Research on D-GERT Model of Joint Operational Situation Assessment Based on Information Flow

Xiao-li ZHANG\textsuperscript{1, a*}, Si-feng LIU\textsuperscript{1} and Zhi-geng FANG\textsuperscript{1}

\textsuperscript{1}School of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, China  
\textsuperscript{a}sweetain@nuaa.edu.cn  
*Corresponding author

Keywords: Joint Operations, Situation Assessment, Operational Information Flow, Graphical Evaluation and Review Technique (GERT).

Abstract. To solve the problem of inaccurate description of battlefield information changes in the process of joint system operations, this paper constructed a joint operational situation assessment D-GERT network model based on information flow, on the basis of the GERT network. And the calculation method of the threat degree, the increment of battlefield information value and the importance of network nodes were given. Finally, MATLAB was used to simulate and verify the model. The results showed that the method in this paper conforms to objective reality and can provide good method support for the commander's combat decision-making.

1. Introduction

Joint theater operations are system operations led by the operational command and control organization and jointly implemented by all forces, driven by operational information flow\cite{1}. Combat information flow is the description of the information value flow of battlefield commands, information, and data. The analysis process of information value flow is the process of transforming information advantages into decision-making and action advantages. Situation assessment refers to predicting the possible trend of the situation on the basis of in-depth analysis of various information on the battlefield, and providing decision support for the commander to adjust and optimize the deployment of actions\cite{2}. Accurately and efficiently assessing the battlefield situation within a limited time is the basis for the command department to execute firepower allocation, and has an important impact on the combat effectiveness of the system.

At present, the research methods of system combat situation assessment at home and abroad mainly include Bayesian network, evidence theory, multi-attribute decision-making model, fuzzy comprehensive evaluation method. The first two have not done in-depth research on the structure of the combat system, and the logical structure relationship between the latter two is not enough to accurately describe the complex logical relationship of the combat system, so this article considers the introduction of graphical review technology to establish the architecture model.

Graphical Evaluation Review Technique (GERT) was first formally proposed by Pritsker in the United States in 1966\cite{3}. It contains various nodes with different logical characteristics, allowing self-loops and loops to exist, which is more suitable for the establishment of the logical structure of the combat system\cite{4}. There are many researches in the field of GERT network. But the existing GERT network also has certain limitations, that is, its transmission signal flow cannot take into account the influence of time factors while considering the value of information. In the process of system operations, battlefield information is constantly changing and updating with time, and information is often constrained by time factors. Therefore, this article uses the battlefield information flow relationship as the transmission signal flow, and adds longitudinal time variables to the network to establish a D-GERT network model to evaluate the battlefield situation.
2. Joint Operation D-GERT Network Model

2.1. Operational Information Value

In the combat system, different equipment subjects are linked together according to the flow of battlefield information, forming a complex GERT network. Each equipment body constitutes the nodes of the GERT network, the battlefield information transmission relationship constitutes the edge of the network, and the battlefield information value constitutes the flow in the network.

According to the principle of information service for combat charges, this article measures the value of information transmitted in the combat system in terms of information completeness, information accuracy, and information timeliness. Information completeness refers to the completeness of information transmission between front and rear equipment nodes. Information accuracy refers to the accuracy of information transmission between front and rear equipment nodes in battlefield situational awareness. Information timeliness refers to the efficiency of information transmission between equipment nodes during system operations.

Definition 1: The information completeness, accuracy and timeliness of joint system operations at time $t$ are respectively:

\[ Comp(t) = \sum_{i=1}^{m} offer_k(t) \] \hspace{1cm} (1)

\[ Accu(t) = \frac{right(t)}{all(t)} \] \hspace{1cm} (2)

\[ Time(t) = \frac{1}{delay_i(t) + delay_j(t)} \] \hspace{1cm} (3)

In the formula, $offer_k(t)$ represents the $k$-th type of information provided by the previous equipment node to the next equipment node; $need_k(t)$ represents the $k$-th complete information required by the next equipment node; $m$ represents the total number of information types required by the next equipment node; $right(t)$ represents the correct amount of information provided by the previous equipment node to the next equipment node; $all(t)$ represents the total amount of information required by the next equipment node. $delay_i(t)$ refers to the information processing time delay of node $i$, and $delay_j(t)$ refers to the information transmission time delay between node $i$ and node $j$.

