Asymmetric Adjustments of U.S. Interest Rates

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Abstract. Literature on dynamics of the term structure of interest rates supports that the adjustment of interest rate spread may be well described by nonlinear time series models. This paper examines the relationship between the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) from January 1962 to November 2015 by employing the Markov-switching VEC model, which allows for the asymmetric adjustment of interest rates to the long run equilibrium across regimes. We find that the high-volatility regime more frequently exists prior to the 1990s, and the low-volatility one occurs more often after 1990s. Secondly, with allowing for the distinct short-run adjustment parameters in the MS-VECM setup, we find that the evidence of asymmetric adjustments of interest rates to the long run equilibrium in difference regimes. More specifically, the long-term interest rate adjusts more quickly towards the long run equilibrium in the high-volatility regimes than in the low-volatility one. Moreover, by comparing the impulses response functions of linear and nonlinear models, we find that empirical results could be misleading if the regime shift is not taken into account.

Introduction

Modeling the dynamics of the term structure of interest rates has always been of fundamental importance to financial economists. According to the expectations hypothesis of term structure of interest rates, the long-term interest rates would be the average of future expected short-term ones. Hence, the expectations hypothesis in the context of the cointegration theory suggests that the long- and short-term interest rates should be cointegrated with parameters (1, -1). There has been a large number of studies strived to test the term structure of interest rates applying cointegration techniques on a linear model (see Engle and Granger (1987), Campbell and Shiller (1991) among others). However, evidences on this hypothesis under the linear framework are mixed.

The empirical relevance of these theoretical implications has been explored by a number of literature. Using the US treasury yield data, Bansal et al. (2004) estimate a regime-shifts term structure model and show that the regimes in the model are related to the NBER business cycle indicator, implying that the term structure regimes confirm and complement the real business cycles. Some other empirical studies investigate the possible nonlinearity in term structure of interest rates by introducing the possibility of structural breaks, or threshold (as in Pfann et al. (1996), and Bec et al. (2008)).

In this paper we consider the possibility that a vector error corrected model with Markov switching in the adjustment parameter (MS-VECM hereafter) would provide a better empirical description of the term structure of interest rates. More specifically, in this MS-VEC model, i) long run equilibrium relationship is kept in linear framework, ii) and Markov-switching dynamics are introduced into the analysis which can be linked to the distinct adjustments across regimes. We choose the MS-VECM to analyze the term structure of interest rates since this model is not only
efficiently captures the dynamics of the interest rates with different maturities in a cointegration context, but also possesses an appealing structural form and provides economically intuitive results. More specifically, by applying this MS-VECM, we can both study the asymmetric adjustment of interest rates in different term structure regimes, and explore the relationship between term structure regimes and business cycle. First, our empirical findings point out the term structure is more likely to be in the high-volatility regime when the economy experiences a recession. Second, the long-term interest rate adjusts asymmetrically towards the long run equilibrium across regimes, whereas the short-term one does not. Third, by comparing the impulses response functions of linear and nonlinear models, we find that empirical results could be misleading if the regime shift is not taken into account.

The paper is organized as follows. Section 2 presents and discusses the methodology used in the empirical analysis. Section 3 describes the implementations of estimating the MS-VECM and discusses the empirical results. Section 4 concludes.

Methodology

Our analysis employs the following MS-VECM,

\[ \Delta X_t = \mu_s + \sum_{k=1}^{p-1} \Gamma_{s-(k)} \Delta X_{t-k} + \Pi_s X_{t-1} + \epsilon_t, \quad t = 1, 2, \ldots, T \]  

where \( p \) is the lag length of the MS-VAR model, \( \epsilon_t \sim N(0, \Omega_{s_t}) \), and \( \Omega_{s_t} \) is a \((2 \times 2)\) positive definite covariance matrix. \( s_t \) presents the unobserved state or regime variable, which is conditional on \( s_{t-1} \), independent of past \( X_t \), and assumed to follow a q-state Markov process. From the essence of economics, two-regime, that corresponds the recession-expansion cycle, is sufficient to describe the dynamic relationship between interest rates with different maturities, we thus set \( q=2 \). Hence the transition probability matrix given by

\[ P = \begin{bmatrix} p_{ii} & p_{ij} \\ p_{ji} & p_{jj} \end{bmatrix}, \quad \sum_{j=1}^{2} p_{ij} = 1 \]

where \( p_{ij} \) represents the probability of transition from \( i \) state to \( j \) state. Besides, the MS-VECM also allows the variance matrix \( \Omega_{s_t} \) to dependent on the regime variable.

