply with the form of the Kalman filter algorithm. Therefore, based on the augmented state method, here we propose another method to eliminate the time delay where exists in the measurement equation. According to the principle of Kalman filter algorithm, for the transformed system equations without time delay, measurement delay Kalman filter algorithms steps are given, and the proposed method is applied to the actual nonlinear measurement photoelectric tracking system to compare the performance of the algorithms. Simulation results show that the proposed method is feasible and can greatly improve the accuracy of the algorithms.

KEYWORDS: Nonlinear filtering; Augmented state; Kalman filtering; Photoelectric tracking system.

1 GENERAL INSTRUCTIONS

Filtering problems has always been the hotspot of the control field, and has been studied for more than sixty years in many kinds of science and engineering applications. Kalman filtering is currently the main research result. It is a kind of optimal filtering based on the linear minimum variance estimation for the systems without delay.

Time delay is a common characteristic of control system, see [1], according to the sources of delay produced, time delay systems can be divided into two categories: one is state delay system, the other one is measurement delay system, refer to [2]. So filtering problems of time delay system can also be divided into two categories.

In recent decades, for the filtering problem of state delay system, scholars mainly use the augmented state method, partial differential Riccati equation and linear matrix inequality method, see [3-5]. However, these methods have some defects, for example, the solving of the partial differential equations is difficult and it is difficult to analyze the performance of the controller and the filter. For the linear matrix inequality method, the structure of the convex optimization problem is difficult and this method can not completely solve the filtering problem in theory. For the filtering problem of measurement delay system, reorganization of innovation analysis method is mainly used, but this method is only applicable to the linear measurement delay system with both instant measurement and delayed measurement refer to [6,7].

For measurement delay system, equations containing measurement delay is the basic reason why the traditional Kalman filtering can not be used directly. In order to explore a simple filtering method that can be directly applied to measurement delay system, here we propose a new algorithm based on augmented state method to eliminate the time delay where exists in the measurement equations, and give the transformed system equations without delay. At the same time, the new Kalman filter algorithms steps for measurement delay system are given, the proposed method is applied to the actual nonlinear measurement photoelectric tracking system to compare the performance of the algorithms. Simulation results show that the proposed method is feasible and can greatly improve the accuracy of the algorithms.

2 STATE ESTIMATION OF MEASUREMENT DELAY SYSTEM

Equations of stochastic nonlinear discrete measurement delay system can be expressed as

\[ x_k = f_{k-1}(x_{k-1}) + w_{k-1} \]  \hspace{1cm} (1)
\[ z_{k-r} = h_{k-r}(x_{k-r}) + v_{k-r} \]  \hfill (2)

Where \( x_k \) is a \( n \)-dimensional state vector, \( z_k \) is a \( m \)-dimensional measurement vector, \( f(\cdot) \) and \( h(\cdot) \) are the state model and measurement model respectively, they are all nonlinear. The subscript \( k \) represents the current time step, \( r \) represents the delay time. Furthermore, \( w_k \in \mathbb{R}^n \) and \( v_k \in \mathbb{R}^m \) are the irrelevant process noise vector and measurement noise vector respectively, they are all white gaussian noise, and the statistical characteristics are given as follows:

\[
E(w_k) = q_k, \text{Cov}(w_k, w_j) = Q_k \delta_{kj}, E(v_k) = r_k, \text{Cov}(v_k, v_j) = R_k \delta_{kj}, \text{Cov}(w_k, v_j) = 0
\]

Where \( Q_k \) is a symmetric non-negative definite matrix, \( R_k \) is a symmetric positive definite matrix.

According to the principle of state estimation, for the measurement delay system, the prediction filtering problem is to use the measurement data sequences \( z_{i-r}, z_{i-r+1}, \ldots, z_{k-r} \) with noise at discrete time steps to estimate the state vector \( x_i \) at time step \( k \).

Therefore, for the measurement delay system described by Eq. (1) and (2), according to the traditional state estimation method, it can only get the state estimates at time step \( k-r \). The fundamental reason why traditional state estimation method cannot be directly applied to systems with delayed measurements are as follow:

1. The measurement equation form of measurement delay system does not meet the form of traditional state estimation method.
2. The measurement sequence of measurement delay system is the measurement at time step \( k-r \) and before.

Therefore, to solve the above problem, the key point is to propose a method to eliminate the delay where exists in the measurement equation, and transform the form of measurement equation to meet traditional prediction filtering algorithm form.

\[ X_k = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-r} \end{bmatrix} + \begin{bmatrix} w_{k-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} f(x_{k-1}) \\ 0 \\ \vdots \\ 0 \end{bmatrix} X_{k-1} + W_{k-1} \]  \hfill (3)

\[ z_{k-r} = h([0 \ldots 0 1]^T) (x_k x_{k-1} \ldots x_{k-r}) + v_{k-r} \]  \hfill (4)

where \( X_k \) and \( Z_k \) are the augmented state vector at time step \( k \) and \( k-1 \) respectively, \( W_{k-1} = [w_{k-1} 0 \ldots 0]^T \) and \( V_k = v_{k-r} \) are the process noise vector and measurement noise vector respectively.

