Price Protection with Consumer’s Policy Behavior

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Abstract: Selling seasonal usually can be divided into two stages by retailers, one of which is the merchandise is sold at a regular price during the regular period and the other is the merchandise sold at a lower price in the discounted period. The price in the discounted period is often lower than the cost price in order to reduce the inventory, which results in a decrease in the profit earned by retailers. Therefore, retailers should take some steps to improve this situation. This paper studies how the price protection strategy carried by retailers influence the retail price, the optimal quantity and profit when the strategic consumers and uncertain demands exist. Price protection is a kind of rebate that the retailer offer consumers price discount when price drops during the selling season, there are three alternative strategies for retailers: no price protection policy, partial price protection policy, full price protection policy. The optimal price and order quantity for retailers are analyzed based on rational exception equilibrium theory for each policy. The result shows that price protection strategies can improve the profitability of retailers, and the full price protection policy is the optimal one.

1. Introduction

The rapid upgrading of products shortens them lifecycle, which aggravates uncertainty of consumers. The product is not emptied within the specified sales period, not only increasing the retailer's inventory costs, but also reducing the value of the product itself, so the retailer has no choice but to reduce the price in order to close out the remaining products. Under these circumstances, strategic consumers believe that there will be a price reduction in the future so that they may delay their purchase. However, consumers must make a trade-off between price and the product, which is the consumers decide to purchase at a regular price, or may purchase nothing after the price declines, because the inventory is limited.

Retailer has proposed price protection strategies to encourage consumers to purchase products at normal price. Price protection strategies refer to a form of the compensation for consumers when the products reduction during the selling period. These strategies can effectively reduce the retailer's expense spends on the inventory. For example, if the price declines within 30 days, Jing-dong Mall promise that: consumers who buy products at normal prices (not including spike, promotional price) can apply for price protection and the excess money will returned to the account paid by consumers; Su-ning Tesco regularly launch a price protection strategy, for consumers who buy the goods can obtain a month of price protection in a certain period of time.

This paper studies three price protection strategies between a retailer and consumers, if the price falls during the selling season after consumers purchase, the retailer will pay compensation to consumers. The standard of compensation is the price fall range. The selling season is divided into two stages: the regular period and the discounted period. According to the amount of price compensation, there are two kinds of price protection policies: (1) partial price protection, compensates consumers for the partial amount of the price difference, (2) full price protection compensates consumers who purchase in the regular period for the full amount of the price difference. The paper analyzes the optimal pricing and the optimal order quantity in consideration of the cost of out-of-stock under different price protection strategies, and then obtains the optimal price protection strategy.
2. Literature Review

Consumers' purchase decisions are affected by many factors such as price and availability in the future. They will buy immediately or later according to the utility of each plan, which we call consumer strategic behavior. Muth (1961) first proposed the consumer’s strategic behavior. He believed that customer behavior was by observing the business strategy and the law to determine their own economic behavior in order to achieve their own benefits to maximize. Coase (1972) did an initiative research on consumer strategy behavior which had an impact on manufacturers of producing durable goods. He found that monopolists were forced to set the price as the marginal cost of the product when the strategic customers realized that the price of the durable goods would fall in the future. Avis, Yossi, Pazgal (2008) studied the impact of strategic customers on the optimal pricing of fashion products. When strategic customers hold information about price discounts of future products, they would purchase them after the price fell. Su, Zhang (2009) found that the strategic consumers sought the largest consumer surplus, and the merchants can reduce the stock surplus by price compensation to increase the profits in the literature. Cachon, Swinney (2011) proposed a number of methods for the capability of rapid response and enhanced design to meet the uncertain need of the strategic consumers in the fashion industries.

At present, the research on price protection is divided into two types: internal price protection and external price protection. The former refers to the retailers commit consumers to sell the goods at the lowest price, otherwise consumers can request for refund, such as Watsons, Carrefour, Wal-Mart Stores, etc. The latter refers to retailers in the sales period will compensate consumers for corresponding difference if the price declines. This paper mainly focuses on external price protection. Lee et al. (2000) studied price protection in the use of personal computers. Price protection at fixed wholesale prices could benefit the entire chain and retailers, but it will be harmful to the manufacturers. Levin et al. (2010) studied the price commitment and its impact on revenue management when retailers decided on dynamic pricing decisions. Xing, Liu (2012) proved that the proper price protection could coordinate the supply chain. Wang, Zhou (2015) studied the price protection strategy based on the rational expectation equilibrium concept.