Empirical Findings

Multiple Cointegration Tests

Our data set comprises monthly observations spanning from 1962:01 to 2015:11 for 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10), coming from the Fed St Louis Database. Table 1 reports selection criterion and multivariate cointegration tests for the VAR(\( p \)) model for the 3-Month Treasury Bill Rate and 10-Year Treasury Rate in levels. Panel A gives the AIC, BIC and Hannan-Quinn (HQ) information criteria. The VAR order is selected as 2 according to AIC and HQ. Panel B reports the multivariate cointegration test of Stock and Watson (1988). Under the null of Stock-Watson cointegration test, we test \( k \) common stochastic trends against \( k-r \) common stochastic trends (or \( r \) cointegration relationships), finding support for cointegration between the two series as well.

Asymmetric Adjustment across Regimes

We estimate both linear VEC and MS-VEC models over the full sample period 1962:01--2015:11 with 647 observations. Table 1 reports estimation results and model selection criteria for the MS-VEC model given in equations (1) and (3). The lag order is selected as 2 according to the
minimum AIC value in a VAR for both linear VEC and MS-VEC models. The MS-VEC model is estimated by Bayesian Monte Carlo Markov Chain (MCMC) method with Gibbs sampling.

First, we use the likelihood ratio statistic to test the nonlinearity, in which the linear VECM as the null hypothesis and MS-VECM as the alternative one. Since unidentified parameters exist under the null, the LR test is not standard. The \( \chi^2 \) p-values (in square brackets) with degrees of freedom equal to the number of restrictions as well as the number of restrictions plus the numbers of parameters unidentified under the null are reported, and the p-value of the Davies' test is presented in squared brackets as well. We find that the linearity is rejected. Henceforth, we focus on discussing the estimates of the nonlinear model; MS-VECM. One of the aims of this paper is to see the asymmetrical adjustments toward the long-run equilibrium across regimes. According to the estimates of the short run correction parameters, \( \alpha \), in Table 2, we could notice that the adjustment of long-term interest rate exhibit significantly asymmetricity across regimes. More specifically, the speed of long-term interest rate adjustment to long-run equilibrium is quicker under high-volatility regime than that under low-volatility one, \( |\alpha_{S=2}| > |\alpha_{S=1}| \). By contrast, we find no evidence supporting the asymmetricity for short-term interest rate.

### Table 1. Multivariate Cointegration Tests.

<table>
<thead>
<tr>
<th>Panel A: VAR order-selection criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag(p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>-2564.6</td>
<td><strong>2572.3</strong></td>
<td>-2567.6</td>
<td>-2555.2</td>
<td>-2540.3</td>
<td>-2540.0</td>
<td>-2537.8</td>
</tr>
<tr>
<td>HQ</td>
<td>-2565.4</td>
<td><strong>2573.7</strong></td>
<td>-2568.6</td>
<td>-2557.7</td>
<td>-2543.8</td>
<td>-2544.6</td>
<td>-2543.6</td>
</tr>
<tr>
<td>BIC</td>
<td><strong>2537.9</strong></td>
<td>-2527.8</td>
<td>-2504.4</td>
<td>-2475.1</td>
<td>-2424.7</td>
<td>-2389.0</td>
<td>-2351.4</td>
</tr>
<tr>
<td>Panel B: Stock-Watson cointegration tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H &gt; q(k, k - r) )</td>
<td>Statistic</td>
<td>Critical value of 5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q(2,0)</td>
<td>-7.356</td>
<td>-17.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q(2,1)</td>
<td>-34.00***</td>
<td>-17.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: ***, **, * represent significance at 1%, 5% and 10% levels, respectively.

### Table 2. Estimation Results for the MS-VEC Model.

<table>
<thead>
<tr>
<th>Model selection criteria</th>
<th>MS(2)-VEC</th>
<th>Linear VEC(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log likelihood</td>
<td>-82.83726</td>
<td>-499.34215</td>
</tr>
<tr>
<td>AIC criterion</td>
<td>-1695.26638</td>
<td>-1308.76149</td>
</tr>
<tr>
<td>HQ criterion</td>
<td>-1713.65929</td>
<td>-1315.82501</td>
</tr>
<tr>
<td>BIC criterion</td>
<td>-1630.81987</td>
<td>-1279.00749</td>
</tr>
<tr>
<td>LR linearity test</td>
<td>833.00977</td>
<td>( \chi^2 (9) = [0.0000]^{***} )</td>
</tr>
<tr>
<td>Short run correction parameters</td>
<td>( \alpha_{S=1} = \begin{pmatrix} -0.02^{**} \ 0.003^{(0.007)} \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \alpha_{S=2} = \begin{pmatrix} -0.06^{(0.027)} \ 0.057^{(0.042)} \end{pmatrix} )</td>
<td></td>
</tr>
<tr>
<td>Transition probability matrix</td>
<td>( P = \begin{bmatrix} 0.955 &amp; 0.1088 \ 0.045 &amp; 0.8912 \end{bmatrix} )</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1. The MCMC estimates are based on 20000 burn-in and 50000 posterior draws. 2. Regime properties include ergodic probability of a regime, observations falling in a regime based on regime probabilities, and average duration of a regime. 3. ***, ** and * represent significance at 1%, 5% and 10% levels, respectively.
Impulse Response Functions

