Dynamic Pricing of Airline Tickets with Strategic and Myopic Passengers

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Abstract. This paper studies the optimal clearance pricing of tickets in the presence of strategic and myopic passengers. We establish a two-period dynamic pricing model, and characterize equilibrium prices and profits using equilibrium theory and backward induction method. We prove that a pure strategy equilibrium fails to exist. Instead, airlines play a mixed strategy with respect to their pricing. Finally, we use numerical examples to explore the effect of various parameters on the performance of optimal strategy.

Introduction

In the airline revenue management, clearance pricing of tickets is a very important problem for selling tickets. However, the strategic passenger behavior makes the tickets mark down in price and reduce airline’s income. In January 2015, the major airlines of China fight in off-season with ultra-low discount ticket strategy to grab market share, which lead to civil aviation total profit loss of nearly 1.5 billion. Directly reducing price is not an effective method to solve the clearance problem, especially under strategic passenger behavior. Traditional strategies enable more strategic passengers to wait for to the sales end of the period, which decrease airline’s profit. How to reasonably price tickets and take competitors’ prices into account are significant for airlines when facing strategic and myopic passengers.

There are two streams of research that are related to our paper: clearance pricing of the perishable product and the strategic consumer behavior. In the clearance pricing of the perishable product literature, Cachon and Kok solve for the optimal order quantity in the newsvendor model assuming rational clearance pricing, they show that the more surplus products, the greater the clearance discount [1]. Koenigsberg et al. study the conditions under which offering a last-minute deal is optimal with the single-price policy, they find that for an intermediate capacity level, the larger the number of segments (that differ in price sensitivity), the longer the duration of the period in which tickets are offered for sale [2]. Dasci and Karakul study dynamic and fixed-ratio pricing policies in a duopoly by building a two-period model, in which capacity constrained firms compete on selling perfectly substitutable and perishable products. They find that the market is more efficient under fixed-ratio pricing [3]. Smith develops optimal clearance prices and inventory management policies that take into account the impact of reduced assortment and seasonal changes on sales rates. These documents make clearance price through the demand and the number of remaining products, which without considering strategic consumer behavior [4]. In fact, consumers exhibit strategic behavior, in that they may strategically time their purchases in anticipation of future price changes. Consumer strategic behavior makes great challenge to firm’s original pricing strategy. Besanko and Winston study find that total revenue of firm will drop about 20% if seller ignore consumer strategic behavior [5].

In the aspect of perishable goods markets with consumers strategic, Aviv and Pazgal research optimal pricing of seasonal products in the presence of strategic consumers, they propose inventory contingent discounting strategies, and announced fixed-discount strategies [6]. Dan and William...
study the effect of strategic consumer behavior on pricing and rationing decisions of a firm selling a single product over two periods. They find that the policy of doing the better of not restricting availability at the clearance price or not offering the product at the clearance price is typically near optimal [7]. Yan and Ke present posterior price matching (PM) and delay posterior price matching (DPM) for dynamic perishable product pricing to consider in strategic consumer behavior. They analyze reduction season price, purchasing equilibrium and regular selling season price using equilibrium theory and backward induction method [8]. However, competition is not considered in these papers. To our knowledge, only Jerath et al. consider the strategic consumer behavior in a competitive environment. The author find that last-minute selling strategy cannot maximize revenue when demand is to deterministic [9]. Ji based on consumer utility function, established a two-stage dynamic pricing model and discussed pricing strategies under customer behavior and market competition. The author finds that the strategic customer behavior reduces both firms' revenue [10].

In Dasci and Karakul model, two different classes of customers come and go in two different periods whereas in our model, customers are divided strategic and myopic passengers who exist in two periods. We start with passenger strategic behavior, research airlines optimal clearance pricing under competition and discuss the equilibrium price. For airlines which facing the intense market competition to take clearance price and revenue management provide theoretical basis.

**Model Descriptions**

We consider two competing airlines, $A$ and $B$, each hold a quantity $k$ of tickets to be sold over two periods, $t = 1, 2$. We assume that there is no vertical differentiation between tickets of the two airlines. The tickets have no value after the flights take off. There are $2\beta$ passengers who arrived at beginning of period. We divide passengers into two categories, strategic and myopic passengers. Each category has the same number $\beta$. This assumption is reasonable, because Ovchinnikov and Milner fund that half of the passengers choose to wait to buy special fares before the flight take off [11]. Myopic passengers will purchase at $t = 1$ as long as the tickets price does not exceed their valuations, or leave the market forever. Strategic passengers may decide to postpone their purchases if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. To simplify the problem, we assume that all passengers have the same valuation $r$ for the tickets.

We use superscript $i(i = A, B)$ denote the airline, subscript $j(j = 1, 2)$ denote the period.