3. The Model Descriptions

The symbols in the model are shown in Table 1.

<table>
<thead>
<tr>
<th>symbols</th>
<th>meanings</th>
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<tbody>
<tr>
<td>$p$</td>
<td>price at regular period</td>
<td>$s$</td>
<td>price at discount period</td>
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<tr>
<td>$Q$</td>
<td>quantity of orders for retailers</td>
<td>$c$</td>
<td>the unit cost of goods</td>
</tr>
<tr>
<td>$g$</td>
<td>unit cost of shortage</td>
<td>$\alpha$ (0 ≤ $\alpha$ ≤ 1)</td>
<td>compensation rate</td>
</tr>
<tr>
<td>$\xi$</td>
<td>the retention value</td>
<td>$\mathcal{P}_\text{prob} (\xi \mid \xi_p)$</td>
<td>the possibility of get goods in the future</td>
</tr>
<tr>
<td>$D$</td>
<td>The demand for the consumer</td>
<td>$f_x$</td>
<td>demand function of consumer</td>
</tr>
<tr>
<td>$F(x)$</td>
<td>distribution function of $x$</td>
<td>$f(x)$</td>
<td>density distribution function of $x$</td>
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</table>

The retailer determines the price and order quantity before the start of the sales season, and there is only one order opportunity, and if the order quantity equals the demand for the regular phase, there is no discount period; if the order quantity is less than the demand, then the retailer has to bear the corresponding loss of stock; if the order quantity is greater than the demand in the regular stage, the retailer will need to enter the discount period to sell the remaining goods at discounted prices.

In order to facilitate the study of the problem, we assume that the retailer's sales period is divided into two stages. The first stage is the regular period, the retailer sells the goods at the normal price $p$, the second stage is the discount period, which is sold at the discounted price $s (p > s)$. 
4. Assumptions

This model has certain conditions, we make the following assumptions:

1. Consumer demand distribution function and probability density function satisfies IFR (increasing failure rate), so \( f(x) / (1 - F(x)) \) is an increasing function, what's more \( f(0) = 0 \)

2. Sellers selling season is very short, businesses will only provide price protection on recent sales of merchandise, such as Jing-dong, provide price protection receive goods within 30 days.

3. The discount price is an exogenous variable, and \( s < V \).

4. As long as the consumer's surplus is greater than zero, the consumer will buy goods.

5. The retailer determines the price compensation rate before the start of the season and predicts the consumer's valuation of goods. And then according to the price compensation rate and forecast information to determine the retail price and order quantity.

6. The possibility of consumers can get goods in the future \( \xi_p \) is the same as \( F(Q) \).

5. Model Establishment Based on Partial Price Protection Strategy

Conclusions:

1. \[ p^* = \frac{\nu - \xi_p \nu + s(\alpha - 1)}{1 - \alpha \xi_p} \]

2. Optimal order quantity can be expressed as:

\[ (p_p + g) F(Q) - sF\left(\frac{sQ_p}{s(1 - \alpha) + \alpha p_p}\right) = (c - s) \]

Proofs:

1. To promote consumers to buy goods at the regular stage, the first stage of the consumers' surplus of purchasing should be bigger than or equal to the second stage, so the retailer decides the price satisfying:

\[ (\nu - p_p) + \alpha (p_p - s) \xi_p \geq (\nu - s) \xi_p \Rightarrow p_p \leq \frac{\nu - \xi_p \nu + s(\alpha - 1)}{1 - \alpha \xi_p} \]

Where consumers' surplus of purchasing in the regular period is \( \nu - p_p \), their surplus in the discount period is \( \nu - s \).

Noticing consumers’ highest price that can be accepted is \( p^* = \frac{\nu - \xi_p \nu + s(\alpha - 1)}{1 - \alpha \xi_p} \) (1)

2. The retailer’s profit function consists of the first and second stage:

\[ \pi_p(Q, p_r, \alpha) = p_r E \min(D, Q) + E \max[sQ_p - D(s - \alpha p_r), 0] - gE \max(D - Q_p) - cQ_p \]

\[ = p_r E \min(D, Q) + E \max[sQ_p - D(s - \alpha p_r), 0] - gE \max(D - Q_p) - cQ_p \]

\[ = p_r E \min(D, Q) - \frac{sQ_p}{s(1 - \alpha) + \alpha p_r} - gE \max(D - Q_p) - (c - s)Q_p \]

\[ = (p_r + g) \int_0^{\nu} x f(x) dx - \int_0^{\nu} Q p f(x) dx + Q_p - g x \]

\[ = \frac{sQ_p}{s(1 - \alpha) + \alpha p_r} + \frac{\nu - sQ_p}{s(1 - \alpha) + \alpha p_r} - \frac{sQ_p}{s(1 - \alpha) + \alpha p_r} \int_0^{\nu} f(x) dx - (c - s)Q_p \]