2.2. Design of Basic Transmission Unit of D-GERT Network Based on Operational Information Flow

According to the information value measurement method in Section 2.1, based on the combat information flow, the basic transmission unit of the D-GERT model is designed as follows:

Definition 2: As shown in Fig. 1, $i$ and $j$ respectively represent network nodes, $H_{ij}(t)$ represents the battlefield information value flow from $i$ to $j$ at time $t$, and $p_{ij}(t)$ represents the realization probability of battlefield information value from node $i$ to node $j$ at time $t$, $Comp_{ij}(t)$, $Accu_{ij}(t)$, $Time_{ij}(t)$ respectively represent the completeness, accuracy and timeliness of information from node $i$ to node $j$. Then the value of the battlefield information transmitted from node $i$ to node $j$ is

\[ Value_{ij}(t) = \alpha_1 Comp_{ij}(t) + \alpha_2 Accu_{ij}(t) + \alpha_3 Time_{ij}(t) \] \hspace{1cm} (4)

Among them, $\alpha_1, \alpha_2, \alpha_3$ are the weights of information completeness, information accuracy, and information timeliness, respectively, and $\sum_{i=1}^{3} \alpha_i(i) = 1$.

Figure 1. The basic unit of joint operations D-GERT network.

Theorem 1: To simplify processing, suppose that the transfer parameters $Comp_{ij}(t)$, $Accu_{ij}(t)$,
Time \( t \) from node \( i \) to node \( j \) are independent of each other, and the moment generating function of each parameter exists, then the moment generating function \( M_{Value} \) of this activity is

\[ M_{Value}(s_1, s_2, s_3) = M_{Comp}(s_1) \cdot M_{Accu}(s_2) \cdot M_{Time}(s_3) \] (5)

Among them, \( s_1, s_2, \) and \( s_3 \) are arbitrary real numbers.

Proof: Since \( Value_i(t) = \alpha_1 Comp_i(t) + \alpha_2 Accu_i(t) + \alpha_3 Time_i(t) \) , and \( Comp_i(t), Accu_i(t), Time_i(t) \) are independent of each other, we can get

\[ M_{Value}(s_1, s_2, s_3) = E(e^{\alpha_1 Comp(t)} \cdot e^{\alpha_2 Accu(t)} \cdot e^{\alpha_3 Time(t)}) = E(e^{\alpha_1 Comp(t)}) \cdot E(e^{\alpha_2 Accu(t)}) \cdot E(e^{\alpha_3 Time(t)}) \]

\[ = E(e^{\alpha_1 Comp(t)}) \cdot e^{\alpha_1 s_1} \cdot e^{\alpha_2 s_2} \cdot e^{\alpha_3 s_3} \]

The proof is complete.

It can be seen that the battlefield information flow transfer function from node \( i \) to node \( j \) in the combat system D-GERT network at time \( t \) is

\[ W(s_1, s_2, s_3, t) = p_x(t) \cdot M_{Value}(s_1, s_2, s_3) = p_x(t) \cdot M_{Comp}(s_1) \cdot M_{Accu}(s_2) \cdot M_{Time}(s_3) \] (7)

According to the Mason formula, the equivalent transfer function of the combat system D-GERT network at time \( t \) can be calculated, and the equivalent transfer probability is as follows:

Theorem 2: The equivalent transmission probability of the combat system D-GERT network at time \( t \) is

\[ p_x(t) = W(s_1, s_2, s_3, t) \mid s_1 = s_2 = s_3 = 0 = W_0(0, 0, 0, t) \] (8)

Proof: According to the characteristics of the moment generating function, when \( s_1 = s_2 = s_3 = 0 \),

\[ W(s_1, s_2, s_3, t) \mid s_1 = s_2 = s_3 = 0 = p_x(t) \cdot M_0(s_1, s_2, s_3) \mid s_1 = s_2 = s_3 = 0 \]

\[ = p_x(t) \cdot M_0(s_1) \cdot M_0(s_2) \cdot M_0(s_3) \mid s_1 = s_2 = s_3 = 0 = p_x(t) \cdot [E(e^s)] \]

The proof is complete.