The following notation is used throughout the paper:

- $p_i^j$: price of airline $i$ at $t = j$
- $k_i^j$: remaining tickets of airline $i$ at $t = 2$
- $D_i^j$: demand of airline $i$ at $t = 2$
- $\Pi_i^j$: expected total profit of airline $i$ at $t = j$
- $U_j$: passenger’s utility at $t = j$
- $\alpha$: the degree of strategicity of a passenger, $\alpha \in [0, 1]$ [12,13].

Let us formally make some assumption for our model:

The airlines and the passengers are homo economicus, that is to say, the airlines pursue the maximum profits and the passengers search for utility maximization;

$\beta \leq k < 2\beta$.

**The Model**

In this section, we study the equilibrium strategies using standard backward induction. Let $\beta_t$ denote the number of passengers who remain in the market at $t = 2$, their valuation is $\alpha r$. Then the number of remaining tickets at $t = 2$ is $k_i^A + k_i^B$. Bikhchandani and Mamer present a model that is closest to ours
They present a single-period inventory liquidation competition between two asymmetric firms. Their model is identical to the second stage subgame of our problem, and therefore their results are utilized in this paper. Depending on the prices, sales realizations of the airlines at $t=2$ (omitting the symmetric cases) are summarized as shown in Table 1 [3].

<table>
<thead>
<tr>
<th>Prices</th>
<th>$D_A^t$</th>
<th>$D_B^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_A^t &lt; p_B^t &lt; \alpha r$</td>
<td>$\min{k_A^t, \beta^t}$</td>
<td>$\min{k_B^t, [\beta^t - k_A^t]}$</td>
</tr>
<tr>
<td>$p_A^t = p_B^t = \alpha r$</td>
<td>$\min{k_A^t, \beta^t}$</td>
<td>$\min{k_B^t, [\beta^t - k_A^t]}$</td>
</tr>
<tr>
<td>$p_A^t &lt; \alpha r &lt; p_B^t$</td>
<td>$\min{k_A^t, \beta^t}$</td>
<td>$\alpha r$</td>
</tr>
<tr>
<td>$p_A^t, p_B^t &gt; \alpha r$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

This game has the following four cases that arise from a comparison of the inventories and the market size at $t=2$:

- **Case 1:** $k_A^t, k_B^t \geq \beta^t$
- **Case 2:** $k_A^t + k_B^t > \beta^t, k_A^t \leq k_B^t < \beta^t$
- **Case 3:** $k_A^t + k_B^t > \beta^t, k_A^t \leq k_B^t < \beta^t$

In Case 1, each airline has sufficient tickets to satisfy the demand by itself, and hence Bertrand’s marginal cost pricing arises. So we just analyze Case 2, and the analysis will be identical for Case 3.

Bikhchandani and Mamer model doesn’t consider the strategic consumer behavior. Based on their model, we introduce the discount factor of value to our model. We obtain the optimal pricing and expected profit of airline $A$ at $t=2$ in Case 1 as follows:

$$p_A^t = \left(\beta^t - k_A^t \right) \left( k_A^t + k_B^t - \beta^t \right) \ln \left( \frac{k_B^t}{\beta^t - k_A^t} \right) \alpha r$$

$$\Pi_A^t = \left( \beta^t - k_A^t \right) k_A^t \alpha r$$

The optimal pricing and expected profit of airline $B$ is:

$$p_B^t = \left(\beta^t - k_B^t \right) \left( k_A^t + k_B^t - \beta^t \right) \ln \left( \frac{k_A^t}{\beta^t - k_B^t} \right) \alpha r$$

$$\Pi_B^t = \left( \beta^t - k_B^t \right) k_B^t \alpha r$$

We next analyze the price game between two airlines at $t=1$. There are four scenarios as follows.

If $p_A^t = \gamma^t, p_B^t = \gamma^t (\gamma^t > \gamma^t)$, then $k_A^t = k_B^t = k$. Two airlines will not sell any tickets at $t=1$, it lead to Case 1, so the total expected profit is:

$$\Pi_A^t = \Pi_B^t = 0$$

If $p_A^t \leq \gamma^t < p_B^t$, only airline $A$ sell tickets at $t=1$, the myopic passengers will purchase immediately as long as $p_A^t \leq \gamma^t$. However, strategic passengers rationally anticipate purchase opportunities at $t=2$ and will buy at $t=1$ only if the price is sufficiently. We have the following lemma.

