The Link of Tax Revenue and Overall Excess Burden

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Abstract. The tax revenue is very important for society and government. However, the increasing tax revenue can cause an amount of excess burden. Overall excess burden may lead to welfare loss of society. Therefore, the quantity link of tax revenue and overall excess burden is crucial for policymaker. To explore the rule existed in the link of tax revenue and excess burden, this study develops algebraic and geometric methods to analyze the dynamic process of tax revenue and excess burden when the tax rate is changed in a specific and a general case. It is found that more tax revenues lead to more excess burden. The increasing rate of excess burden exceeds that of tax revenue.

Introduction

In the theory of optimal commodity taxation, the general assumption is a minimum of excess burden and no lump sum taxes with a fixed amount of national tax revenues. It is further assumed that an individual consumes only two unrelated commodities, X and Y, and leisure l with a fixed time endowment. Furthermore, it concludes that a uniform tax rate across all commodities, including leisure, is equivalent to a lump sum tax that has no excess burden according to a mathematical deduction. However, leisure time cannot be taxed; only commodities X and Y can [3-5]. Hence, overall excess burden is inevitable to some extent. Theoretically, the objective of optimal commodity taxation relies on minimizing the excess burden for a certain amount of tax revenue.

Therefore, study on the link of excess burden and tax revenue is crucial for not only deriving the Ramsey Rule but also justifying the theory of optimal commodity taxation. This study develops an algebraic and geometrical method that certifies each other to show the dynamic relation of excess burden and tax revenue.

Links Between Tax Revenue (TR) and Excess Burden (EB)

For simplicity, a specific linear function of demand and supply is used for analysis. Meanwhile, a unit tax of $1 is levied on commodities, X and Y. We assume that the demand and supply curves for X can be sketched as $Q^d = 50 - 5P$ and $Q' = -10 + 5P$, respectively. In figure 1, the equilibrium of quantity and price is at the point E (20, 6). When a unit tax is imposed on the supply side, the equilibrium changes to point G (17.5, 6.5) and the triangular region shaded in light Grey represents excess burden. The rectangular region shaded in dark Grey represents tax revenue. Because of a unit tax, the new supply curve becomes $Q'^s = -15 + 5P$, which is above the original curve $Q^s = -10 + 5P$ by a vertical distance of $1. So EB = 1.25 and TR = 17.5 when the tax equals $1.

A general case is introduced to show TR and EB if tax is changed. Assuming the demand and supply curves are $Q^d = a - b \cdot P$ and $Q'^s = -c + d \cdot P$ $(a, b, c, d > 0)$, respectively, the formula $Q'^s = -(c + d \cdot t_X) + d \cdot P$ is derived for the new equilibrium if $t_X$ is imposed on commodity X. According to this case, we define $a = 50, b = 5, c = 10, d = 5$. 

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Therefore, \( TR = 20t_X - \frac{5}{2}t_X^2 \), \( EB = \frac{5}{4}t_X^2 \), as derived by plugging the related numbers into the equations for \( TR \) and \( EB \).

The objective of formulating an expression for \( TR \) and \( EB \) is to show the link between \( TR \) and \( EB \) clearly. A table and diagram show this link. Table 1 describes change in value of \( TR \) and \( EB \) with the change in \( t_X \). \( TR \) increases when \( t_X \) increases 0 to 4, and decreases when \( t_X \) increases from 4 to 8. The relationship is symmetrical because \( TR \) is a quadratic function with a maximum value of 40. \( EB \) increases steadily with increases in \( t_X \). We define \( \mu = EB/TR \). A higher \( \mu \) implies that \( EB \) increases at a higher rate than \( TR \) with \( t_X \). Table 1 also uses the values of \( \frac{d(EB)}{d(TR)} \) and \( \frac{d^2(EB)}{d(TR)^2} \) to reveal the same. We derive \( \frac{d(EB)}{d(TR)} > 0 \) and \( \frac{d^2(EB)}{d(TR)^2} > 0 \) if \( t_X \in [0, 4) \). The latter is always greater than zero. This result shows that \( EB \) increases with \( TR \) and at a faster rate than \( TR \), consistent with intuition.

We also obtain the algebraic results \( \frac{d(EB)}{d(TR)} = \frac{t_X}{8-2t_X} > 0 \) and \( \frac{d^2(EB)}{d(TR)^2} = \frac{1}{20(4-\frac{1}{10}TR)^3} > 0 \) if \( t_X \in [0, 4) \). These results are consistent with the geometrical ones and the conclusions that previous scholars drew [1].

Table 1. Value of \( TR \) and \( EB \) with the Different \( t_X \).

<table>
<thead>
<tr>
<th>( t_X )</th>
<th>( TR )</th>
<th>( EB )</th>
<th>( \mu = EB/TR )</th>
<th>( \frac{d(EB)}{d(TR)} )</th>
<th>( \frac{d^2(EB)}{d(TR)^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>0</td>
<td>0.00625</td>
</tr>
<tr>
<td>1</td>
<td>17.5</td>
<td>1.25</td>
<td>0.0714</td>
<td>0.1667</td>
<td>0.00649</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>5</td>
<td>0.167</td>
<td>0.5</td>
<td>0.00675</td>
</tr>
<tr>
<td>3</td>
<td>37.5</td>
<td>11.25</td>
<td>0.3</td>
<td>1.5</td>
<td>0.00703</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>20</td>
<td>0.5</td>
<td>--</td>
<td>0.00732</td>
</tr>
<tr>
<td>5</td>
<td>37.5</td>
<td>31.25</td>
<td>0.833</td>
<td>-2.5</td>
<td>0.00764</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>45</td>
<td>1.5</td>
<td>-1.5</td>
<td>0.00798</td>
</tr>
<tr>
<td>7</td>
<td>17.5</td>
<td>61.25</td>
<td>3.5</td>
<td>-1.167</td>
<td>0.00834</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>80</td>
<td>--</td>
<td>-1</td>
<td>0.00873</td>
</tr>
</tbody>
</table>
In figure 2, the geometrical trend shows that \( \frac{d(EB)}{d(TR)} > 0 \) and \( \frac{d^2(EB)}{d(TR)^2} > 0 \) for EB values corresponding to \( t_X \) from 0 to 4. TR does not decrease in practice for \( t_X \) ranging from 4 to 8. According to the figure, EB is a quadratic function of TR. The maximum value of TR is 40. EB always has a positive value. The shape of curve is similar to the Laffer curve, which shows the existence of an optimal tax that may be 2.667 when the difference between TR and EB is maximized in this case. Certainly, the most important result is \( \frac{d(EB)}{d(TR)} > 0 \) and \( \frac{d^2(EB)}{d(TR)^2} > 0 \). This result emphasizes the importance of analyzing the principle of minimizing overall EB, as done in the next section.

Figure 2. The link of TR and EB.

Conclusion

This study analyzes the relationships among tax rate, tax revenue, and excess burden. The excess burden increases steadily regardless of tax revenue increasing or decreasing. However, tax revenue increases at first and decreases later when tax rate increases. Furthermore, the increase of excess burden will exceed the increase of tax revenue. Therefore, algebraically and geometrically, we obtain an important result that \( \frac{d(EB)}{d(TR)} > 0 \) and \( \frac{d^2(EB)}{d(TR)^2} > 0 \) for a relevant range of tax. This result shows that the more tax revenue is not a good consequence for whole society. The cost is increase of excess burden that declines the wealth of a nation if government ignores this fact [2].

Reference


