The Empirical Study on the Correlation Among International Stock Markets Based on the Multi-scale Analysis

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Abstract. In this paper, it was the multi-scale analysis method that was used to decompose the stock index of international stock markets into a long-term trend signal and two short-term wave signals. Then, by the principal component analysis method, all of the stock markets were divided into several different categories. The study demonstrated that the developed countries and the emerging economies were divided into two different categories in the long term of trend. For the four days’ short-term fluctuations, the United States and Europe were the same category, while Asia-Pacific countries were divided into the another category. For the two days’ short-term fluctuations, all the stock markets are divided into three categories. European and American stock markets were classified into the same category. Besides, the Asia Pacific stock markets were divided into the same category except HK stock market and Shanghai stock market, which were another category. Finally, the article put forward that in order to reduce the investment risk, the investment capital should be dispersed into different categories of the stock markets.

Introduction

Compared to the hundreds of years’ history of the United States stock market, it was very short of the 30 years’ history of Chinese stock market. But the development of Chinese stock market was unquiet. From December 1990 to the present, Chinese stock market has experienced eight upheavals, including the establishment of the China Securities Regulatory Commission, non-tradable shares reform, the implementation of the QFII and QDII in China, and so on. In recent years, it was these factors, such as rapid progress of computer technology and the continuous openness to the world that made Chinese stock market closer with the rest of the world. Meanwhile, Chinese stock market was inevitably affected by fluctuations of international stock markets. Therefore, the study on stock market’s risk control was important to stock market’s regulators and investors. It was generally known that a method of controlling financial risk was diversification of capital, which meant putting eggs into more than one basket. If investors put their capital into stock markets without strong correlation, the risk were scattered. If the capital was put into two stock markets with strong correlation, the investment risk would not be scattered. It was why the correlation among international stock markets was of great significance to the international stock market’s regulators and investors.

Literature Review

Aviral Kumar Tiwari, Arif Billah Dar, Niyati Bhanja and Aasif Shah (2013) studied stock Markets’ integration in Asian countries. They used methods of the multiple wavelet correlation and multiple cross correlation to evaluate the linkage of Asian stock markets. The result showed that correlation among the low frequency data of stock markets were stronger than those among the high frequency data. In order to study volatility of Shenzhen stock market and Shanghai stock market, Dandan Xi(2012) decomposed the closing price index which was from 1990 to 2010 to different scales data,
and analyzed the symbolization of these data. Finally her distinguished the principal patterns and abnormal changes in the different time scales, which provided risk management strategy to different types of investors.

Throughout the domestic and foreign literature, it was rough of the researches on the correlation among different stock markets in the past. Some of them were correlation analysis of the stock markets’ returns, and the others were just about the correlation of the stock markets return in different time scales. Wavelet multi-scale analysis can analyzed the correlation of the stock price from different time scales and frequencies, which can provide reference for investors with different investment strategies. In this paper, the principal component analysis was applied to get the principal components of the data which was decomposed into different scales. The stock markets were strong correlated if their factor loading were large in the same principal component. In order to reduce the risk, investors should divide their assets into stock markets of different principal component.

Methodology

Multi-scale Analysis

Multi-scale analysis was an important part of wavelet analysis theory. It was established on the basis of functional space and Fourier transformation. Through the multi-scale analysis, the signal was decomposed in different function space, that is, the different scale space.

Continuous Wavelet Transformation. Wavelet transformation can be described as a projection of signal \( f(t) \) on wavelet basis function \( \{\Psi_{a,\tau}(t)\}_{a>0,\tau \in \mathbb{R}} \). This projection process was a function of inner product: \( \forall f(t) \in L^2(\mathbb{R}) \), the inner product of functions was:

\[
WT_f(a, \tau) = \langle f(t), \Psi_{a,\tau}(t) \rangle = \frac{1}{\sqrt{a}} \int_{\mathbb{R}} f(t) \Psi^*(\frac{t-\tau}{a}) dt .
\] (1)

The inner product above was the continuous wavelet transformation of function \( f(t) \), which was called wavelet transformation coefficient. If \( a \) and \( \tau \) (\( a \) means scale factor and \( \tau \) means displacement factor) varied, then the basis function \( \Psi_{a,\tau}(t) \) should vary. Changing the basis function would bring out change of the projection of function \( f(t) \) in the direction of different vectors, thus the original signal was decomposed into different directions.