Based on the above definition, the basic structure of the joint operations D-GERT network based on the random process time trajectory is shown in Fig. 2 (the real numbers \( s_1, s_2, \) and \( s_3 \) are omitted in the figure).

![Figure 2. Schematic diagram of the basic structure of D-GERT network.](image)

The calculation method of the equivalent transfer function of the D-GERT network model is the same as that of the static model, and the introduction of time parameters has no effect on the calculation method of the equivalent transfer function and the equivalent moment generating function.
3. Joint Operational Situation Assessment Method Based on D-GERT Network

3.1. Calculation of Target Threat Degree in Joint Operations

The essence of situation assessment is to establish a functional relationship between situation characteristics and the degree of completion of combat missions. The characteristics of the combat situation are usually described by the status indicators of the combat system. This article describes the battlefield situation of the combat system with the degree of target threat, the increment of battlefield information value and the importance of network nodes.

The target threat level refers to the possibility that the opponent's forces will successfully attack our combat system. The incoming target poses a threat to our system, and it can only occur when our combat system fails to successfully complete the combat mission. Therefore, the definition of the threat level of joint operations targets is as follows:

Definition 3: The threat of the battlefield target at time \( t \) in joint operations is:

\[
\lambda(t) = 1 - p_s(t)
\]  

Among them, \( p_s(t) \) is the probability of the system successfully completing the combat mission. In the D-GERT network model,

\[
p_s(t) = W_{s}(s_1, s_2, s_3, t) |_{s_2 = s_3 = 0}
\]  

Therefore, the battlefield threat level at time \( t \) of the joint operation D-GERT model is:

\[
\lambda(t) = 1 - p_s(t) = 1 - W_{s}(s_1, s_2, s_3, t) |_{s_2 = s_3 = 0}
\]  

3.2. Incremental Calculation of Information Value of Joint Operations

The value of battlefield information refers to the extent to which combat information as a resource meets the needs of various information users such as commanders. The value of battlefield information gradually increases with the system's combat process, and its increment represents the ability of the combat system to accumulate information value. Therefore, the calculation method for the increment of information value of joint operations is defined as follows:

Definition 4: The increment of the \( k \)-th information value in the network value transmission at time \( t \) is

\[
E_k(t) = \frac{1}{\alpha_k} \left. \frac{\partial}{\partial E_i} \left( W_{s}(s_1, s_2, s_3, t) \right) \right|_{E_i = 0}
\]  

Therefore, the total value increment of the battlefield information value flow at time \( t \) is

\[
E(t) = E[\alpha_{Comp}(t) + \alpha_{Accu}(t) + \alpha_{Time}(t)] = \sum_{i=1}^{n} \frac{\partial}{\partial E_i} \left( W_{s}(s_1, s_2, s_3, t) \right) |_{E_i = 0}
\]  

3.3. Calculation of Node Importance of Joint Operations D-GERT Network

Researching and calculating the importance of nodes in the joint operations network can effectively observe and control the changes in the importance of each node in the network. It is then used to determine whether key equipment in the combat system has changed over time, providing a basis for subsequent development and equipment replacement. The importance of network nodes in the situation assessment model is defined as follows:

Definition 5: For all activities flowing from node \( i \), let \( M_i(c, s_1, s_2, s_3, t) = c \cdot M_i(s_1, s_2, s_3, t) \), \( s_1, s_2, s_3 \), and \( c \) be any real numbers, and calculate the equivalent transfer function \( W_{s}(c, s_1, s_2, s_3, t) \). Then the importance of node \( i \) at time \( t \) in the combat system D-GERT network is:
\[
\eta(t) = \left[ \frac{\partial M_s(c,t)}{\partial c} \right]_{c=0} \sum_i \left[ \frac{\partial M_s(c,t)}{\partial c} \right]_{c=0}
\]

Among them, 
\[
M_s(c,t) = M_s(c,s_1,s_2,s_3,t) \bigg|_{s_1=s_2=s_3=0} = \begin{bmatrix} W_s(c,s_1,s_2,s_3,t) \\ W_s(0,0,0,t) \end{bmatrix} \bigg|_{s_1=s_2=s_3=0}.
\]

4. Case Application and Analysis

4.1. Operational Situation Assessment of a Joint System

According to the operational scenario, assume that the GERT random network diagram of a certain operational system during joint operations is shown in Fig. 3 (For simplification, the time axis is omitted in the figure). The meaning of each node is shown in Table 1.