The first-order derivative on \( Q_p \) of \( \pi_p(Q, p_r, \alpha) \) is:

\[ \frac{\partial \pi_p}{\partial Q_p} = (p_r + g) F(Q_p) - sF\left(\frac{sQ_p}{s(1 - \alpha) + \alpha p_r}\right) - (c - s) \]

By proving \( \frac{\partial \pi_p}{\partial Q_p} = 0 \) exists one positive solution and the \( Q_p \) satisfies
Denote $\frac{F(Q)}{Q}$ as $1-F(Q)$

Where $p^* = \frac{v - s}{\alpha} p \frac{(1-s)}{(1-s)}$, so the optimal order quantity satisfies

\[ \frac{v - s}{\alpha} p \frac{(1-s)}{(1-s)} + g \frac{F(Q_p)}{Q_p} - s \frac{F(\alpha s + (1-s)\xi_p)}{Q_p} = (c-s) \]

(2)

Then we prove

\[ \frac{\partial^2 \pi}{\partial Q^2} = -(p+g) \frac{f(Q_p)}{Q_p} + \frac{s^2}{Q_p} \frac{f(Q_p)}{(1-\alpha) + \alpha p} \]

= \left[ (1-\alpha) + \alpha p \right] \left[ \frac{-2}{(1-\alpha) + \alpha p} f(Q_p) + \frac{s^2}{(1-\alpha) + \alpha p} \frac{f(Q_p)}{Q_p} \right] \]

\[ \leq (1-\alpha) + \alpha p \left[ -f(Q_p) + \frac{s^2}{(1-\alpha) + \alpha p} \frac{s}{Q_p} \right] \]

< 0

So the optimal order quantity is unique.

Definition 1. When $\alpha = 0$, from Equation 1 and Equation 2, we know:

1) $p^* = \frac{v - s}{\xi_p}$

2) $F_q(Q) = \frac{v - s}{\xi_p}$

Proof 1. (1) In this section, if consumers choose to purchase immediately, their expected surplus is $\frac{v - s}{Q_p}$. Consumers will buy at price $p^*$ if and only if $\frac{v - s}{Q_p} \geq \frac{v - s}{\xi_p}$. So, in this section, the retailer’s price is

\[ p^* = \frac{v - s}{\xi_p} \]

(3)

The retailer’s expected profit function is

$\pi_0 = pE \min(D,Q) + sQ_0 - E \min(x,Q_0) - gE \max(D-Q_0,0) - cQ_0$

$= p \int_0^{Q_0} \min(D,Q_0) f(x)dx + sQ_0 - \int_0^{Q_0} \min(D,Q_0) f(x)dx - g \int_0^{Q_0} \max(D-Q_0,0) f(x)dx - cQ_0$

$= p \int_0^{Q_0} f(x)dx - \int_0^{Q_0} Q_0 f(x)dx + Q_0 + s \int_0^{Q_0} f(x)dx + g \int_0^{Q_0} f(x)dx - g[x-Q_0] - \int_0^{Q_0} f(x)dx + \int_0^{Q_0} Q_0 f(x)dx - cQ_0$

The first-order derivative on $Q_0$ of $\pi_0$ is:

\[ \frac{\partial \pi_0}{\partial Q_0} = -p \int_0^{Q_0} f(x)dx + p + s \int_0^{Q_0} f(x)dx + g \int_0^{Q_0} f(x)dx - c \]

\[ \frac{\partial \pi_0}{\partial Q_0} = -(p-s+g)(1-\int_0^{Q_0} f(x)dx) - (c-s) \]

Combining Equation 3 and Equation 4,

\[ F(Q_0) = \frac{c-s}{v-(v-s)\xi_p} \]

\[ \frac{c-s}{v-(v-s)\xi_p} \]

is proved.

