Transit Network Design Based on the Bi-Level Programming

Min SUN
School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China
msun_bjtu@163.com

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Abstract. In this work, we study the transit network design problem on the basis of present urban road network from the perspective of programming and optimizing. So we propose a bi-level programming model to design the transit network. Considering the generation of bus lines, the number of stops along the lines, the interval of departing time, the number of buses on each line every day and so on, planners make endeavors to minimize running cost of transit system in the upper programming model. The lower level programming model is mainly from the angle of the travelers, taking travel time that is also called general travel cost, including in-vehicle travel time and transfer time, as the optimization goal. Finally, a simple experiment is performed to demonstrate the efficiency of the bi-level model above.

1. Introduction

With the rapid development of the society economy and the promotion of the urbanization, there are increasingly numbers of motor vehicles in our country. By the end of 2015, the total number of the civil vehicles that increased by 11.5% than the quantity in 2014, has reached 172.28 million (including three-wheeled vehicles and low-speed trucks, 9.55 million), of which the number of private vehicles is 143.99 million with the growth rate of 14.4%. Simultaneously, the total number of civil cars is 95.08 million with the growth rate of 14.6%, including the private cars number 87.93 million whose growth rate is 15.8% than last year’s. The rapid increase of the number of the vehicles makes the congestions severer, which seriously affects the residents’ life, impedes the development of the society economy and accelerates the deterioration of the ecological environment. Compared with private cars, public transport has a big advantage in saving resources, protecting the environment, improving traffic congestion and enhancing traffic efficiency.


2. Transit Network Design

2.1 Upper programming

In the upper programming, we consider the fixed costs of all the bus lines and the stops along all the lines. Table 1 lists general sets, parameters and decision variables used in optimization model appeared in this part.
Table 1. Main parameters used in the model.

<table>
<thead>
<tr>
<th>Sets</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>The set of all the stops on the transit network.</td>
</tr>
<tr>
<td>$D$</td>
<td>The set of bus routes levels.</td>
</tr>
<tr>
<td>$Q$</td>
<td>The set of middle stops along the bus lines.</td>
</tr>
<tr>
<td>$C_i$</td>
<td>The volume of departure station $i$.</td>
</tr>
<tr>
<td>$C_j$</td>
<td>The volume of terminal station $j$.</td>
</tr>
</tbody>
</table>

Parameters

- $c_{ij,d}$: The fixed cost of the level $d$ bus line between $i$ and $j$.
- $N_q$: The maximum number of bus lines can be docked on the stop $q$.
- $n_{ij,d}$: The number of the buses is needed to purchase on the level $d$ bus line between $i$ and $j$.

Decision variables

- $x_{ij,d}^q = 1$ if the level $d$ bus line include the stop $q$ between $(i, j)$; $= 0$ otherwise.
- $x_{ij,d} = 1$ if there exists the level $d$ bus line between $(i, j)$; $= 0$, otherwise.

The following is the objective function and the related constraints:

$$
\min Z = \sum_{i \in V} \sum_{j \in V} \sum_{d \in D} c_{ij,d} x_{ij,d} + \sum_{i \in V} \sum_{d \in D} \sum_{q \in Q} c_{ij,d}^q x_{ij,d}^q + Z(X)
$$

(1)

St.

1. $\sum_{j} \sum_{d} x_{ij,d}^q \geq 1, \quad q \in V \setminus \{i, j\}$  

(1.1)

2. $\sum_{j} \sum_{d} x_{ij,d} \leq N_q$  

(1.2)

3. $\sum_{j} \sum_{d} x_{ij,d}^q \leq N_q^d$  

(1.3)

4. $\sum_{j} x_{ij,d} n_{ij,d} \leq C_i, \quad i \in V$  

(1.4)

5. $\sum_{j} x_{ij,d} n_{ij,d} \leq C_j, \quad j \in V$  

(1.5)

Eq.1.1 ensures that every stop will be passed by the bus routes. Eq.1.2 represents the capacity of some stop. Eq.1.3 represents the upper number of kinds of bus route plan in some stop. Eq.1.4 and Eq.1.6 respectively means that the number of vehicles parked in the origin station and terminal station is limited. The relationship between two decision variables as follow:

$$
x_{ij,d}^q \leq x_{ij,d}, \quad i, j \in V, \quad q \in Q, \quad d \in D
$$

(2)

2.2 Lower programming

Table 2 has listed the related parameters in this part.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{uv}$</td>
<td>The passenger flow between $u$ and $v$.</td>
</tr>
<tr>
<td>$z_{uv}$</td>
<td>The impedance of the route between $u$ and $v$.</td>
</tr>
<tr>
<td>$t_{uv}^{wa}$</td>
<td>The passenger’s waiting time between $u$ and $v$.</td>
</tr>
<tr>
<td>$t_{uv}^{in}$</td>
<td>The time that passenger spends in car.</td>
</tr>
<tr>
<td>$t_{uv}^{trans}$</td>
<td>The transfer time of passengers between $u$ and $v$.</td>
</tr>
</tbody>
</table>
The length of the route between \( u \) and \( v \).

\( v_{u,v} \) The running speed of the bus between \( u \) and \( v \).

\( t_{u,v}^k \) The time of the bus stopping at the stop \( k \) between \( u \) and \( v \).

\( t_{u,v}^q \) Passenger transfer time at the stop \( q \) between \( u \) and \( v \).

The followings are the objective function and related conditions:

\[
Z(X) = \sum_{u,v} \sum_l f_{u,v} x_{u,v}
\]  

St.

\[
z_{u,v} = t_{u,v}^{Run} + t_{u,v}^{Stop} + t_{u,v}^{Trans}
\]  

\[
t_{u,v}^{Run} = \frac{l_{u,v}}{v_{u,v}}
\]  

\[
t_{u,v}^{Stop} = \sum_k t_{u,v}^k
\]  

\[
t_{u,v}^{Trans} = \sum_q t_{u,v}^q
\]

Eq.3.2, Eq.3.3 and Eq.3.4 separately represents the running time, stopping time and transfer time. The lower programming aims to calculate the total travel time of all the passengers.

3. Analysis of Model

Based on the bi-level programming model, we set the running cost of the buses and the travel cost of all the passengers as our purpose to design and optimize the transit network. At first, we set the stop along the bus line and combine all the plans to serve to the transit network. All the solutions are calculated according to the objective function of the bi-level programming which is limited by the constraints.

The primary solutions are gotten from the upper programming, which can make us calculate the lower objective function. Combine the two costs together to get the result of the solution 1. Randomly select the variables of decisions, then we can obscure the second result of solution 2. In a similar fashion, we can get a series of solutions and results. Finally, we can pick out the best solution as our most optimal result.

4. Conclusion

The bi-level model is conducted to design and optimize the transit network. However, there still exists some practical restricts and conditions that we have ignored. The paper aims to obscure the theoretical solutions. There still a long distance between the theoretical and practical solutions. So we need the further research about the transit network to make the model more feasible.

References