Discrete Wavelet Transformation. In the daily life, most signals were discrete, and the computer was base on digital processing mode, which was suitable for processing discrete signals. So the continuous wavelet transformation was necessary to be discretized. Discrete wavelet transformation is the discrete treatment of independent variables \( a \) and \( \tau \) in essence.

There were many ways of discretization of \( a \) and \( \tau \), which were widely accepted and applied in the following ways:

Discretized the scale factor \( a \) in the form of power series. That meant \( a = a_0^m \).

At the same scale, the displacement factor \( \tau \) was uniformly discretized. That meant \( \tau = k a_0^m \tau_0 \).

Where, \( a_0, \tau_0 \in \mathbb{R}, \text{and } a_0 > 0, \tau_0 > 0 \). \( m, k \) were integers. Then the wavelet basis functions and wavelet transformation could be expressed like this

\[
\Psi_{a_0^m, k \tau_0} = a_0^{-m} \tau \Psi(a_0^{-m} \tau t - k \tau_0) \quad (2)
\]

\[
WT_f(a_0^m, k \tau_0) = \langle f(t), \Psi_{a_0^m, k \tau_0}(t) \rangle = a_0^{-m} \tau \int_{\mathbb{R}} f(t) \Psi^*(a_0^{-m} \tau t - k) dt \quad (3)
\]
Assumed that $a_0 = 2$, $\tau_0 = 1$, then the wavelet basis was simplified as:

$$\Psi_{m,k} = 2^{-m/2} \Psi(2^{-m} t - k) \quad (4)$$

This wavelet base was commonly applied in engineering applications. And the discrete wavelet transformation becomes the following form,

$$WT_f (m, f) = \langle f(t), \Psi_{m,f}(t) \rangle = 2^{-m/2} \int_{R} f(t) \Psi^*(2^{-m} t - k) dt \quad (5)$$

The above discretization scheme was a kind of dyadic discrete scheme with good property.

**Multiscale Analysis Derivation from the Angle of the Filter Bank.** Assuming the signal was $x(t)$, the sampled signal $x(n)$ was passed through a low pass filter $H$. This filter could strain off the high frequency signals which was set in advance, and retains the low frequency signals. Similarly, after passing the sampling signal through a high filter $G$, there will be a high frequency signal to be output. Then on the frequency band $0 \sim \pi$, the frequency spectrum of the signal will be decomposed into the low frequency in $\pi/2 \sim 0$ and high frequency signals in $\pi/2 \sim \pi$. Low frequency signal could be interpreted as the smooth part of the original signal whose rough had been erased, and the high frequency part was the details of the original signal.

![Figure 1: Signal decomposition in the frequency domain](image)

After passing two filters $G$ or $H$, the bandwidth of the output signal was reduced to 1/2 of the original bandwidth, so the sampling rate could be reduced to 1/2 of the original signal sampling rate without losing information.

**Principal Component Analysis**

The basic idea of Principal Component Analysis (PCA): conversion of massive relevant random variables into less random variables which are irrelevant to each other; In the study of Algebra, conversion of former Multivariate covariance matrix into diagonal matrix with less variables; In the study of Geometry, dimension reduction for high dimensional variables; in the study which’s prospective abstracted according to features, PCA can be taken as an abstract method which based on Minimum mean square error.