Definition 2. When $\alpha = 1$, From Equation 1 and Equation 2, we know: (1) $p^* = v$;
\[ (v + g) \bar{F}(Q_1^*) - sF\left(\frac{sQ_1^*}{\nu}\right) = (c - s) \]  

(2)

Proof 2. The same way as full price protection, we solve:

(1) \((v - p_i^*) + \xi_i(p_i^* - s) \geq \xi_i(v - s) \Rightarrow p_i^* = v\)

(2) In this part, Retailers will consider price protection strategies only if

\[
\pi(Q, p_i) = p_iE\min(D, Q_i) + E\max\{s(Q_i - D) - (p_i - s)D, 0\} - gE\max(D - Q_i, 0) - cQ_i 
\]

\[
= p_iE\min(D, Q_i) + E\max(-p_iD, -sQ_i) + sQ_i - gE\max(D - Q_i, 0) - cQ_i 
\]

\[
= p_iE\min(D, Q_i) - p_iE\min(D, s\frac{Q_i}{p_i}) - gE\max(D - Q_i, 0) - (c - s)Q_i
\]

= \((p_i + g)\int_0^{Q_i} (sQ_1^* - s\int_0^{Q_i} f(x)dx) - (v - s)\int_0^{Q_i} g(x)dx -(c - s)Q_i\)

Like last part, we can solve that the optimal order quantity satisfies

\[
(p_i + g)\bar{F}(Q_1^*) - sF\left(\frac{sQ_1^*}{p_i}\right) = (c - s) \]

Then we prove

\[
\frac{\partial^2 \pi_i}{\partial Q_i^2} = -(p_i + g)f(Q_i) + \frac{s^2}{p_i^3} f\left(\frac{S^2}{p_i}\right) \leq 0
\]

Combining all above: since \(f(x) \geq f'(x)\) and \(p_i f(Q_i) \geq \frac{s^2}{p_i^3} f\left(\frac{S^2}{p_i}\right)\) is undoubted, we can conclude \(\frac{\partial^2 \pi_i}{\partial Q_i^2} \leq 0\). So the optimal order quantity is unique. Put \(p_i^* = v\) into Equation 5, we know that in this section the optimal order quantity satisfies

\[
(v + g)\bar{F}(Q_1^*) - sF\left(\frac{sQ_1^*}{\nu}\right) = (c - s) 
\]

6. Model Analysis of Three Kinds of Price Protection

Conclusions:

(1) Price protection policy may facilitate consumers to purchase goods in the regular stage, with the rate of price compensate increase, retailers can gradually increase retail prices to increase profits, that is \(p_0^* \leq p_i^* \leq p_1^*\).

(2) When there is a price protection strategy, the retailer can reduce the order quantity, prompting the consumer to purchase in the normal stage, that is \(Q_0^* \geq Q_i^* \geq Q_1^*\).

(3) In the presence of price protection, the retailer's profit will increase as the price compensation rate increases, that is \(\pi_0 \leq \pi_p \leq \pi_1\).

Proves:

The optimal price and optimal order quantity for the three different situations are shown in Table 2:
Table 2.

<table>
<thead>
<tr>
<th>Condition</th>
<th>No price protection ($\alpha = 0$)</th>
<th>Partial price protection ($0 &lt; \alpha &lt; 1$)</th>
<th>Full price protection ($\alpha = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The optimal price in the regular stage</td>
<td>$p^*_0 = v - (v - s)\bar{\xi}_p$</td>
<td>$p^*_p = \frac{v - \bar{\xi}_p[v + s(\alpha - 1)]}{1 - \alpha \bar{\xi}_p}$</td>
<td>$p^*_1 = v$</td>
</tr>
<tr>
<td>Optimal order quantity</td>
<td>$F^<em>(Q^</em>_0) = \frac{c - s}{v - s + g + (s - v)\bar{\xi}_p}$</td>
<td>$F^<em>(Q^</em>_p) = \frac{(v - \bar{\xi}_p)[v + s(\alpha - 1)]}{1 - \alpha \bar{\xi}_p} + gF(Q^*_p)$</td>
<td>$(v + g)F(Q^*_1)$</td>
</tr>
<tr>
<td></td>
<td>$= -sF\left(\frac{s\bar{Q}^*_p}{[s(1 - \alpha) + \alpha \bar{\xi}_p[v + s(\alpha - 1)]}]\right)$</td>
<td>$= -sF\left(\frac{\bar{Q}^*_0}{v}\right) = (c - s)$</td>
<td></td>
</tr>
</tbody>
</table>

