A Model of Orders Decision-making Cross Supply Chains Under Cloud Manufacturing

Xing-jian ZHOU\textsuperscript{1,2,a,*}, Xiang ZHOU\textsuperscript{3,b}, Yan FENG\textsuperscript{1,c} and Zhi-jie CHEN\textsuperscript{1,2,d}

\textsuperscript{1}School of Management, Wuhan Textile University, Wuhan, 430073, China
\textsuperscript{2}Research center of enterprise decision support, Research Base for Humanities and Social Sciences in Hubei Province, Wuhan, 430073, China
\textsuperscript{3}Law and Business School, Wuhan Institute of Technology, Wuhan, 430205, China
\textsuperscript{a}wuliuwtu@163.com, \textsuperscript{b}wuliuwuse@sina.com, \textsuperscript{c}sumfruit@sina.com, \textsuperscript{d}logiswtu@163.com

*Corresponding author

Keywords: Cross-chain; Orders Decision; Cloud Manufacturing; Lagrange Algorithm

Abstract. For the single chain, an enterprise usually deals with the customer order by extending working hours or simply outsourcing when facing the capacity shortage. However, potential operational risks will increase as well. Considering the background of cloud manufacturing, the cluster supply chain consists of two parallel single chains we build the basic model to assign the orders priority within each capacity. Then, considering the inter-chain horizontal cooperation, the extended model is proposed to parallel allocation of cross-chain orders as the orders exceeding one single-chain’s capacity. Lagrange algorithm is implemented, and the simulation analysis shown that the opportunity cost of rejected orders factor and cross-chain orders manufacturing cost factor have significant impacts on orders’ allocation decision, and there is a critical point in the combinations of those two factors. Through combinations, the cluster supply chain can make the acceptance decisions policy and production schedules of priority orders and cross-chain orders, so that customers’ satisfaction and the cluster supply chain’s total profits achieve the best situations.

Introduction

MTO firms usually organize and coordinate the operations with orders-driven, and use the way of flexible specialization to deal with various production processes for the orders with multi frequency, small batch and personalized products. Because the main driver in MTO operations is customer orders, it is vital to coordinate operations and sales functions for effective use of available resources by managing the demand placed on the system\cite{1} (Mehmet and Sridharan, 2005). Therefore, when making the order decisions, they usually use the way of work overtime or subcontracting, which on the contrary leads to higher operation costs.

Through specialization and cooperation with each other, MTO firms often gather in a particular area and form the industry clusters, based on which a kind of network with multi supply chains comes into being as we called cluster supply chain\cite{2} (CSC)(Li, 2006). Under cloud manufacturing, the MTO firms in CSC can collaborate between supply chains, for example, the firms in different supply chain manages allocate inventory and collaborative purchase\cite{3,4} (Liu, 2011,2013). Similarly, the firms in different supply chain (is also called single supply chain) can also process the customer orders together, namely the problem is how to make the order decision in multi supply chains.

Problem Description

A cluster supply chain is composed by a number of single supply chains, and each single supply chain contains MTO firms, sellers, and customers. The customer orders are collected by sellers and sent to MTO firms from the downstream to upstream of a single supply chain, then a job shop in a MTO firm with a set of customer orders is considered. The decision to make is which customer orders
to accept and how to schedule it in order to maximize the profit and to fulfill the accepted orders by
the due date, as well as how to process the rejected orders in order to maximize customer satisfaction
and to allocate the rejected orders in multiple supply chains. Both decisions should be made
simultaneously, otherwise an order may be accepted but the available residual capacity may not
permit on-time delivery.

Each customer order has a set of operations to be processed with linear precedence constraint and
deterministic processing times, a fixed due-date, and a known sales price. Tardy deliveries are not
allowed. There are multiple resource types; each resource type has one or more machines. Job
recirculation is allowed (i.e. the jobs can visit the same resource more than once). The objective
considered is to maximize the operational profit over a planning horizon considering only the sales
price and the manufacturing costs by accepting a subset of customer orders. The planning horizon is
discreted into time buckets of equal length know as time periods. Without loss of generality, each
time period is assumed as one day. Furthermore each day is divided into two shifts namely regular
time and overtime. Overtime is typically expensive. The decision of accepting or rejecting the orders
is done at the beginning of the day.