**Lemma 1** If $p_A^t \leq \gamma^t < p_B^t$, then $U_2 > U_1$, all strategic passengers will wait until $t=2$.

**Proof:**

When $p_A^t \leq \gamma^t < p_B^t$, the airline $A$ sells tickets but the airline $B$ doesn’t sell at $t=1$. If the price $p_A^t < \gamma^t$ of airline $A$ is more attractive for strategic passengers to buy at $t=1$, the remaining tickets of market at $t=1$ is $k_A^t = k$, it contradicts to the condition $k_A^t + k_B^t > \beta^t, k_A^t \leq k_B^t < \beta^t$. Suppose that the
airline A sells $\beta$ tickets, the remaining tickets at $t=2$ are $k^A_2 = k - \beta$. And the remaining tickets of the airline B are $k^B_2 = k$, then the price of airline B must below A, the strategic passengers will buy from B. As discussed above, the optimal pricing of airline A at $t=1$ is $r$, and makes a profit of $r\beta$. Therefore, we obtain $U_1 = 0$.

Let Eq. 1 - Eq. 2

\[
p^A_i - p^B_i = \left[ \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \ln \frac{k^A_i}{k^B_i} \right] \alpha r \left[ \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \left( \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \ln \frac{k^A_i}{k^B_i} \right) \right] \alpha r > 0
\]

So $U_2 = \alpha r - p^B_i = \alpha r - \left[ \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \left( \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \ln \frac{k^A_i}{k^B_i} \right) \right] \alpha r$.

Where $k^A_i = k - \beta, k^B_i = k$, we have

\[
U_2 = \alpha r \left[ 1 - \frac{\beta_i + \beta - k}{2k - \beta_i} \ln \frac{k}{\beta_i + \beta - k} - \frac{\beta_i - k^A_i}{k^A_i + k^B_i - \beta_i} \ln \frac{k^A_i}{k^B_i} \right] > 0
\]

$U_2 > U_1$, all strategic passengers will wait until $t=2$, this implies $\beta_i = \beta$.

This completes the proof.

From lemma 1, the expected profit for airline A and B are:

\[
\Pi^A = r\beta + \left( \frac{2\beta - k}{k} \right) (k - \beta) \alpha r
\]

\[
\Pi^B = (2\beta - k) \alpha r
\]

$p_i^A = p_i^B \leq r$, the myopic passengers will purchase immediately. If the price $p_i^A < r$ of airline A is more attractive for strategic passengers to buy at $t=1$, it obtains expected profit $\Pi^A = \Pi^B = p_i^B \beta$. Now either airline has an incentive to deviate $p_i^A$, and then Bertrand outcome arises. So the optimal pricing of airline A at $t=1$ is $r$, all strategic passengers will wait until $t=2$. Then the passenger’s utility is:

\[
U_2 = \alpha r - p^A_i = \alpha r - \left[ \frac{k - \beta}{2k - \beta} \ln \frac{k/2}{\beta - k/2} \right] \alpha r \left[ \frac{k - \beta}{2k - \beta} \ln \frac{k/2}{\beta - k/2} \right] > 0
\]

Since $U_2 > U_1$, the expected profit for airline A and B are:

\[
\Pi^A = \Pi^B = r\beta / 2 + (\beta / 2 - k)^2 \alpha r
\]

If $p_i^B \leq r < p_i^A$, the analysis is identical to the $p_i^A \leq r < p_i^B$, and therefore, it is omitted. The expected profit for airline A and B are:

\[
\Pi^A = (2\beta - k) \alpha r
\]

\[
\Pi^B = r\beta + \left( \frac{2\beta - k}{k} \right) (k - \beta) \alpha r
\]

Based on the four situations, we have the following theorem.

**Theorem1.** There is no pure-strategy equilibrium of the airlines; 2) There are only mixed strategy equilibrium, both airlines quote $r$ with probability $\omega$ and $r$ with probability $1 - \omega$ at $t=1$, where
\[ \omega = \frac{\beta + \alpha (k-\beta)(2\beta-k)/k}{(2\beta-k)-(k-\beta)(2\beta-k)/k-(3\beta-2k)^+}\alpha + 3\beta/2 \]

and receive an expected total profit of

\[ \Pi_{\infty} = (2\beta-k) \alpha \omega \]  \( \text{(9)} \)

Proof:
Suppose that both airlines quote \( p_i \leq r \), then airlines sell \( \beta/2 \) tickets at \( t=1 \) and end up with \( k-\beta/2 \) tickets at the beginning of \( t=2 \). Hence, both airlines make an expected profit of

\[ \Pi (p_i, p_i) = p_i \beta/2 + (\beta/2-k)^+ \alpha r \]

However, an airline may deviate and set the price at \( p_i - c \) for an expected profit of

\[ \Pi (p_i - c, p_i) = (p_i - c) \beta + (2\beta-k)/k \alpha r \]

as \( c \to 0 \), \( \Pi (p_i - c, p_i) = p_i \beta + (2\beta-k)/k \alpha r \)

or it may choose not to sell in the first period by quoting \( r \) for an expected profit of

\[ \Pi (r, p_i) = (2\beta-k) \alpha r \]

If \( k > 3\beta/2 \), then \( \Pi (p_i, p_i) \leq \Pi (p_i - c, p_i) \) for \( p_i \geq 0 \). Let us prove by contradiction that \( \Pi (p_i, p_i) \leq \max \{ \Pi (p_i - c, p_i), \Pi (r, p_i) \} \) when \( \beta \leq k < 3\beta/2 \).