![GERT network structure diagram](image)

Figure 3. GERT network structure diagram of a certain combat system joint combat process.

Table 1. The meaning of GERT network nodes in a joint operation process of a combat system.

<table>
<thead>
<tr>
<th>Node number</th>
<th>Meaning</th>
<th>Node number</th>
<th>Meaning</th>
<th>Node number</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X-band reconnaissance radar</td>
<td>4</td>
<td>Fire control system</td>
<td>7</td>
<td>Standard-3 missile</td>
</tr>
<tr>
<td>2</td>
<td>Communication information</td>
<td>5</td>
<td>GBI interceptor</td>
<td>8</td>
<td>Hit success</td>
</tr>
<tr>
<td>3</td>
<td>Command and control center</td>
<td>6</td>
<td>Tracking guidance radar</td>
<td>9</td>
<td>Hit failed</td>
</tr>
</tbody>
</table>

According to the calculation method in Section 2.1, the distribution of three kinds of information value is simulated according to the simulation data, and the probability and information value distribution of each program in the system's combat process are shown in Table 2, where \( p_1(t) \), \( p_2(t) \), \( p_3(t) \) is the probability value that changes with time.

Table 2. D-GERT network parameter table of a certain combat system joint combat process.

<table>
<thead>
<tr>
<th>Arc</th>
<th>Probability</th>
<th>Completeness</th>
<th>Accuracy</th>
<th>Timeliness</th>
<th>Arc</th>
<th>Probability</th>
<th>Completeness</th>
<th>Accuracy</th>
<th>Timeliness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>1</td>
<td>6(8.1.5)</td>
<td>5(2.1.3)</td>
<td>7(5.1.10)</td>
<td>5-6</td>
<td>1</td>
<td>4(0.0.5)</td>
<td>6(4.1.1)</td>
<td>5(5.2.10)</td>
</tr>
<tr>
<td>2-3</td>
<td>1</td>
<td>8(0.1.4)</td>
<td>6(5.1.9)</td>
<td>7(6.1.3)</td>
<td>5-5</td>
<td>1</td>
<td>1(1.0.3)</td>
<td>1(0.0.3)</td>
<td>3(1.1.4)</td>
</tr>
<tr>
<td>3-4</td>
<td>1</td>
<td>7(2.2.3)</td>
<td>4(2.1.3)</td>
<td>5(8.2.1)</td>
<td>6-7</td>
<td>1</td>
<td>6(5.1.1)</td>
<td>5(0.2.1)</td>
<td>6(0.1.2)</td>
</tr>
<tr>
<td>4-5</td>
<td>( p_1(t) )</td>
<td>5(4.1.2)</td>
<td>7(7.1.6)</td>
<td>3(5.0.7)</td>
<td>7-8</td>
<td>( p_3(t) )</td>
<td>7(1.0.9)</td>
<td>5(3.1.9)</td>
<td>4(9.1.5)</td>
</tr>
<tr>
<td>4-2</td>
<td>( 1- p_1(t) )</td>
<td>5(1.8.0.2)</td>
<td>7(1.3.0.1)</td>
<td>2(7.0.4)</td>
<td>7-9</td>
<td>( 1- p_3(t) )</td>
<td>2(2.0.4)</td>
<td>4(0.0.7)</td>
<td>1(8.0.7)</td>
</tr>
</tbody>
</table>

According to the Mason formula, the equivalent transfer function from source node 1 to node 8 is

\[
W_k = \frac{W_{12}W_{23}W_{34}W_{45}W_{56}W_{67}W_{78}}{1-W_{12}W_{23}W_{34}W_{45}W_{56}W_{67}W_{78}}
\]

