Determine of Shale Tensile Strength by Using Brazilian Disc Test

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Abstract. The disc specimen subjected to compression is proposed for testing tensile strength of brittle materials such as rocks. The Brazilian disc test is an indirect measurement method of mechanical properties of rock, which theoretical foundation is the elasticity solution of stress of pressed disc. However, the distribution of contact load acting on the disc is not exactly known. Based on the complex function theory for elastic plane problem, the complex solution of stress is given under the condition that the distribution of contact load is general distribution. The theoretical formula of tensile strength is also presented. Discuss the influence of seven kinds of contact load distributions and different contact angles on stress field. Through the Brazilian disc test for shale from the depth of eight hundred, we obtain the tensile strength.

Introduction

The Hydraulic fracturing has recently become one of the most importance technologies to increase yield in the shale gas exploitation. Tensile strength is a key parameter of rock mechanics to model and analysis hydrofracturing in shale reservoir.

For testing tensile strength of rock and concrete materials, the Brazilian test or call splitting tension test, as a simple experiment technology in an indirect way, is widely applied in rock engineering. The test is performed by compression with diametrically opposite concentrated loads on a disc specimen [1-3]. However, Fairhurst [4] pointed that the failure of the Brazilian disc under concentrated load sometimes initiates at the loading points instead of the center of disc. Research efforts should conduct to widen the scope of application for the Brazilian test or improve its performance. Until now, there are two standards of testing procedure for measuring brittle materials indirect tensile test, call as ISRM and ASTM, which was suggested by International Society for Rock Mechanics [5,6] and by American Society for Testing and Materials [7], respectively.

The British disc test is an indirect testing method to test the tensile strength of specimen, Its theoretical basis is the solution of stress for pressed disc specimen. The stress solution of the Brazilian disc under concentrated load was studied much earlier by Hertz [8] in 1883, and some years later by Michell [9]. Later, Timoshenko gave the stress solution on the diameter of force-crossing and vertical, and Muskhelishvili established bipolar coordinate system giving
the full-filed analytic expression of stress by complex function method. Along with the application of the Brazilian test, the Brazilian disc under radially uniform pressure more and more came to people’s consideration. Initially, Hondors [10] analyzed the stress field of the Brazilian disc under radially uniform pressure giving the full-field stresses in series solutions by using the series expansion technique. Later, Wijk [11] amended Hondors’ 2D solution, which gives the display three dimensional stress solutions on $x=0$. Meanwhile, Markides et al. [12] using complex potentials and divisional method also gave the explicit close-full-field solutions for stresses and displacements in the Brazilian disc under uniformly distributed radial load. Kourkoulis et al. [13] considered the boundary load is parabolically varying, and gave a closed form full-field displacement solution of the Brazilian disc which is in good agreement with the experimental results made from the 3D digital image correlation technique.

For the Brazilian disc test based on the standard of ISRM, the situation of contact load distribution between the Brazilian disc specimen and clamp holders is not exactly known. This paper will give the stress solution under the condition of contact load generally distribution. Analyze the influence of seven kinds of contact load distribution and different contact angles on stress field. Through the Brazilian disc test for shale from the depth of eight hundred, the tensile strength of shale is given.

Theoretical Analysis

Stress Solutions for General Distribution of Contact Load

In order to set up the analysis, it is assumed that the material of the disc is homogeneous, isotropic and linear elastic. The contact length between the disc and the loading jaws is considered a finite arc rather than a single point. It is also assumed that the contact between the disc and the loading jaws is simulated by the symmetric radial compressive pressure acting on two arcs of the perimeter of the disc, symmetric about $x$-axis and $y$-axis (Fig.1). The only stress at the contact area is radial compress. Any friction forces developed at the disc-jaws interface are ignored. Finally, the problem is considered to be a plane one.

As shown in Fig.1, under these assumption the mathematics problem approaching the real experiment is considered as a problem of linear elasticity for disc of radius $R$, under a distributed radial load $q(\theta)$, symmetrical about $\theta$, acting on two symmetric arcs each one of non-zero length $2R\alpha$. The resultant force over the arc be denoted as
\[ F = Rt \int_{-\alpha}^{\alpha} q(\theta) \cos \theta d\theta. \] (1)

For planar deformation either plane strain or plane stress in a given polar coordinate system \((r, \theta)\), the in-plane stress components are denoted by \(\sigma_r, \sigma_\theta, \text{ and } \tau_{r\theta}\), respectively. By introducing a complex variable \(z = re^{i\theta}\) in the region of the disc, there exist two complex stress functions \(\Phi(z)\) and \(\Psi(z)\), which are analytic for \(z\), such that the stresses can be expressed by \(\Phi(z)\) and \(\Psi(z)\) as [9]

\[
\begin{align*}
\sigma_r + \sigma_\theta &= 4 \text{Re} \Phi'(z) \\
\sigma_\theta - \sigma_r + 2i\tau_{r\theta} &= 2e^{i2\theta} \left[ \overline{\Phi''(z)} + \Psi''(z) \right].
\end{align*}
\] (2)

As shown in Fig.1, a circular disc loaded by two diametrically opposing distributed compressions located on the top and bottom of the disc is analyzed in this paper.

The stress boundary conditions of this problem are

\[
\sigma_r \bigg|_{r=R} = \begin{cases} 
-q(\theta) & (-\alpha \leq \theta \leq \alpha) \\
-q(\theta - \pi) & (\pi - \alpha \leq \theta \leq \pi + \alpha), \\
0 & (\text{the others})
\end{cases}, \quad \tau_{r\theta}(R,\theta) = 0,
\] (3)

where \(q(\theta)\) is the applied distributed traction to be expressed as pressure and symmetrical about \(\theta\), \(q(\theta) = q_{\text{max}} f(\theta)\) (\(q_{\text{max}}\) is the maximal pressure over the arc), \(2\alpha\) is the angle at the origin subtended by the loaded section of the rim, \(t\) and \(R\) is respectively thickness and radius of the disc.