The processing of order decision in a cluster supply chain (a CSC comprised of \( n \) single supply
chains, which is defined as \( \text{CSC} = \sum_{i=1}^{n} \text{SC}_i \) ) involves three steps.

Step 1, when customer orders arrive to a single supply chain \( \text{SC}_i \), considering of the production
capacities and customer satisfactions, one part of the orders will be accepted priority by the MTO firm
(defined as “directed orders”), another part of the order will not be rejected directly, but be accepted
temporarily (defined as " reserved orders");

Step 2, the reserved orders of \( \text{SC}_i \) are put into the next single supply chain \( \text{SC}_{i+1} \) and made orders
decisions again as the first stage. Similarly, there are some of the orders will be accepted as
“cross-chain orders”, and the remaining orders will be still accepted temporarily again as reserved
orders;

Step 3, the reserved orders of \( \text{SC}_{i+1} \) are put into \( \text{SC}_{i+2} \) and repeat Step 2 until the reserved orders of
\( \text{SC}_n \) become cross-chain orders or rejected orders (defined as "rejected orders", the main reason for
rejection is that the capacities of CSC are not enough to complete such orders).

**Literature Review**

About order decisions, the order acceptance, lead-time or due date quotation, pricing and capacity
planning are closely related. In the absence of differential pricing, RM becomes a capacity allocation
and order acceptance problem.

Slotnick and Morton\(^5\) (2007) model a manufacturing facility that considers a pool of orders, and
chooses for processing a subset that results in the highest profit. In addition to the problem
characteristics in Slotnick and Morton\(^6\) (2009) they consider customer weight. The objective is to
maximize profit, which is the sum of per-job revenues minus total weighted tardiness. They propose
two approaches: separation of sequencing and job acceptance decisions, utilizing a property of the
problem that is exploited to good advantage in the analogous problem with weighted lateness and a
joint consideration of sequencing and acceptance, using relaxation. They state that the joint approach
is far superior to the first. Yano\(^7\) (2010) research the order decisions by minimizing the expected total
inventory holding costs and delay costs with the order delivery lead time as decision variable.
Mehmet\(^8\) (2010) studied order decisions and orders of the production planning problems under the
income management; So and Song\(^9\) (2010) considered short-term decision-making factors such as
price, delivery time and capacity expansion level to make order decisions; Ebben\(^10\) (2012) and
Reitman\(^11\) (2013) proposed order decision problems when the customer demand is sensitive of the
price and delivery time. Weng\(^12\) (2015) discussed the order production planning from the point of
view of to maximize the expected income.
In the existing literature, order decisions involves only a single MTO firms in the same supply chain.

**Model definition**

**Mathematical formulation**

The notation used in the formulation is presented below. The symbols and meanings of the set and the parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>set</th>
<th>symbol meaning</th>
<th>parameter</th>
<th>symbol meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { r \in R } )</td>
<td>production equipment</td>
<td>RTL(_r)</td>
<td>length of shift time in ( SC_i )</td>
</tr>
<tr>
<td>( { t \in T } )</td>
<td>time periods</td>
<td>MC(_r)</td>
<td>production capacity of ( r ) within the time interval ( t ) in shift time ( s )</td>
</tr>
<tr>
<td>( s \in S = { 1, 2 } )</td>
<td>shift time, 1 is regular time (RT), and 2 is overtime (OT).</td>
<td>MT(_r)</td>
<td>production operation of order ( j )</td>
</tr>
<tr>
<td>( { j \in J } = { 1, 2, 3 } )</td>
<td>orders / products (1 orders only 1 products)</td>
<td>LT(_j)</td>
<td>lead time of order ( j ) in ( SC_i )</td>
</tr>
<tr>
<td>( { o \in O } )</td>
<td>production operations</td>
<td>MCT(_r)</td>
<td>the cost of using equipment ( r ) in the shift time ( s ) in ( SC_i )</td>
</tr>
<tr>
<td>( i \in I = { 1, 2 } )</td>
<td>the single supply chain ( i )</td>
<td>( \alpha_{ij} )</td>
<td>the opportunity cost weight factor for rejected order ( j ), ( 0 \leq \alpha_{ij} \leq 1 )</td>
</tr>
</tbody>
</table>

