Research on Risk of Stock Index Futures Market Based on EGARCH Model

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Abstract. Volatility is an important indicator to measure the risk of financial market. The EGARCH model is established by using the logarithmic yield of the China Securities Index (CSI) 300 stock index futures from January 2014 to August 2017, then the volatility and yield of the data in the last three months are back tested to confirm the reliability of the model. Furthermore, based on the established EGARCH model, the future volatility and yield are forecasted, and the value at risk of stock index futures contract is calculated. At last, the error of our EGARCH model is determined by compare the predicted yield with actual value.

Introduction

It is well known that futures markets can play important roles such as price discovery and risk management in an economy. It can also contribute to the market efficiency. Commodity futures markets have existed for a long time, but the stock index futures has been introduced in the developed economies not long ago, and it is still new or even non-existent for most emerging economies.

Since the stock index futures were listed in China on April 16, 2010, the market size has continued to increase, and the trading volume has even surpassed the total volume of the stock market. The issuance of stock index futures makes our financial derivatives more complete and increases investors’ speculation. Stock index futures are more flexible than stocks and it can make short trades, so there were more profit opportunities for investors. The $T + 0$ trading mode of stock index futures also makes the market more liquid than the $T + 1$ trading mode of stocks. These advantages attract more speculators and hedgers. The margin trading maybe not only increase investors' profits but also expands their losses, so the risk measurement of such highly leveraged financial derivatives is particularly important. It is of great significance to study which kind of stock index futures should investors purchase and what size of their risks and profits.

China’s futures market started late, but Chinese domestic scholars have done a lot of research on the futures market (see Yang et al. [1], Hou and Li [2,3]). Value at risk (VaR) is the main instrument to measure the market risk. Jorion [4] defined VaR as the maximum loss of a financial position in a given period of time with a given probability. Therefore, VaR has become a standard method to measure market risk. For the study of volatility, scholars have focused on the GARCH models. Under different hypothesis distributions, Zhang and Lan [5] gave the GARCH-type model with continuous and jump variation for stock volatility and its empirical study in China. Zhou et al. [6] studied the measurements accuracy of VaR based on GARCH models using spot transactions in Shanghai and London gold markets as objects, and they concluded that the Shanghai gold market was more risky than the London gold market. Lin [7] used the GARCH model to analyze Growth Enterprises Market (GEM) volatility and concluded that the AR(1)-GARCH(1,1) model is more effective in predicting GEM volatility. Liu and Shi [8] applied the ARCH model whose residuals obey t distribution to foreign exchange risk, and founded that the VaR value predicted by the ARCH model can be fitted to the actual situation of the daily fluctuation rate of USD/RMB exchange rate.
In this paper, the EGARCH model was established by using the logarithmic yield of the CSI 300 stock index futures from January 2014 to August 2017, then the volatility and yield of the data in the last three months were back tested to confirm the reliability of the model. Furthermore, based on the established EGARCH model, the future volatility and yield are forecasted, and the value at risk of stock index futures contract is calculated to provide investors with decision-making assistance.

This paper is organized as follows. In Section 2, we give the brief introduction to the theory. In Section 3, we give empirical analysis. We establish the EGARCH model, forecast the future volatility and yield, and calculate the value at risk of stock index futures contract. The last part, Section 4, is the summary.

Preliminaries

Logarithmic Yield
The natural logarithm of simple gross yield of an asset is called a continuous compound yield or a logarithmic yield. Its formula is as follows:

\[ r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}}. \]  

(1)

When dealing with financial data, we generally use logarithmic yield mainly because we have this formula:

\[ r_t[k] = \ln(1 + R_t[k]) = \ln[(1 + R_t)(1 + R_{t-1})\cdots(1 + R_{t-k+1})] = \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) = r_t + r_{t-1} + \cdots + r_{t-k+1}, \]  

(2)

that is, the continuous compound multi-period yield is the sum of the consecutive composite single-period yields. In addition, the logarithmic yield makes the data smoother and more easily to be processed.

ARMA Model
Autoregressive moving average (ARMA) model is introduced by Box and Jenkins [9], whose specific structure is as follows:

\[ r_t - \varphi_0 r_{t-1} - \cdots - \varphi_p r_{t-p} = \varphi_0 + a_t - \theta_0 a_{t-1} - \cdots - \theta_q a_{t-q}, \]  

(3)

where \( a_t \) is the white noise. The ARMA model can be selected when there is no obvious truncation character in the autocorrelation function (ACF) map and the partial autocorrelation function (PACF) map of stationary sequence. The concept of ARMA model is closely related to volatility modeling, which is an important connection for fitting.

EGARCH Model
Since Engle [10] first proposed the autoregressive conditional heteroscedasticity (ARCH) model, many of scholars have further developed and popularized it to make the ARCH model widely used. The generalized autoregressive conditional heteroscedasticity (GARCH), proposed by Bollerslev [11], provides an effective method of describing volatility and analyzing the financial time series. It is by far the most common and convenient fitting model of heteroscedasticity sequence, but it also has some shortages. In order to expand the range of GARCH model and improve the fitting accuracy of GARCH model, Nelson [12] proposed the EGARCH model, which includes mean equation, distribution hypothesis, and conditional variance equation, as follows:
\[
\begin{align*}
\begin{cases}
x_t = \varphi + \sum_{i=1}^{m} \varphi_i x_{t-i} + \sum_{j=1}^{m} \theta_j a_{t-j} + a_t, \\
a_t = \sigma_t \epsilon_t, \\
\ln(\sigma_t^2) = \alpha_0 + \left[ \sum_{i=1}^{m} \alpha_i \frac{a_{t-i}}{\sigma_t} + \sum_{i=1}^{m} \gamma_i \left| \frac{a_{t-i}}{\sigma_t} \right| \right] + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-j}^2),
\end{cases}
\end{align*}
\]
where \( \epsilon_t \sim N(0, \sigma^2) \).