For the measurement delay system as Eq. (3) and (4) describe, Eq. (3) still conforms to the system equation form of traditional prediction filtering algorithm, but Eq.(4) does not conform to the measurement equation form. Therefore the state augmented method can not solve the problem. The key point is still need to propose a method to transform the form of the measurement delay system to meet traditional prediction filtering algorithm form. Namely, the expression of the relationship between the measurement vector \( z_k \) and the state vector \( X_k \) at the time step \( k \).

The Eq. (4) describes a relationship between the measurement vector \( z_{k-r} \) at the time step \( k-r \) and the state vector \( X_k \) at the time step \( k \), but the essence is still the relationship between the measurement vector \( z_{k-r} \) and the state vector \( X_{k-r} \) at the time step \( k-r \). Therefore, here the measurement vector is redefined as \( Z_k = [0 \ldots 0 1]^T z_k z_{k-1} \ldots z_{k-r} \), and the measurement delay system as Eq. (3) and (4) describe can be expressed as follow:

\[ X_k = f^*(X_{k-1}) + W_k \]  \hfill (5)

\[ Z_k = h^*(X_k) + V_k \]  \hfill (6)

where \( X_k \) is the augmented state vector at time step \( k \), \( Z_k \) is the measurement vector at time step \( k \), \( W_k \) and \( V_k \) are the irrelevant process noise vector and measurement noise vector respectively, the nonlinear mapping relationship between \( f^*(\cdot) \) and \( h^*(\cdot) \) is same to Eq. (3) and (4).

Here the system equations as Eq. (5) and (6) describe are the system model equations without measurement delay, so the form of the transformed system equations meet traditional prediction filtering algorithm form.

3.2 Augmented state Kalman filter algorithm of the system without time delay

3.2.1 Augmented state EKF algorithm

Extend Kalman Filter(EKF) is a effective method to solve the nonlinear system filtering problem. The
system as Eq. (5) and (6) describe is still a nonlinear system, therefore here we combined the EKF algorithm with the augmented state method, get the augmented state EKF algorithm. First, develop the nonlinear function \( f^* (\cdot) \) and \( h^* (\cdot) \) into a Taylor series around the filter value \( \hat{X}_{k-1} \), and omit the second order and higher order terms, then the transformed system equations as Eq. (7) and (8) describe can be expressed as follow:

\[
X_k = \Phi_{k|k-1}X_{k-1} + W_k \\
Z_k = H_k X_k + V_k
\]  

(7) and (8)

Where \( X_k \) and \( X_{k-1} \) are the augmented state vectors at time step \( k \) and \( k-1 \) respectively. \( W_{k,i} = [w_{k,i} 0 \cdots 0]^T \) and \( V_{k,i} = v_{k,i} \) are the process noise vector and measurement noise vector respectively. The relationship between the corresponding noise variance matrix and the original system noise variance matrix is \( R_k = R_i \). \( Q_i = diag(Q_i)_{i \times i} \).

According to the principle of Kalman filter, the state estimation specific steps based on the augmented state method are given as:

\[
\hat{X}_{k|k-1} = \Phi_{k|k-1} \hat{X}_{k-1} + f^* (\hat{X}_{k-1}) \\
P_{k|k-1} = \Phi_{k|k-1} P_{k-1|k-1} \Phi^T_{k|k-1} + Q^*_i \\
K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R^*_i)^{-1} \\
\hat{X}_k = \hat{X}_{k|k-1} + K_k (Z_k - h^* (\hat{X}_{k|k-1})) \\
P_k = (I - K_k H_k) P_{k|k-1}
\]  

(9) to (13)

For the delay system as Eq. (1) and (2) describe, by using the augmented state EKF algorithm, the measurement delay system is transformed into no delay system, then according to the state values \( x_0 \cdots x_r \), and variance \( P_r \), the augmented state estimation \( \hat{X}_k \) at time step \( k \) can be recursive calculated.

The original system state estimation \( \hat{x}_k \) at time step \( k \) can be calculated by the Eq. (14).

\[
\hat{x}_k = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix} \hat{X}_k
\]  

(14)

3.2.2 Augmented state UKF algorithm

Because of the core idea of the EKF algorithm is to make the nonlinear model into the first-order linearization, and omit the second order and higher order terms, this leads to the EKF algorithm has a low accuracy in the application of high order system, the augmented state EKF algorithm still inherited this defect. In order to overcome it, here we combine the augmented state method with UKF algorithm, its core idea is to use the UT transform instead of the first-order linearization of EKF algorithm, its accuracy is much higher than EKF algorithm.