The decision variable are as follow.

\[
PT_{jors} = \text{hours of operation of order } j \text{ processed on resource } r \text{ in shift time } s \text{ of period } t; \\
pP\_{jors} = \begin{cases} 1, & \text{if operation of order } j \text{ processed on equipment } r \text{ in shift time } s \text{ of time period } t; \\
0, & \text{otherwise}; \\
pA\_j = \begin{cases} 1, & \text{if order } j \text{ accepted}; \\
0, & \text{otherwise}. \
\end{cases}
\]

**Decision model for single supply chain order (basic model)**

Generally, when the customer orders arrive to the MTO firms in CSC, considering of production capabilities, and opportunity cost of rejection orders, the manager accept one part of orders (directed orders) and reject another orders (rejected orders) to pursue their own profit maximization. In the process of order decisions, the resources allocated within the single supply chain, and there is no cooperation between supply chains.

The mathematical formulation proposed for the basic model is presented below.

\[
\begin{align*}
\text{Maximize} & \quad Z = \sum_{nI} \sum_{j \in J} \sum_{o \in O} \sum_{r \in R} \sum_{s \in S} \sum_{t \in T} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} (1 - PA_j) \cdot P\_j \cdot MCT\_r \cdot PT\_{jors} \\
\text{Subject to} & \quad \sum_{j \in J} \sum_{o \in O} \sum_{r \in R} \sum_{s \in S} \sum_{t \in T} \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} PT\_{jors} \leq MC\_r, \forall i \in I, r \in R, t \in T, s \in S \\
& \quad \sum_{nI} \sum_{s \in S} \sum_{t \in T} PT\_{jors} = MT\_r \cdot PA\_j, \forall i \in I, j \in J, o \in O, r \in R
\end{align*}
\]
Objective (1) is formulated to maximize the total profit of CSC, and consisted of two parts: the first term is the total sales revenue minas the production cost, and the second term is the opportunity cost of refused orders. \( \alpha \) is the weight factor of the opportunity cost, in the short term, the order’s opportunity cost is not more than the order’s sale revenue, namely \( 0 \leq \alpha \leq 1 \).

Constraint set (2) ensures that the production capacity of equipment \( r \) of shift time \( s \) in time period \( t \) is not violated. Constraint set (3) ensures that adequate production equipments are allocated to process operation \( o \) of order \( j \). The total hours allocated to process an operation should be equal to its processing time. Constraint set (4) ensures that each operation of an order is processed for no more than \( \text{RTL}_o \) hours in each shift time during each time period. Constraint set (5) and (6) set the \( PP_{ijorts} \) decision variables to either 1 or 0. It takes a value of 1 when \( PT_{ijorts} > 0 \), indicating that operation \( o \) of order \( j \) is scheduled for processing on equipment \( r \) of shift time \( s \) in time period \( t \); otherwise it takes a value of 0. The \( PP_{ijorts} \) variables are used to ensure the precedence relationship. The parameter \( s \) in constraint (5) indicates that whenever an operation is processed on an equipment it should be processed for at least \( s \) units of time. Constraint set (7) ensures that when an order is accepted, the completion time of the last operation of that order does not exceed the order due date. Constraint set (8) ensures that operation \( o \) of order \( j \) can be processed in period \( t \) during regular hours only after completing operation \( (o-1) \). Constraint set (9) ensures that operation \( o \) of order \( j \) can be processed in period \( t \) during overtime only after completing operation \( (o-1) \). Constraint sets (10) impose the non-negativity restrictions and binary restrictions on the decision variables.

