On the Probability Density Function and Market Risk

Li-jiang ZENG
Research Centre of Zunyi Normal College, Zunyi, Guizhou, China
ZLJ4383@sina.com

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Abstract. In the real life, the market plays an important role to our all kinds of business. Every day we cannot leave the markets and the economy, the market investors always want to report. However, the market investment has high risk. In this paper, we studied the market risk of investment based on the mathematical probability density function, mathematical model and various viewpoints, and obtained a very interesting conclusion.

1. Introduction

In real life, every day we cannot leave the markets and the economy [1-7], for example, production and sales of chemical products, production and sales of building materials, production and sales of daily supplies, input and output of construction [8-13], and so on. However, as the market investors, they always want to report, they asked for higher returns can make little or no customer, and thus low return rate [14-17], on the other hand, if the reservation in advance is too low, the rate of return on the same leads to lower returns, we based on the probability density function of mathematics, mathematical model [18-22] and a variety of point of view, to do the research of market risk of investment.

2. Motivations of Market Investment

In this paper, we start from the knowledge accumulated on the characterization of marginal distributions of asset returns. This knowledge combined with adequate representations of the dependence structure between assets described in the following chapters can then be used to fully define the multivariate risks. The present chapter thus reviews the bricks of individual asset risks which can then be combined with the help of copulas to build the multivariate risk edifice. The emphasis is put on the determination of the precise shape of the tail of the distribution of returns of a given asset, which is a major issue both from a practical and from an academic point of view. Indeed, for practitioners, it is crucial to accurately estimate the high and low quantiles of the distribution of returns (profit and loss) because they are involved in almost all the modern risk management methods while from an academic perspective, many economic and financial theories rely on a specific parameterization of the distributions whose parameters are intended to represent the “macrovariables” influencing the agents.

For the purpose of practical market risk management, one typically needs to assess tail risks associated with the distribution of returns or profit and losses. Following are the recommendations of the BIS (Bank for International Settlements). The BIS is an international organization which fosters cooperation among central banks and other agencies in pursuit of monetary and financial stability. Its banking services are provided exclusively to central banks and international organizations.), one has to focus on risks associated with positions held for 10 days. Therefore, this takes into account the distributions of 10-day returns. However, at such a large time scale, the number of (non-overlapping) historical observations dramatically decreases. Even over a century, one can only collect 2500 data points, or so, per asset. Therefore, the assessment of risks associated with high quantiles is particularly unreliable.
3. Mathematical Models

Recently, the use of high frequency data has allowed for an accurate estimation of the very far tails of the distributions of returns. Indeed, using samples of one to 10 million points enables one to efficiently calibrate probability distributions up to probability levels of order 99.9995%. Then, one can hope to reconstruct the distribution of returns at a larger time scale by convolution. It is the stance taken by many researchers advocating the use of Levy processes to model the dynamics of asset prices. The recent study shows the relevance of this approach, at least for fluctuations of moderate sizes. However, for large fluctuations, this approach is not really accurate, as shown in Fig.1, which compares the probability density function (pdf) of raw 60-minute returns of the Standard & Poor’s 500 index with the hypothetical pdf obtained by 60 convolution iterates of the pdf of the 1-minute returns; it is clear that the former exhibits significantly fatter tails than the latter.

Figure 1. Kernel density estimates of the raw 60-minute returns and the density obtained by 60 convolutions iterates of the raw 1-minute returns kernel density for the Standard & Poor’s 500.

This phenomenon derives naturally from the fact that asset returns can not be merely described by independent random variables, as assumed when prices are modeled by Levy processes. In fact, independence is too strong an assumption. For instance, the no free-lunch condition only implies the absence of linear time dependence since the best linear predictor of future (discounted) prices is then simply the current price. Volatility clustering, also known as the ARCH effect, is a clear manifestation of the existence of nonlinear dependences between returns observed at different lags. These dependences prevent the use of convolution for estimating tail risks with sufficient accuracy. Fig.1. illustrates the important observation that fat tails of asset return distributions owe their origin, at least in part, to the existence of volatility correlations. In the example of Fig.1, a given 60-minute return is the sum of sixty 1-minute returns. If there was no dependence between these sixty 1-minute returns, the 60-minute return could be seen as the sum of 60 independent random variables; hence, its probability density could be calculated exactly by taking 60 convolutions of the probability density of the 1-minute returns.

4. Marginal Distributions of Returns

Note that this 60-fold convolution is equivalent to estimating the density of 60-minute returns in which their sixty 1-minute returns have been reshuffled randomly to remove any possible correlation. Fig.1. shows a faster decay of the pdf of these reshuffled 60-minute returns compared with the pdf of the true empirical 60-minute returns. Thus, assessing extreme risks at large time scales (1 or 10 days) by simple convolution of the distribution of returns at time scales of 1 or of 5 minutes leads to crude approximations and to dramatic underestimations of the amount of risk really incurred. The role of the dependence between successive returns is even more important in times of crashes: very large drawdowns (amplitudes of runs of losses) have been shown to result
from anomalous transient dependences between a few successive days; as a consequence, they cannot be explained or modeled by the distribution calibrated for the bulk (99%) of the rest of the sample of drawdowns. These extreme events have been termed “outliers”, “kings” or “black swans”.

5. Conclusions

The only way to reliably aggregate high-frequency data is to have a consistent model at one’s disposal. By consistent model is meant a model that accounts for the complex time structure of asset returns. The multifractal models or any other stochastic volatility model can be used for this purpose, but none of them is yet universally recognized since they do not rely on well-established founding of economic principles. As a consequence, one is exposed to model error: for instance a simple GARCH model still underestimates the tail risks since it underestimates the long-range dependence of the volatility.

Reference