Apply the augmented state method to the nonlinear system as Eq.(5) and (6) describe, the algorithm steps are as follow:

1. The initial state statistical properties are

\[
\begin{bmatrix} \hat{X}_0 & P_0 \end{bmatrix} = E[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^T]
\]

(15)

2. Select the Sigma point sampling strategy of the UT transform.

3. Time update equations. According to the Sigma sampling strategy, the Sigma point \( \xi_{i,k-1} \) \((i=0, 1, \cdots, L)\) can be calculated by using \( \tilde{X}_{i,k-1} \) and \( P_{i,k-1} \), and \( \gamma_{i,k-1} \) can be calculated by the nonlinear state function \( f_k (\cdot) + q_{k-1} \) with the Sigma point \( \xi_{i,k-1} \) \((i=0, 1, \cdots, L)\), then the one-step state predictive value \( \tilde{X}_{i,k|k-1} \) and the error covariance matrix \( P_{i,k-1} \) can be obtained by the \( \gamma_{i,k-1} \).

\[
r_{i,k-1} = f^* (\xi_i,k-1) + q_{i,k-1}
\]

(16)

\[
\hat{X}_{k|k-1} = \sum_{i=0}^{L} W_{i}^m \gamma_{i,k-1} = \sum_{i=0}^{L} W_{i}^m f^* (\xi_i,k-1) + q_{i,k-1}
\]

(17)

\[
P_{k|k-1} = \sum_{i=0}^{L} W_i^r (\gamma_{i,k-1} - \hat{X}_{k|k-1})(\gamma_{i,k-1} - \hat{X}_{k|k-1})^T + Q^*_i
\]

(18)

4. Measurement update equations. Similarly, according to the Sigma sampling strategy, the Sigma point \( \xi_{i,k} \) \((i=0, 1, \cdots, L)\) can be calculated by using \( \tilde{X}_{i,k-1} \) and \( P_{i,k-1} \), and \( \gamma_{i,k} \) can be calculated by the nonlinear measurement function \( h_k (\cdot) + r_{k-1} \) with Sigma point \( \xi_{i,k} \) \((i=0, 1, \cdots, L)\), then the output prediction \( \tilde{Z}_{i,k|k-1} \), the auto-covariance matrix \( P_{\tilde{Z}_i} \) and the cross-covariance matrix \( P_{\tilde{Z}_i, X_i} \) can be obtained by the \( \gamma_{i,k-1} \).

\[
\tilde{Z}_{i,k} = \sum_{i=0}^{L} W_{i}^m \tilde{X}_{i,k|k-1} = \sum_{i=0}^{L} W_{i}^m h^* (\xi_i,k) + r_{i,k}
\]

(19)

\[
P_{\tilde{Z}_i} = \sum_{i=0}^{L} W_{i}^r (\tilde{Z}_{i,k|k-1} - \hat{Z}_{i,k|k-1})(\tilde{Z}_{i,k|k-1} - \hat{Z}_{i,k|k-1})^T + R^*_k
\]

(20)

\[
P_{\tilde{Z}_i, X_i} = \sum_{i=0}^{L} (\xi_{i,k} - \hat{X}_{i,k|k-1})(\tilde{Z}_{i,k|k-1} - \hat{Z}_{i,k|k-1})^T + R^*_k
\]

(21)

Update the filtering measurement after get the new measurement \( Z_k \).
\[
\begin{align*}
\dot{X}_k &= \dot{X}_{k|k-1} + K_k (Z_k - \hat{Z}_{k|k-1}) \\
&= \dot{X}_{k|k-1} + K_k (Z_k - \hat{Z}_{k|k-1}) \\
K_k &= P_{\dot{X}_k|k-1} P_{Z_k}^{-1} \\
P_k &= P_{\dot{X}_{k|k-1}} - K_k P_{\hat{Z}_k} K_k^T
\end{align*}
\] (23)

Similar to the augmented state EKF algorithm described above, according to the state value \(x_0, \ldots, x_r\) and variance \(P_r\), the augmented state estimation \(\hat{X}_k\) at time step \(k\) can be recursive calculated. Similarly, the original system state estimation \(\hat{x}_k\) at time step \(k\) can be calculated by the Eq. (14).

