Optimal Hedging Model Based on Gibbs Sampling

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Abstract. For estimating the optimal hedging model of futures and possible risks, In this paper, we use the static method of ordinary least squares model (OLS model) and dynamic generalized autoregressive conditional heteroskedasticity model (GARCH model) to analysis. And based on the Gibbs sampling method to make empirical analysis of the rebar futures market’s optimal hedging model. The results show that dynamic hedging is better than static hedging, Statistical model based on the Gibbs sampling better than the average frequency. Finally, according to the analysis results, this paper gives some suggestions for the future hedging of rebar futures.

Introduction

Steel industry is an important basic industry in China's economy, which can effectively measure the level of development of China's national economy and comprehensive strength. China is also the world's largest consumer of steel products. Due to the changes of macroeconomic policies, the price fluctuation of steel is very large, and the profit of enterprises is gradually decreasing. Especially in the increasingly competitive market environment, the risk of fluctuations in steel prices is difficult to estimate, so in order to avoid risks, it is necessary to use the futures market hedging to reduce economic losses.

After the reform and opening up, China's rapid economic development, which plays a more and more important role in the futures market. The hedging problem has been a hot issue that many researchers pay close attention to. The application for the rebar futures hedging function and research is not deep enough, in the hedging model research of the steel, this paper is concerned with empirical analysis in Gibbs sampling under the steel futures hedging model.

Since March 2009, the Shanghai futures exchange officially launched rebar futures, rebar futures in the futures market has an important degree of participation. The volume and turnover of rebar futures has become the largest futures market futures varieties within a year, trading is very active. Over the past two years, the steel market volatility is very significant, steel prices continued to fall, construction of the hedging model is especially important in market application.

By constructing the optimal hedging model based on the Gibbs model, this paper analyzes the hedging strategy, and gives some suggestions on the hedging of the steel enterprises. The traditional analysis of hedging using a simple model, without considering the existence of certainty, so the error is larger. Based on the literature support, using static OLS model to study the steel futures hedging, and get to the Gibbs sampling results and dynamic GARCH model to analyze the results and draw conclusions, and effectively solve the problem of risk.

Model theory development

The ordinary least square model is one of the most commonly used models in econometric analysis, which is used to express the correlation between the two variables.

\[ y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + u \]  

(1)

The GARCH model is based on the ARCH model, which is called the generalized autoregressive conditional heteroskedasticity model, with GARCH (1,1) as an example. GARCH has two important equations:
The above formula is conditional mean equation and conditional variance equation, alpha is arch and beta is garch.

**Gibbs sampling**

The Gibbs sampling algorithm is an observation sample algorithm which used to obtain a series of approximations equal to the specified multidimensional probability distribution (for example, the joint probability distribution of two or more random variables) in Markov Monte Carlo theory.

Suppose there are three unknown parameters $g$, $u$, and $b$. Assuming that the joint distribution of parameters is represented by $\pi(a)$ which can be obtained by means of a posteriori of three variables $P(g \mid u, b)$, $P(U \mid g, b)$, $P(B \mid g, U)$. The calculation method of Gibbs Sampling algorithm is as follows:

1. Assigning initial values to parameters ($G_0$, $U_0$, $B_0$);
2. Using $P(g \mid U_0, B_0)$ get $g_1$;
3. Using $P(u \mid G_1, B_0)$ get $u_1$;
4. Using $P(b \mid G_1, U_1)$ get $b_1$;

Repeat step second to four steps, get a full Markov chain $\{a_1, a_2, ..., a_i, a_{i+1}, ..., a_t\}$ which can effectively estimate the unknown parameters according to the convergence.

**Optimal hedge ratio**

The key to study the hedging model is whether the hedge ratio and hedging performance are good. In this paper, we use the minimum variance hedge ratio:

$$h = \frac{\text{cov}(r^s, r^f)}{\text{var}(r^f)} = \frac{\rho \sigma_s}{\sigma_f}$$

$\rho$ is the correlation coefficient between futures and spot, $\sigma_s$ and $\sigma_f$ is the corresponding standard deviation of spot and futures. In the OLS model, $h$ is the coefficient before the variable $\beta$. When establishing the GARCH model, the regression coefficient of the conditional mean model is $h$. After the hedge ratio is obtained, the return variance is used to represent the hedging performance:

$$\text{var}(H) = \text{var}(S) + h^2 \text{var}(F) - 2h \text{cov}(S, F)$$

The smaller the variance is, the smaller the risk is.

**Model Empirical Analysis**

In the selection of rebar futures price data, we chose the rebar futures (continuous) day closing price. Because of the existence of futures delivery problems, so in order to better analysis, we choose the Futures (continuous) data, rather than the main contract. About the selection of the spot, the average price is the national spot (rebar: $\phi25mm$). Futures and spot time interval from April 1, 2009 to April 29, 2016.