According to expert knowledge, assuming that the weights of information completeness, accuracy and timeliness are \( \alpha_1=0.3, \alpha_2=0.3, \alpha_3=0.4 \), then the moment generating function from node \( i \) to node \( j \) is

\[
M_{\text{distribution}}(s_1,s_2,s_3) = M_{\text{completeness}}(\alpha_1) \cdot M_{\text{accuracy}}(\alpha_2) \cdot M_{\text{timeliness}}(\alpha_3) = M_{\text{completeness}}(0.3) \cdot M_{\text{accuracy}}(0.3) \cdot M_{\text{timeliness}}(0.4)
\]

(17)

1) Target threat. According to the equivalent transfer function from node 1 to node 8, the equivalent transfer probability of successfully completing the joint operation process at time \( t \) can be calculated as

\[
p_k(t) = \left. W_k(s_1,s_2,s_3,t) \right|_{s_1=s_2=s_3=0} = p_3(t)
\]

(18)
From formula (10), the target threat degree of the system is

$$\lambda(t) = 1 - p_x(t) = 1 - p_i(t)$$

(19)

2) Battlefield information value increment. From formula (14), the total value increment of the battlefield information value flow at time \( t \) is

$$E(t) = \sum_{i=1}^{n} \left[ W_{(c,s_1,s_2,s_3,t)} - W_{(0,0,0,0)} \right]$$

(20)

3) Network node importance. From formula (15), the importance of each node at time \( t \) can be obtained. Take node 5 as an example,

$$M_s^5(c,t) = M_s^5(c,s_1,s_2,s_3,t)_{W(c,s_1,s_2,s_3,t)} = \frac{W_{(c,s_1,s_2,s_3,t)}}{W_{(0,0,0,0)}} = \frac{p_c e^t}{1-(1-p_i)}$$

(21)

Then the importance of node 5 at time \( t \) is:

$$\eta(5,t) = \frac{\partial M_s^5(c,t)}{\partial c} = \frac{p_i}{3p_i p_1 + p_1 + 3p_2}$$

(22)

In the same way, the importance of each equipment of the combat system is shown in Table 3.

Table 3. The importance of each equipment in the combat process of a joint system.

<table>
<thead>
<tr>
<th>Node ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance ( \eta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
<td>( \Delta )</td>
</tr>
</tbody>
</table>

In the table, \( \Delta = \frac{p_i p_2}{3p_1 p_2 + p_1 + 3p_2} \).

4.2. Simulation Analysis

Assuming \( p_i(t) = 0.9 \), \( p_1(t) = 0.02 + 0.7 \), \( p_2(t) = -0.12 + 0.23 + 0.8 \), when \( t = 0,1,2,\ldots,10,\ldots \), use MATLAB simulation to consider the influence of time factors on the threat of joint combat targets, the increase in battlefield information value and the importance of nodes (the importance of nodes is represented by node 5), such as shown in Fig. 4.

![Simulation Results](image)

Figure 4. The influence of time factor on target threat degree, battlefield information value increment, and node importance.

From the simulation results, we can clearly see the degree of threat of joint combat targets, the increment of battlefield information value, and the process of network node importance over time. As time progresses, the threat of combat targets has gradually increased, the increase in the value of battlefield information has gradually decreased, and the importance of some nodes has gradually decreased. Therefore, under the scenario given in this example, the combat system should promptly improve the combat plan and combat plan, and adopt methods such as replacing equipment or improving the system structure to improve the overall combat capability of the system, providing a basis for the development of the battlefield situation and future situation planning.
5. Conclusion

The battlefield situation is constantly changing, and battlefield information is constantly updated. The traditional GERT network model can no longer meet the needs of the battle system situation assessment. Therefore, this paper adds a time axis to the GERT network model, establishes a D-GERT network model based on combat information flow, and builds a situation assessment model on this basis to calculate the threat level of joint combat targets, the increment of battlefield information value and the importance of network nodes. Simulation experiments show that the method proposed in this paper can correctly evaluate the situation of joint system operations and provide commanders with a basis for command decision-making.

References