**Value at Risk**

Value at risk (VaR) introduced by Jorion [4], gives the maximum loss of an asset at a certain time period under a given confidence level, as follows:

\[
P\{R_t > -\text{VaR}_t\} = 1 - \alpha,
\]

where \( R_t \) is the yield of asset in \( t \) period, and \( \text{VaR}_t \) means the VaR value at \( \alpha \) level in \( t \) period and the value is positive. According to the above VaR, the VaR of EGARCH model can be obtained:

\[
\text{VaR} = Z_\alpha \hat{\sigma}_t - \mu_t,
\]

where \( Z_\alpha \) represents the \( \alpha \) quantile under standard normal distribution and \( \mu_t \) is the mean.

**Empirical Analysis**

We adopt the closing price of main contract of China Securities Index (CSI) 300 stock index futures from January 2, 2014 to August 30, 2017, and obtain the logarithmic yield of this data based on the closing price. The main contract is the futures contract with the most volume and open interest in the month. Therefore, it is better to choose the main contract of the month to reflect the price fluctuations of the stock index futures market. This paper uses R to analyze and disposal data.

**Stability Test**

First, we use R to draw a sequence diagram of the logarithmic return rate of the stock index futures.

![Sequence diagram](image)

Figure 1. Sequence diagram.

From Figure 1, we can see that the series is basically stable. Then the white noise test is performed.
From Figure 2, the P value is less than the significance level $\alpha$, so the null hypothesis is rejected and the sequence is a non-white noise sequence. Based on the above judgments, the sequence can be modeled. When the ACF map and the PACF map of the sequence are found to be truncated, it is more reasonable to construct an ARMA model.

ARCH Test

It can be seen from Figure 1 that the fluctuation between May 2015 and August 2015 continued to be large, and the fluctuation between October 2016 to May 2017 continued to be small, these showing the clustering effect. The clustering effect means that the variance of the sequence is homogenous in the whole sequence observation period, but it is significantly different from the expectation variance in a few time quantum, so we can use the conditional heteroscedasticity model. The Portmanteau Q test was performed on the sequence.

```
> for(i in 1:2) print(Box.test(ty,lag=6*i))

Box-Pierce test
data:  ty
X-squared = 38.427, df = 6, p-value = 9.268e-07

Box-Pierce test
data:  ty
X-squared = 58.517, df = 12, p-value = 4.2e-06
```

Figure 2. White Noise Test.

ARCH Test

From Figure 3, the P value obtained was less than the significance level $\alpha$, and the original hypothesis was rejected. The variance of the sequence was considered non-homogeneous and relevant.

EGARCH Model

Because of the variance of EGARCH model is exponential, the variance is always positive, and no constraints can be applied to reduce the calculation. Secondly, the positive and negative values of stochastic disturbance $\varepsilon_t$ can make different effects, which can better reflect the fluctuation of the sequence. Based on above advantages, fitting EGARCH model and the results are as follows:

\[
\begin{align*}
\ln(\sigma_t^2) &= -0.023 + 0.023 \frac{\varepsilon_{t-1}}{\sigma_t} + 0.147 \ln(\sigma_{t-1}) + 0.996 \ln(\sigma_{t-1}), \\
\sigma_t &= \sigma_{t-1} a_t, \\
x_t &= 0.000724 - 0.498 x_{t-1} - 1.007 x_{t-2} - 0.048 x_{t-3} - 0.02 x_{t-4} + a_t + 0.466 a_{t-1} + 0.988 a_{t-2}, \\
\end{align*}
\]

Model Back Testing

There are 895 stock index futures data selected in this paper, and the last 100 data are selected as out-of-sample data, and the rest are as sample data. Through backtesting the last 100 data, we can
examine the prediction effect and reduce the over-fitting of the historical data curve. The following two graphs are the rolling forecast of yield and volatility. In Figure 4, the predict yield range basically contains the real rate of return. And in Figure 5, the volatility is basically consistent with the actual fluctuation rate.

Figure 4. Back test Yield.                              Figure 5. Back test Volatility.

Value at Risk
Observing the historical data and histogram (see Figure 6), we can see that its distribution is similar to normal distribution from Figure 6. Therefore, it can be assumed that the future yield may also comply with the normal distribution. Then we can obtain the VaR at the 95% confidence level though Formula (6). From the 100 data obtained, it can be seen that VaR is concentrated in (0.022; 0.034), and the loss of investors is increasing as time goes by.

Figure 6. Histogram.

Forecast the Yield
The EGARCH model not only predicts the volatility, but also predicts the yield (logarithmic rate of return) over the next 100 days. According to Formula (1), the forecast value of yield can be obtained, the following Table 1 was shown comparation of real rate of return and forecast value of yield:
Table 1. Forecast of Yield.

<table>
<thead>
<tr>
<th>Actual value</th>
<th>Forecast value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.034552382</td>
<td>0.0449809</td>
<td>0.301817629</td>
</tr>
<tr>
<td>0.071433213</td>
<td>0.0725036</td>
<td>0.014984444</td>
</tr>
</tbody>
</table>

For the forecast of yield, we note that the more sample data makes the more accurate prediction.

Summary

According to the model simulation and parameter estimation, we find that stock index futures has volatility aggregation, the fluctuation is stable, the yield of CSI 300 stock index futures has heteroskedasticity. By constructing the EGARCH model, we eliminate the ARCH effect of the stock index futures yield, and solve the volatility cluster. Based on our EGARCH model, both the volatility and yield were predicted in actual range. Therefore, we conclude that the EGARCH model was effective.

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