Assuming in a real question, there are $p$ index, marked as data matrix with $x_1, x_2, \cdots, x_p$, $n$ samples.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix} = \begin{bmatrix} X_1 & X_2 & \cdots & X_p \end{bmatrix} \quad (6)$$
PCA is to convert $p$ observed variables into $p$ new variables by combination of linear transformation.

$$
\begin{align*}
F_1 &= a_{11}X_1 + a_{12}X_2 + \cdots + a_{1p}X_p \\
F_2 &= a_{21}X_1 + a_{22}X_2 + \cdots + a_{2p}X_p \\
& \quad \cdots \\
F_p &= a_{p1}X_1 + a_{p2}X_2 + \cdots + a_{pp}X_p
\end{align*}
$$

(7)

Then, $F_1$ is the first principal component, $F_2$ as the second, and so on.

**Empirical Analysis**

This paper selects the most representative stock index from NPC (November 8, 2012) to October 22, 2015 of the different countries or regions, including the composite stock index on the Shanghai Stock Exchange, the Taiwan weighted index, the Jakarta index, Nikkei, Hang Seng Index, South Korea's Kospi index, the FTSE 100 index, Germany's DAX index, the standard & Poor's 500 index, Australian Standard & Poor's 500 index. Then seek the yield of these data, and the formula is:

$$R_t = 100 \times \ln(P_t/P_{t-1})$$

(8)

In this paper, MATLAB is used to analyze the yield. The wavelet basis selected to decompose the data was DB6 wavelet. After decomposed, the data was turned into low frequency signal $a_i$ and high frequency signal $d_1, d_2$. $d_1$ corresponds to 2 days fluctuations of the data, which often reflects the operation of short-term speculators. $d_2$ corresponds to 4 days fluctuations of the data, which generally reflects the operation of long-term investors. What’s more, low frequency approximately represents the long-term trend of the stock.

Take a case study of Shanghai Stock Exchange Composite Index yield. Firstly, after 2 layers decomposition of data, the result turn out to be $a_i, d_1$ and $d_2$. Low frequency signal $a_i$ and high frequency signal $d_1$ are obtained by wavelet decomposition of the original signal. After $a_i$ being decomposed, the decomposing result $d_2$ is the high frequency part of $a_i$, which describes the investment strategy of long-term investors. Then analyze the principal component of $a_i, d_1$ and $d_2$, respectively.

Use SPSS software to execute the principal component analysis to $a_i, d_1$ and $d_2$. The information about the independent variables of each principal component can be obtained by calculating the rotation factor load matrix. Generally, the independent variables were categorized into the main component, in which the load value of independent variable is the largest. Table 1 illustrates rotation factor load matrix.
Table 1. Rotated component matrix of $a_i$, $d_i$ and $d_2$.

<table>
<thead>
<tr>
<th></th>
<th>$a_i$</th>
<th>$d_i$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Component 1</td>
<td>Component 2</td>
<td>Component 1</td>
</tr>
<tr>
<td>China</td>
<td>-0.038</td>
<td>0.773</td>
<td>0.036</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.471</td>
<td>0.647</td>
<td>0.786</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.335</td>
<td>0.558</td>
<td>0.615</td>
</tr>
<tr>
<td>Japan</td>
<td>0.573</td>
<td>0.379</td>
<td>0.661</td>
</tr>
<tr>
<td>HK</td>
<td>0.350</td>
<td>0.802</td>
<td>0.527</td>
</tr>
<tr>
<td>Korea</td>
<td>0.378</td>
<td>0.707</td>
<td>0.817</td>
</tr>
<tr>
<td>UK</td>
<td>0.876</td>
<td>0.287</td>
<td>0.213</td>
</tr>
<tr>
<td>Germany</td>
<td>0.860</td>
<td>0.168</td>
<td>0.115</td>
</tr>
<tr>
<td>USA</td>
<td>0.870</td>
<td>0.163</td>
<td>-0.003</td>
</tr>
<tr>
<td>Australia</td>
<td>0.573</td>
<td>0.446</td>
<td>0.678</td>
</tr>
</tbody>
</table>

Take the result of $a_i$ as an example. The calculation indicates that the load value of Japan, UK, Germany, USA and Australia is relatively great in the first principal component, which means that the linkage of the stock market in these countries is relatively strong. And the second principal component get grater load value in China, Taiwan, Indonesia, HK and Korea, which shows that the stock market of 5 countries or regions has strong correlation.