**Data descriptive statistics**

Descriptive statistics analysis of the sample data based on R, and draw the histogram can get the following conclusions: The logarithmic yield of spot and futures prices of rebar is not subject to normal distribution (The skewness -0.14 and -0.13 Can indicate that there is a right deviation) and sample showed leptokurtic. According to the P value of the coefficient can be shown that the futures and spot price returns are stable (p-value<0.05). Secondly, since the DW test value is close to 2 (respectively, 1.35 and 2.04), there is no autocorrelation.
Static hedging model

Because the P value of the unit root test for spot price of rebar was not significant, we use the futures return rate(lnf) and stock return rate(lns) (first-order logarithmic difference) which has strong correlation and the correlation matrix is

\[
\rho_{\ln s, \ln f} = \begin{pmatrix}
1 & 0.5744 \\
0.5744 & 1
\end{pmatrix}
\]

(6)

So we can construct a linear regression model, OLS model between futures and spot:

\[
\Delta s = -0.000998 + 0.314624 \Delta F + \mu
\]

(7)

The hedging ratio of 0.3146 (beta), a representative of each rebar spot with 0.3146 positions to reverse the futures positions to hedge risk, and we get the model fitness value of 0.32, which is not very high.

To the hedging performance of formula, the hedging performance:

\[
\text{var}(H) = \text{var}(d \ln s) + 0.3146^2 \text{var}(d \ln f) - 2 \times 0.3146 \times \text{cov}(d \ln s, d \ln f) = 0.3301
\]

(8)

Modeling based on Gibbs sampling

After constructing the OLS regression model based on the sample data, we can use Gibbs sampling to estimate the parameters, The prior distribution is assumed as follows:

\[
d \ln s[i] \sim \text{dnrom}(mu[i], \tau), \alpha \sim N(\alpha, 0.5), \beta \sim N(\beta, 0.5), \tau \sim \text{gamma}(0.1, 0.1)
\]

(9)

\[
mu[i] = \alpha + \beta \times d \ln f[i]
\]

(10)

\[
\pi(\alpha, \beta, \tau) = \pi(\alpha) \times \pi(\beta) \times \pi(\tau)
\]

(11)

\[
L(d \ln s|\alpha, \beta, \tau) = \prod_{i=1}^{n} \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(d \ln s[i] - \alpha - \beta \times d \ln f[i])^2}{2\sigma^2} \right)
\]

(12)

Based on WinBUGS, draw Gibbs sampling and parameter distribution as follows:

![Gibbs sampling principle diagram.](image)

Through the initial value of the assignment, and then 10000 times to get the parameters of the estimated results are as follows:
The sampling results of the obtained coefficients are as follows: the hedge ratio (Beta coefficient 0.3148) is brought into the performance formula to get the variance:

\[
\text{var}(H) = \text{var}(d \ln s) + 0.3148^2 \text{var}(d \ln s) - 2 \times 0.3148 \times \text{cov}(d \ln s, d \ln f) = 0.3298
\]  

(13)

**Dynamic hedging model**

For the above OLS model heteroscedasticity test (P = 0.809). OLS model has Heteroscedasticity problem. Based on the two conditional equations of the GARCH model:

\[
\Delta S_t = -0.016 + 0.4831 \Delta F + \mu_t
\]  

(14)

\[
\sigma_t^2 = 1.5e - 6 + 0.7392 \mu_{t-1}^2
\]  

(15)

GARCH (1, 1) and cannot meet the requirements of the premise \( \alpha + \beta < 1 \), by determining the nature of the sample, through the establishment of GARCH (1, 0) can effectively estimate the parameters, and the results are significant, P values are close to 0. Check the stability of the residual \( P<2.2e-16 \), to prove the GARCH model of excellence. The hedge ratio was 0.4831, which was significantly higher than the OLS model. Hedging performance formula available:

\[
\text{var}(H) = \text{var}(d \ln s) + 0.4831^2 \text{var}(d \ln f) - 2 \times 0.4831 \times \text{cov}(d \ln s, d \ln f) = 0.2353
\]  

(16)

**Summary**

The results show that the static hedging model (OLS ratio:0.3146 variance:0.3301) is lower than the dynamic model (GARCH ratio:0.4831 variance:0.2353). And the variance of dynamic model is smaller. It can be seen that the dynamic model is better than the static model in the analysis of the steel futures hedging. However, the effectiveness of Hedging under Gibbs sampling (Gibbs ratio: 0.3148 variance: 0.3298) is higher than that of frequency sampling. It is proved that the model based on Gibbs sampling can avoid the estimation risk, and how to solve these problems effectively is a problem for investors.

Finally, combined with the Gibbs sampling model results of some suggestions on the problems of rebar hedging hedging, such as popular in the business of hedge theory, and the selection of outstanding futures brokerage. We also can improve the futures market trading laws and regulations and other measures to improve the delivery system, reduce delivery costs. At the same time, along with the stable development of the option market in our country, the enterprise can use the futures contract to carry on the trend hedging analysis, and it can avoid the risk and create a good environment.
References