**Decision model for multiple supply chain orders (extended model)**

Considering of the collaboration between supply chains, the rejected orders in single supply chain will be accepted temporarily as the reserved orders. Then those orders enter into the next supply chain and be made order decisions again.

The mathematical formulation proposed for the extended model is presented below.
\[
\begin{align*}
\text{Maximize } & \quad \sum_{j \in J} \left( \sum_{i \in I} P_i^j \cdot PA_j - \sum_{i \in I, o \in O, r \in R, t \in T, s \in S} MCT_{ir}, PT_{ojr} \right) \\
& + \sum_{j \in J} \left( \sum_{i \in I} P_i^j' \cdot PA_j' - \sum_{i \in I, o \in O, r \in R, t \in T, s \in S} \beta_j \cdot MCT_{ir}, PT_{ojr}' \right) \\
& - \sum_{i \in I, o \in O} \alpha_i (1 - PA_j - PA_j') P_i \\
\end{align*}
\]

Subject to constraints (2)-(10), and

\[
\sum_{j \in J} \sum_{i \in I, r \in R, t \in T, s \in S} PT_{ojr} + \sum_{j \in J} \sum_{i \in I, r \in R, t \in T, s \in S} PT_{ojr}' \leq MC_{ir}, \forall i \in I, r \in R, t \in T, s \in S
\]

\[
\sum_{j \in J} \sum_{i \in I, r \in R, t \in T, s \in S} PT_{ojr} = \sum_{j \in J} \sum_{i \in I, o \in O, r \in R} M \left( PA_j + PA_j' \right), \forall i \in I, j \in J, o \in O, r \in R
\]

\[
\sum_{j \in J} \sum_{i \in I, o \in O, r \in R, t \in T, s \in S} PT_{ojr} \leq R_{T, ir}, \forall i \in I, j \in J, t \in T, s \in S
\]

\[
\sum_{r \in R} PP_{ojr} \leq LT_j PA_j, \forall i \in I, j \in J, t \in T, s \in S
\]

\[
PT_{ojr} \geq 0, PP_{ojr} \in \{0, 1\}, \forall i \in I, j \in J, o \in O, r \in R, t \in T, s \in S
\]

\[
PP_{ojr} \in \{0, 1\}, \forall i \in I, j \in J, o \in O, r \in R, t \in T, s \in S
\]

\[
PA_j + PA_j' \leq 1, \forall i \in I, j \in J
\]

Objective (11) consists of three parts: the first part is the profits of directed orders; the second part is the profits of cross-chain orders; the last part is the opportunity cost of rejected orders. \( \beta_{ij} \) is the production cost factor of cross-chain orders, obviously, \( \beta_{ij} \geq 1 \). When \( \beta_{ij} = 1 \), there is a high level of cooperation between supply chains, and cross-chain production cost is equivalent to the single supply chain. On the contrary, if \( \beta_{ij} \rightarrow +\infty \), which shows the collaboration level between supply chains is very low, and then the orders processed in another supply chain need more time to coordinate people and equipments, so the production cost is very high. \( \beta_{ij} \) ensures the accepted orders processing sequence, first is directed orders, followed by cross-chain orders, which comply with the actual operation norms in CSC. Moreover, we add three decision variables \( PT_{ojr}^\prime, PP_{ojr}^\prime \), and \( PA_j^\prime \). \( PT_{ojr}^\prime \) is the time of operation \( o \) of cross-chain order \( j \) processed on resource \( r \) in shift time \( s \) of period \( t \); \( PP_{ojr}^\prime \) is 1 if operation \( o \) of order \( j \) processed on equipment \( r \) in shift time \( s \) of time period \( t \), otherwise is 0; \( PA_j^\prime \) is 1 if cross-chain order \( j \) accepted, otherwise is 0.