We analyze the impulse responses for both linear VECM and nonlinear MS-VECM. Figure 1 reports 1 to 20 step impulse responses to 1 standard deviation shocks in linear VECM, and Figures 2 and 3 for nonlinear MS-VEC models. Impulse responses appear as black lines and 95 percent confidence intervals appear as blue lines. Firstly, we compare the different effects of a shock to the long-term interest rate on DSG10, between linear VECM and nonlinear MS-VECM. From the left upper graph of Figure 1, we see that the impulse responses for the linear VECM show a significant positive effect of long-term 10-year Treasury shock on DSG10, that is stable at about 0.35 over time. However, The impulse responses for the MS-VECM model show an asymmetric significant positive effect of long-term 10-year Treasury shock on DSG10, which is plotted in the upper two graphs of Figure 2. More specifically, in low-volatility regime (regime 1) the significant positive effect of long-term 10-year Treasury shock on DSG10 decreases over time, while in high-volatility regime (regime 2) the effect increases over time. The asymmetricity indicates that the effect of a long-term 10-year Treasury shock on DSG10 is enhanced in high-volatility regime, or say during the recession period. Secondly, the asymmetric effects of long-term 10-year Treasury shock on DTB3 across regimes are confirmed as well, as can be seen clearly from the lower two graphs of Figure 2. We find that the impulse responses of long-term 10-year Treasury shock on DTB3 are strongly larger and stable at about 1.4 over time in high-volatility regime (regime 2), which is four times as large as those in low volatility regime (regime1). It worth noting that the impulse responses are quite stable in both regimes even though their magnitudes are uneven, besides the magnitude of impulse responses from linear framework lie between them (referring to the left lower graph of Figure 1).

Lastly, the effects of short-term treasury bill shock on DTB3 are simulated and depicted. Regarding to the impulse responses under nonlinear MS-VECM, plotted in the lower graphs of Figure 3, their magnitudes exhibit no significant difference between two regimes at the beginning, however, the impulse responses stay stable in the low-volatility regime but decrease in the high-volatility regime. Moreover, the magnitudes of the impulse responses under nonlinear MS-VECM is larger than the magnitudes of the impulse responses under linear VECM, which is plotted in the right lower graph of Figure 1.

In sum, our findings point out the impulse response show asymmetricity across two regimes, besides, the impulse responses from nonlinear VECM are quite different from linear one both in magnitude and movements, implying the importance of considering two regimes when modeling the dynamics of the term structure.

Figure 1. Impulse Response of Linear VEC Model.
Impulse Responses to 10 Year Treasury Shock

Figure 2. Impulse Response to 10-Year Shock in MS-VEC Models.

Impulse Responses to 3 Month Treasury Bill Shock

Figure 3. Impulse Response to 3-Month Treasury Bill Shock in MS-VEC Models.

Conclusion

We find empirical evidences that the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) exhibit non-stationary characteristic. Since these two series are proved to be cointegrated, thus leading to the methodology of the MS-VECM. We find that the high-volatility regime more frequently exists prior to the 1990s, and the low-volatility one occurs more often after 1990s. Secondly, with allowing for the distinct short-run adjustment parameters in the MS-VECM setup, we find that the evidence of asymmetric adjustments of interest rates to the long run equilibrium in difference regimes. More specifically, the long-term interest rate adjusts more quickly towards the long run equilibrium in the high-volatility regimes than in the low-volatility one. Thirdly, using the NBER business cycle dates, we examine the relationship between the recessions and the smoothed probability of a high-volatility regime identified from MS-VECM. We find it more likely that the economy experiences a recession when high-volatility regime occurs, while the economy is more possible stays in an expansion period when low-volatility regime takes place. This finding suggests that the regimes in the model are related to the NBER business cycle indicator, implying that the term structure regimes confirm and complement the real business cycles.
Acknowledgement
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References