4 SIMULATION OF THE NONLINEAR MEASUREMENT DELAY PHOTOELECTRIC TRACKING SYSTEM

For the nonlinear measurement delay photoelectric tracking system, in order to achieve the target motion parameters experiment by using the augmented state EKF algorithm and the augmented state UKF algorithm, make the following design:

1) To emphasize nonlinear measurement, here a second-order constant speed (CV) linear model and a third-order constant acceleration (CA) linear model with random interference respectively are selected as the target motion model, which take the target motion parameters (position, velocity, acceleration) as the state variables of the system. Here system noise is assumed to be uncorrelated white Gaussian noise;

2) The measurement model in polar coordinates is converted into a nonlinear measurement model in cartesian coordinates, and it is seen as the photoelectric tracking target measurement model, the discrete photoelectric tracking target measurement delay model can be expressed as :

\[
Z_k = \begin{bmatrix} r_k \\ a_k \\ e_k \end{bmatrix} = h(X_k) + V_k
\]

\[
\begin{bmatrix}
\sqrt{\left(x_k^2 + y_k^2 + z_k^2\right)} \\
\arctan \frac{y_k}{x_k} \\
\arctan \frac{z_k}{\left(x_k^2 + y_k^2\right)^{1/2}}
\end{bmatrix}
= \begin{bmatrix} v_r \\ v_a \\ v_e \end{bmatrix}
\] (24)

The target measurement value \(Z\) is composed of distance \(r\), azimuth angle \(a\) and pitching angle \(e\), and assuming that \(v_r, v_a, v_e\) are uncorrelated white Gaussian noise, \(\sigma_r^2, \sigma_a^2, \sigma_e^2\) are the variances respectively, \(R_k = \text{diag}\{\sigma_r^2, \sigma_a^2, \sigma_e^2\}\) is the measurement noise variance.

3) The measurement data is obtained from tracking the actual random moving target, and the sampling period is \(T=0.00625s\), the measurement error is \(\sigma_r=5m, \sigma_a=\sigma_e=3"\), here make the experiment last for ten seconds, and obtain the experiment data.

4) Compare the performance of the augmented state EKF algorithm and the augmented state UKF algorithm. The table 1 and table 2 are the root-mean-square error and mean error statistical data of the two filtering algorithms about the CV and CA model, the figure 1 and figure 2 are the prediction error curve respectively of the two filtering algorithms about the CV and CA model.

| Table 1. The estimation performance comparison of two filtering algorithm about CV model. |
|---------------------------------|----------|----------|
| algorithm | EKF | UKF |
| Distance RMSE(km) | 0.1250 | 0.0110 |
| Distance ME(km) | 0.0970 | 0.0010 |
| Azimuth angle RMSE(deg) | 0.1163 | 0.0969 |
| Azimuth angle ME(deg) | 0.0979 | 0.0960 |
| Pitching angle RMSE(deg) | 0.1151 | 0.1110 |
| Pitching angle ME(deg) | 0.1403 | 0.1394 |

| Table 2. The estimation performance comparison of two filtering algorithm about CA model. |
|---------------------------------|----------|----------|
| algorithm | EKF | UKF |
| Distance RMSE(km) | 0.1560 | 0.0110 |
| Distance ME(km) | 0.0200 | 0.0010 |
| Azimuth angle RMSE(deg) | 0.1104 | 0.0965 |
| Azimuth angle ME(deg) | 0.0980 | 0.0977 |
| Pitching angle RMSE(deg) | 0.1138 | 0.1107 |
| Pitching angle ME(deg) | 0.1371 | 0.1384 |

Figure 1. The prediction error curve of CV model.
From the error curve we can see: for the two the target motion models, the prediction error curves of augmented state EKF algorithm and augmented state UKF algorithm are convergent, the settling time of augmented state EKF algorithm is long, and the filtering initial error is large. In the premise of ensuring the convergence of the error curve, compared with the augmented state EKF algorithm, the settling time of augmented state UKF algorithm is shorter, and the distance error is greatly reduced, the pitch angle error and azimuth angle error are also effectively improved. Through the analysis of the table data we can see, the performances of the augmented state UKF algorithm are better than that of the augmented state EKF algorithm. The above experiment results show that for the prediction filtering problem of measurement delay systems, the proposed augmented state EKF algorithm and the augmented state UKF algorithm are feasible and effective, which augmented state UKF algorithm filtering performance is better.

5 CONCLUSION

In practice, measurement equations with time delay is very common, such kind of system is more complex than the system without time delay, so traditional Kalman filter algorithm cannot be directly applied to it. In this paper, based on the augmented state method, the augmented state Kalman filter algorithm structure is proposed for nonlinear measurement delay system, and for the nonlinear measurement delay photoelectric tracking system, the tracking random moving target experiments are made.

6 ACKNOWLEDGEMENTS

Thanks for the support of the project named Theoretical Study of Prediction Filtering Algorithm For Measurement Time Delay System of Provincial Natural Science Foundation of China (No. 20150101048JC) and the project named Nonlinear Predictive Control and Multi-Objective Energy Saving Optimization of Boiler and Turbine System of Natural Science Foundation of China (No.61503072).

REFERENCES