Observe that

\[ p_i \geq (2\beta - k) \alpha r \]
\[ \frac{1}{2} p_i \beta \geq \frac{1}{2} (2\beta - k) \alpha r \beta - \frac{1}{2} \alpha r \beta \]
\[ p_i \beta + \left( \frac{2\beta-k}{k} \right) (k-\beta) \alpha r \geq \frac{p_i \beta}{2} + \left( \frac{3\beta-k}{2} \right) \alpha r \]
\[ \Pi (p_i - c, p_i) \geq \Pi (p_i, p_i) \]

Similarly, observe that

\[ p_i \leq (2\beta - k) \alpha r \]
\[ \frac{p_i \beta}{2} + \left( \frac{3\beta-k}{2} \right) \alpha r \leq (2\beta-k) \alpha r \]
\[ \Pi (p_i, p_i) \leq \Pi (r, p_i) \]

As above discussed, the first stage game does not have a pure strategy equilibrium, and hence we look for an equilibrium in mixed strategies.

Summing up Eq. 3 ~ Eq. 8, we can reduce the first-stage game to a game in bimatrix form as shown in Table2.
Table 2. The first-stage game.

<table>
<thead>
<tr>
<th>Airline A</th>
<th>Airline B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$r$</td>
</tr>
<tr>
<td>$r$</td>
<td>$r$</td>
</tr>
</tbody>
</table>

The equilibrium condition is simply that the airlines receive equal expected profit from playing either strategy. Therefore, by playing $r$ or $r'$, an airline should make identical expected profit, i.e.,

$$
\omega \left[ \frac{r \beta}{2} + \left( \frac{3 \beta - k}{2} \right) \alpha \right] + (1-\omega) \left[ r \beta + \left( \frac{2 \beta - k}{k} \right) (k-\beta) \alpha \right] = \omega (2\beta - k) \alpha
$$

from which we obtain $\omega$ as

$$
\omega = \frac{\beta + \alpha(k-\beta)(2\beta-k)/k}{\left[ (2\beta-k) - (k-\beta)(2\beta-k)/k -(3\beta-2k) \right] \alpha + \beta/2}
$$

This completes the proof.

**Numerical Experiments**

We present a numerical experiment in this section to illustrate how the proposed approach works in a constructed example. First we will illustrate the profit $\prod$ on the performance of the demand $\beta$ when passengers have fully strategic behavior, $\alpha = 1$. Assuming each airline has 100 units of tickets, without loss of generality, as demonstrated in Fig.1. We can see that the profit $\prod$ is increasing in $\beta$. Obviously, the greater the demand, the more benefits obtained by the airlines.

![Figure 1. The airline profit $\prod$ as a function of the demand $\beta$.](image)

Next, we consider the profit $\prod$ on the performance of the strategicity parameter $\alpha$, as demonstrated in Fig.2. Three values of $\alpha$ are examined: $1/2, 2/3, 4/5$. We can see that the profit $\prod$ is increasing in $\alpha$. The reason is intuitive, a large $\alpha$ means the probability of strategic passengers waiting is large, there will be much more tickets at $t=2$ for those passengers in order to gain more profit through dynamic pricing.
Conclusions

In this paper we research clearance pricing policy in a duopoly, in which airlines facing strategic and myopic passengers. The problem is modeled as dynamic game with complete information and subgame-perfect Nash equilibrium concept is used to characterize the solutions. To analyze this issue we propose a two-period dynamic model in which two airlines compete for passengers by selling a limited inventory of tickets. We prove that there does not have a pure strategy equilibrium of the airlines, only exists mixed-strategy equilibrium under the condition of $\alpha \leq k < 2\beta$. Finally, we use numerical experiments to explore the effect of various parameters on the performance of optimal strategy. The main conclusion is that the strategic passenger behavior can be relieved by dynamic pricing.

Our research can provide theoretical basis for the optimal pricing decision to airlines in clearance pricing when facing strategic and myopic passengers. There are a number of important research avenues that can be pursued. One natural extension is to increase the number of periods, and hence the number of opportunities airlines can change prices. Besides, passengers’ purchase process may be endogenously derived as a function of stochastic passenger arrival process.

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References

