Empirical Research of the Pricing of Shanghai 50 ETF Options Based on Volatility and Fractional B-S Model

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Abstract. By analyzing the closing price of Shanghai 50ETF options from 2017 to 2018, the authors find that the logarithmic return of 50ETF options has fractional characteristics. The fractional B-S model is used to analyze 50ETF option pricing. Firstly, the authors make R/S analysis of 50ETF option logarithmic return and get the Hurst index of the fractional B-S model. Secondly, the authors build the GARCH (1,1) model to characterize the volatility of 50ETF option’s yield and use the R software to get the historical volatility. Then the authors use fractional B-S model and the Matlab software to get the implied volatility. The authors make an empirical analysis on the pricing of 50ETF option based on the two kinds of volatility. The authors calculate the AMSE of results from the models with the market price, and compare them fully. It is found that the fractional B-S models based on the two kinds of volatility have good fitting effect on the pricing of 50ETF option, and the implicit volatility model has better fitting effect on the pricing of 50ETF option than the historical volatility model.

Introduction

With the listing of Shanghai 50ETF options in February 9, 2015, capital market of China entered the era of options. Investors can make use of options to carry out risk management. The pricing of option has always been a hot issue in various fields.

The most important variable in pricing model is volatility. How to describe the volatility has become a hot issue in various fields. Zhang Chen et al. (2015)\textsuperscript{[1]} applied the GARCH model and fractional Brownian motion to the study of carbon options. The volatility is fitted by GARCH model. Wang Peng and Yang Xinglin (2016)\textsuperscript{[2]} revised the defect of deviation of return distribution by using Sigma shape polynomial, and constructed B-S model with time-varying volatility. Through empirical research, we find that time-varying volatility has higher pricing accuracy than the constant volatility in classical B-S mode. Hao Meng et al. (2017)\textsuperscript{[3]} combined GARCH model with generalized hyperbolic distribution, established GARCH-GH model to describe the return and volatility characteristics of Shanghai 50ETF options. Fang Yan et al. (2017)\textsuperscript{[4]} found that both B-S-M option pricing model and Monte Carlo simulation method can accurately and effectively simulate the 50 ETF option price of Shanghai Stock Exchange.

Compared with the existing literatures, this paper uses implicit and historical volatility model to price Shanghai 50ETF options, and compares the differences between the results from the implicit volatility model and the results from historical volatility model, explaining the reasons for the differences in information.

Models

Under the complete market with no arbitrage and risk neutral conditions, the execution price is X, the maturity date is T, the price of underlying asset at time t is S, and the price of standard European call option at time t is:
\[ C = SN(d_1) - X e^{-\gamma(T-t)} N(d_2), \]  
\[ d_1 = \frac{\ln(S/X) + r(T-t) + \frac{1}{2} \sigma^2 (T^{2H} - t^{2H})}{\sigma \sqrt{T^{2H} - t^{2H}}}, \]  
\[ d_2 = \frac{\ln(S/X) + r(T-t) - \frac{1}{2} \sigma^2 (T^{2H} - t^{2H})}{\sigma \sqrt{T^{2H} - t^{2H}}} = d_1 - \sigma \sqrt{T^{2H} - t^{2H}}. \]

In this paper, we use the GARCH (1,1) model to predict volatility.

\[ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \]  

Empirical Analysis

Sample Selection

We select the daily closing prices of the Shanghai 50ETF option from January 3th, 2017 to March 30th, 2018. The 303 data are from wind information.

The Characteristic Test of the Logarithmic Yield of Shanghai 50ETF Option

Table 1 lists the logarithmic return rate series of Shanghai 50ETF option. From the table 1, we can know that the logarithmic return rate sequence of Shanghai 50ETF option does not obey the normal distribution assumption, that is, there is a phenomenon of “spikes and thick tails”.

<table>
<thead>
<tr>
<th>Statistical index</th>
<th>mean</th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>ADF statistic</th>
<th>J-B statistics</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics</td>
<td>0.000598</td>
<td>0.008785</td>
<td>-0.789768</td>
<td>6.519782</td>
<td>-3.779</td>
<td>187.2878</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

It is known from table 1 that the ADF statistic is -3.779. Therefore, we have a sufficient reason to explain that the logarithmic return rate sequence of Shanghai 50 ETF option is a stable sequence.

Using R/S Method to Test Fractional Characteristics

We use \( \log(R/S)_n = \log P + H \log n \) to get the coefficient H. We regard \( \log(R/S)_n \) as dependent variable and \( \log n \) as independent variable and then regress them by getting data. We can get the linear equation.

\[ \log(R/S)_n = -0.0031735 + 0.5162218 \log n, \]  

The H value is 0.5162218 and is bigger than the critical value of 0.5. Therefore, the Shanghai 50ETF option market has long memory and fractional features.