Constraint set (12) ensures the production capacities meet with the directed orders and cross-chain orders; Constraint set (13) ensures the equipments are enough for processing operation \( o \) of order \( j \), and total hours allocated to process an operation should be equal to \( MT_{io,r} \); Constraint set (14) ensures that each operation of the directed orders and cross-chain orders is processed for no more than \( R_{T, ir} \) hours in each shift time during each time period; Constraint set (15) ensures that when a cross-chain order is accepted, the completion time of the last operation of that order does not exceed the leading time \( LT_j \); Constraint set (16)-(17) is the same as constraint set(10), but constraint set (18) ensures the order is accepted by the SC \( i \) to be unique.
Algorithm Design

The extended model are all a mixed integer nonlinear programming problem (MINLP). To solve this problem, an effective method is calculating lower bounds, and using upper and lower bounds to evaluate the algorithm \(^{13,14}\). Lagrange Relaxation is an effective method for solving the lower bound. Because Lagrange relaxation is relatively simple and has good properties, it can not only be used to evaluate the effect of the algorithm, but also improve the efficiency of the algorithm. The basic principle of Lagrange algorithm using Lagrange multiplier to relax the difficult constraints in the original problem, so it is relatively easy to solve Lagrange's problem, and through calculate the Lagrange dual problem and gradually approaching to obtain the optimal solution of the original problem.

A better lower bound for the Lagrange relaxation problem also should be similar to the optimal solution of the IP problem. With such logic, Lagrange heuristic algorithm is generated. A Lagrange heuristic algorithm mainly includes two parts: the first part is a Lagrange sub gradient optimization, but the result maybe not the necessarily feasible solution, so the second part is making the feasible solution based on the first part.

Step 1. Initialization, determine the value of the parameters according to the actual situation, set initial value Lagrange multiplier \( \lambda^0_i = k_i \), \( k = 0, k \in T \).

Step 2. For a given \( \lambda^k_i \), calculate \( z_{lr}(\lambda^k) \).

Step 3. The feasible solution set of LR is composed of a finite number of integer points, and the pole is \( x^* \) ( \( x \) is the all decision variables in the model). Then \( z_{lr}(\lambda) = \max(ax^* + \lambda^*b) \) ( \( a \) and \( b \) are all parameters in the model). Set \( I = \{ t \vert z_{lr}(\lambda) = ax^* + \lambda^*b \} \), for \( t \in T \), calculate sub gradient \( s^i = bx^i \).

Step 4. Choose a sub-gradient \( s^k (k \in T) \) from step 3, if \( s^k = 0 \), \( \lambda^k \) is the optimal solution and the calculation will be stopped; otherwise, go to step 5.

Step 5. Design equation \( \lambda^{k+1} = \min\{ \lambda^k + \theta^k, s^k, 0 \} \), \( k := k + 1 \), and \( \sum_{k=1}^{\infty} \theta^k = \infty \), \( \theta_k \to 0, k \to \infty \).

Repeat Step 5.

Step 6. Algorithm termination principle: the value of \( \lambda^k \) is no more than the given value in a specified number of steps, at this time the target value is not likely to change or change very little.

Summary

Considering the three dimensions of customer satisfaction, enterprise resources and supply chain collaboration, we establish a decision model for cross-chain orders based on collaborations in supply chains. The numerical calculation and result analysis shows that the cross-chain order decisions is more flexible than the non-cross-chain’s case. At the same time, the order rejection opportunity cost factor \( \alpha \) and cross-chain order production cost factor \( \beta \) have an affect on order decisions. through designing the value combination of \( (\alpha, \beta) \), the managers can make appropriate order accepted decisions and production plans, which make the customer satisfaction and total profit of cluster supply chain to reach the optimal value.

Acknowledgement

The authors thank Xiang ZHOU, Yan FENG, and Zhi-jie CHEN, the editor, and the reviewers for their insightful comments and encouragement that have helped improve this paper greatly. Their thanks are also due to this research has been supported by the National Natural Science Foundation for Young Scientists of China (71501146), Hubei Provincial Department of Education Research Program in 2016 (B2016069), General research project of Humanities and Social Sciences in Hubei
province in 2016 (20160371), and Research center of enterprise decision support for general projects of Research base for Humanities and social sciences in Hubei Province (DSS20170109).

References