The results of empirical analysis could be summarized as Table 2.

Table 2. Summary of the results of empirical analysis.

<table>
<thead>
<tr>
<th>Signal type</th>
<th>Component</th>
<th>Stock market</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
<td>First principal component</td>
<td>Japan, UK, Germany, USA, Australia</td>
</tr>
<tr>
<td></td>
<td>Second principal component</td>
<td>China, Taiwan, Indonesia, HK, Korea</td>
</tr>
<tr>
<td>$d_1$</td>
<td>First principal component</td>
<td>Taiwan, Indonesia, Japan, Korea, Australia</td>
</tr>
<tr>
<td></td>
<td>Second principal component</td>
<td>UK, Germany, USA</td>
</tr>
<tr>
<td></td>
<td>Third principal component</td>
<td>China, HK</td>
</tr>
<tr>
<td>$d_2$</td>
<td>First principal component</td>
<td>China, Taiwan, Indonesia, Japan, HK, Korea, Australia</td>
</tr>
<tr>
<td></td>
<td>Second principal component</td>
<td>UK, Germany, USA</td>
</tr>
</tbody>
</table>

A Discuss on Linkage of Different Stock Markets and Advice for Investors

PCS on long-term trend rate of stock returns, act of short-term speculators, act of long-term investor can give us the results of the relevance of different stock marks. According to the result of empirical experiment, we can have the conclusion as followed.

Based on long-term trend of stock returns rate, the stock markets of Japan, UK, Germany, USA and Australia were of high relevance. It was obvious that the real economy of mentioned country had higher connections with many currency policies. Real Economy and currency policies had most powerful affections on the long-term trend of stock markets. Meanwhile, the stock markets of China, Taiwan, HK, Indonesia and Korea were of high relevance. For these Asian Country and region’s economy and currency policies were closely related to China. Among mentioned 10 economies, China and USA were of the most importance and they were of strongest influence. Incidentally, they were spread into different PC. This was mainly caused by the difference of Chinese and USA currency policies, for instance, in 2015, Chinese Central Bank reduced benchmark deposit rate 3 times, but Federal Reserve demanded a increase on interest rate. That was why it was easy to understand why the turnover of them was enormous meanwhile the long-term trend of stock markets were not of high relevance. For long-term investor, to reducing risk, it is advised to invest in stock markets of different PC.
Based on short-term speculators’ acts, stock markets of Taiwan, Indonesia, Japan, Korea and Australia are of high relevance. UK, Germany, USA’s stock markets were of more obvious relevance. Chinese and HK’s relevance were more obvious among others. The Short-term changes of high relevance indicated that the info-exchange speeds were swift. From the result of classification, Asian stock markets, China and HK were of class 1, which meant the info-exchange speed between them was smooth. That was directly connected with deeper degree of capital openness between China and HK. The other stock markets in Asia and Occidental stock markets were of class 2, which indicated that info-openness were higher and capital flow speed were quicker among Asian stock markets, compared to those between Asian and Occidental stock markets. On the other hand, among Occidental stock markets, info-openness were higher and capital flow speed were quicker, compared to those between them and Asian stock market. Thus, short-term speculators should spread capital into all 3 classes’ stock markets to reduce risk.

Based on acts of relative long-term investors, China, Taiwan, Indonesia, Japan, HK, Korea and Australia can be classified as class 1, while UK, Germany and USA can be classified as class 2. Relative long-term investors’ strategies would be affected by many factors combined, include long-term factors and short-term factors, for instance, the conformity between real economy and currency policies, the capital flow speed between markets and info-exchange speed, etc. The result of classification affirmed this phenomenon that markets of Asia and Occident were obviously different class. Also it affirmed that markets of close region had higher relevance according to long-term investors’ act. If investors should intend to have a long-term investment and reduce the risk, they should invest the capital into Asian and Occident stock market.

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References