Estimating the Volatility

Historical Volatility Based on GARCH Model

In the financial market, the prices of asset will show periodicity and clustering property, which will make the variance of asset’s prices show heteroscedasticity. Therefore, ARCH effect test is needed to fit GARCH model. So we use the LM test method to carry out the ARCH-LM test for OLS residuals from one to five lag phases. The results are as follows.
Table 2. ARCH effect test.

<table>
<thead>
<tr>
<th>lags(p)</th>
<th>chi2</th>
<th>df</th>
<th>Prob&gt;chi2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.040</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>23.166</td>
<td>2</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>25.840</td>
<td>3</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>26.403</td>
<td>4</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>26.525</td>
<td>5</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The results of ARCH test show that there is a ARCH effect in the yield sequence. So we build the GARCH (1,1) model to characterize the volatility of Shanghai 50ETF option’s yield. The specific models are as follows.

\[
\begin{align*}
\sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\
\varepsilon_t &= \sigma_t u_t, \quad u_t \sim i.i.d.N(0,1)
\end{align*}
\]

(4)

We use R to estimate the model’s parameters and find significant parameters. The formula is as follows.

\[
\sigma_t^2 = 0.00000248 + 0.1172194 \varepsilon_{t-1}^2 + 0.8566416 \sigma_{t-1}^2,
\]

(5)

Because \( \alpha_1 + \beta_1 = 0.1172194 + 0.8566416 = 0.973861 < 1 \), it meets the constraint conditions of building a GARCH model.

We use the rolling window method to make dynamic prediction. The result are shown in Table 3.

Table 3. Volatility based on GARCH (partial data).

<table>
<thead>
<tr>
<th>time</th>
<th>volatility based on GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-03-05</td>
<td>0.1867</td>
</tr>
<tr>
<td>2018-03-15</td>
<td>0.1007</td>
</tr>
<tr>
<td>2018-03-30</td>
<td>0.0448</td>
</tr>
</tbody>
</table>

**Implicit Volatility**

The implied volatility is the volatility deduced from the fractional B-S model. As long as the four basic parameters and the real market price of options are brought into the B-S pricing formula, the implied volatility can be calculated. The results are shown in Table 4.

Table 4. Implicit volatility (partial data).

<table>
<thead>
<tr>
<th>time</th>
<th>Implied volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-03-05</td>
<td>0.2017</td>
</tr>
<tr>
<td>2018-03-15</td>
<td>0.1717</td>
</tr>
<tr>
<td>2018-03-30</td>
<td>0.1206</td>
</tr>
</tbody>
</table>

**Forecast Results and Analysis**

This article selects 20 trading days from March 2th, 2018 to March 29th, 2018, and selects the Shanghai 50ETF options which expire in April and have a exercise price of 2.65 yuan as the research objects. Through the above calculation and estimation, we can determine the parameters’ values as follows. \( X = 2.65 \), \( S \) is the closing price of the 20 trading days from March 5th to March 30th, \( T-t \) is known, Table 3 gives the historic volatilities based on GARCH(1,1) model, Table 4 gives the implicit volatilities. We bring data into the fraction B-S option pricing formula to obtain the theoretical prices, as shown in Table 5.
Among them, forecast price P1 is calculated by using the historical volatility, and forecast price P2 is calculated by using the implicit volatility.

In this paper, the percentage AMSE of mean square difference is selected as the evaluation formula for two kinds of volatility.

\[
\text{AMSE} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{C_i - \hat{C}_i}{C_i} \right)^2.
\]

(6)

The AMSE values from the two kinds of volatility are shown in Table 6.

Table 6. AMSE values for two kinds of volatility.

<table>
<thead>
<tr>
<th>model</th>
<th>AMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>historical volatility</td>
<td>0.029865884</td>
</tr>
<tr>
<td>implicit volatility</td>
<td>0.00885706</td>
</tr>
</tbody>
</table>

According to the definition of AMSE, the smaller the value of AMSE is, the better the model fits. Therefore, the implicit volatility is better than the historical volatility for option pricing. The reason is that the volatility fitted by GARCH model will incorporate the error of this period into the prediction of the next period. In this way, with the extension of the prediction period, the error will become larger and larger, and the prediction effect will also become worse. However, the implied volatility contains more market information and more reflects the price changes.

Summary

This paper uses fractional B-S modes with historical volatility and implicit volatility to price Shanghai 50ETF options. We find that the implicit volatility pricing is more accurate because the implicit volatility contains more information on prices, and can reflect the price change. Historical volatility does not reflect the current price of options well because it does not take into account the latest information and changes in the market environment. In the following research, how to fit time-varying volatility and how to price options with high-frequency return rate becomes the focus.

Acknowledgments

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References