Part of the price and no price protection for comparison:

$$p^*_0 \leq p^*_p$$

So $p^*_0 \leq p^*_p \leq p^*_1$. Similarly, $p^*_1 - p^*_0 = \frac{\bar{\xi}_p(v - s)(1 - \alpha)}{1 - \alpha \bar{\xi}_p} \geq 0$. Then $p^*_0 \leq p^*_p \leq p^*_1$

Optimal order quantity and price compensation rate:

Another way of expressing the price by assuming (6):

$$p^*_p = \frac{v - F(Q_p)[v + s(\alpha - 1)]}{1 - \alpha F(Q_p)} = \frac{(v - s)[1 - F(Q_p)] + s}{1 - \alpha F(Q_p)}$$

And since

$$\frac{\delta F(Q_p)}{\delta \alpha} = \frac{\bar{\xi}_p[1 - \bar{\xi}_p(v - s)]}{(1 - \alpha \bar{\xi}_p)^2} = \frac{\delta p}{\delta Q} \frac{\delta Q}{\delta \alpha} \geq 0$$

So $\frac{\delta Q}{\delta \alpha} \leq 0$. That is $Q^*_0 \geq Q^*_p \geq Q^*_1$. This paper also establishes a numerical model to explore the relationship between optimal order quantity and price discount rate. In this numerical example, the retention value $v = 8$, the sale price $s = 4$, the possibility of getting goods in the future $\bar{\xi}_p = 0.5$, the cost $c = 5$, and set three different unit cost of shortage $g = 2$, $g = 3$, $g = 4$, to study the order quantity changes with the price compensation rate. As shown in Figure 1.
Figure 1. The relationship between the optimal order quantity and the price compensation rate.

Figure 1 shows that the optimal order quantity of the retailer decreases as the price compensation rate increases. When faced with strategic consumers, the higher price compensation will prompt consumers to purchase in the regular period. Retailers will reduce the order quantity as much as possible, to reduce the conventional commodity surplus.

(3) Analyze retailer profit and price compensation rates, Set part of the price protection under the maximum profit for $\pi^* = \pi^*(Q^*, p^*, \alpha)$

$$\frac{\partial \pi^*}{\partial \alpha} = \frac{\partial \pi}{\partial Q} \frac{\partial Q}{\partial \alpha} + \frac{\partial \pi}{\partial p} \frac{\partial p}{\partial \alpha}$$

$$= [(p + g)F(Q) - sF\left(\frac{sQ}{s(1-\alpha) + \alpha p}\right) - (c - s)] \frac{\partial Q}{\partial \alpha} + \frac{\partial \pi}{\partial p} \frac{\partial p}{\partial \alpha}$$

When $Q = Q^*$, $(p + g)F(Q) - sF\left(\frac{sQ}{s(1-\alpha) + \alpha p}\right) - (c - s) = 0$, $\frac{\partial \pi}{\partial p} \geq 0$, $\frac{\partial p}{\partial \alpha} \geq 0$. So $\frac{\partial \pi}{\partial \alpha} \geq 0$.

The retailer's profit increases as the price compensation rate increases, that is $\pi_0 \leq \pi_p \leq \pi_1$.

From the above we can find that the full price is the optimal protection strategy, because the retailer can get the maximum profit. At the full price protection the retailer order the least amount, which prompts consumers to buy goods in the regular period. At the same time, at full price protection, the retail price is the highest, and the marginal profit of every product increases, which results in the increase of retailer’s profit. So the firm can increase the profit by increasing the retail price and decreasing the order quantity.

7. Conclusion

With the enhancement of the consumer's strategic awareness, the retailer must take steps to reduce the loss caused by strategic consumers. The price protection strategy is mutually beneficial to consumers and retailers, so it is widely used in e-commerce marketing. This paper shows that retailers can develop higher retail prices to achieve higher profits by adopting consumer protection strategies (including partial and all) in the face of consumer strategy behavior. More importantly, consumers do not have to bear the risk of a possible decline after purchase and they are willing to buy in advance, which makes the retailer more profitable. Of mentioned above, we find that the full price protection strategy is most beneficial to the retailer by calculating and the online retailer can take a full price protection strategy to increase profits.
With the deepening of price protection research, this paper also has following shortcomings:

(1) This paper assumes that the utility of consumer protection against price is linear, but the utility of consumer protection for price may also be non-linear.

(2) This paper only examines the price protection of one retailer, which is not involved of two or more retailers.

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References