The two analytic function \(\Phi'(z)\) and \(\Psi'(z)\) can be expressed as power series in the region \(r < R\), meanwhile the \(f(\theta)\) can be expanded as Fourier series of pole angle \(\theta\) in the boundary \(r=R\).

When \(r=R\), from Eqs. (2) and (3) unknown analytic function \(\Phi'(z)\) and \(\Psi'(z)\) can be uniquely determined as

\[
\Phi'(z) = -q_{\text{max}} \left( \frac{C_0}{2} + \sum_{n=1}^{\infty} C_{2n} \left( \frac{z}{R} \right)^{2n} \right), \quad \Psi'(z) = 2q_{\text{max}} \sum_{n=0}^{\infty} (n+1) C_{2[n+1]} \left( \frac{z}{R} \right)^{2n},
\] (4)

where the coefficients of the above power series

\[
C_{2n} = \frac{2}{\pi q_{\text{max}}} \int_{0}^{\alpha} q(\theta) \cos 2n\theta d\theta, \quad (n = 0, 1, 2, \ldots).
\] (5)

Instituting the Eq. 4 into Eq. 2, we obtain the solution of stress within the circular disc as
\[
\sigma_r(r, \theta) = -q_{\text{max}} \left[ C_0 + 2 \sum_{n=1}^{\infty} C_{2n} \left( n-(n-1) \left( \frac{r}{R} \right)^2 \right) \left( \frac{r}{R} \right)^{2n-2} \cos 2n\theta \right]
\]

\[
\sigma_\theta(r, \theta) = -q_{\text{max}} \left[ C_0 - 2 \sum_{n=1}^{\infty} C_{2n} \left( n-(n+1) \left( \frac{r}{R} \right)^2 \right) \left( \frac{r}{R} \right)^{2n-2} \cos 2n\theta \right].
\]  

\[
\tau_{r\theta}(r, \theta) = 2q_{\text{max}} \sum_{n=1}^{\infty} C_{2n} n \left( 1 - \left( \frac{r}{R} \right)^2 \right) \left( \frac{r}{R} \right)^{2n-2} \sin 2n\theta
\]

**The Expression of Tensile Strength by Compressed Disc**

The tensile strength \( \sigma_r = \max(\sigma_G) \), in which equivalent stress \( \sigma_G \) is determined by the Griffith failure criterion, that is, the failure occurs when

\[
\sigma_G = \begin{cases} 
\sigma_1 & \text{if } 3\sigma_1 + \sigma_3 \geq 0 \\
\frac{(\sigma_1 - \sigma_3)^2}{8(\sigma_1 + \sigma_3)} & \text{if } 3\sigma_1 + \sigma_3 < 0
\end{cases}
\]

where \( \sigma_1 \) and \( \sigma_3 \) is the major and the minor principal stress, respectively.

**Analysis for Various Case of Contacted Load Distributions**

According to the theory of contact mechanics [14], the boundary conditions are not exactly known. To examine the influence of distribution form for contact loads on the full-field stress and tensile strength, here we consider the seven cases of contact load distribution: uniform, semiellipse, biquadratic parabola, catenary, quadratic parabola, hyperbola and cosine distributions, as follows:

**Case I**: uniform distribution of distribution of contact load

\[
q(\theta) = \frac{q_0}{2 \sin \alpha}, \quad (q_0=F/Rt)
\]

**Case II**: semiellipse function distribution of contact load

\[
q(\theta) = \frac{q_0}{\pi J_1(\alpha)} \sqrt{1 - \frac{\theta^2}{\alpha^2}}, \quad J_1(\alpha) \text{ is the Bessel function of the first kind}
\]

**Case III**: biquadratic parabola distribution of contact load

\[
q(\theta) = \frac{q_0 \alpha^4}{8 \left( \alpha \cos \alpha \left( 6 - \alpha^2 \right) + 3 \sin \alpha \left( \alpha^2 - 2 \right) \right)} \left( 1 - \left( \frac{\theta}{\alpha} \right)^4 \right)
\]

**Case IV**: catenary distribution of contact load

\[
q(\theta) = q_0 \left( \cosh \frac{\alpha}{k} - \cosh \frac{\theta}{k} \right)
\]
Case V: quadratic parabola distribution of contact load

\[ q(\theta) = \frac{q_0 \alpha^2}{4(\sin \alpha - \alpha \cos \alpha)} \left(1 - \frac{\theta^2}{\alpha^2}\right), \]

Case VI: hyperbola distribution of contact load

\[ q(\theta) = \frac{q_0 \left(\sqrt{2} - \sqrt{1 + (\theta/\alpha)^2}\right)}{\alpha \left(\sqrt{2} - \ln(1 + \sqrt{2})\right)}. \]

Case VII: cosine function distribution of contact load

\[ q(\theta) = \frac{q_0 \left(\pi^2 - 4\alpha^2\right)}{4\pi \alpha \cos \alpha} \cos \frac{\pi \theta}{2\alpha}. \]

Then, we discuss the influence of different contact load distribution \(q(\theta)\) on stress distribution within the circular disc. Fig. 2 and Fig. 3 show the normalized radial stress \(\sigma_r/(F/\pi R t)\) and hoop stress \(\sigma_\theta/(F/\pi R t)\) profiles versus the normalized radius \(r/R\) for various the contact load distribution \(q(\theta)\) and contact angle \(2\alpha\), respectively. As shown in Fig. 3 and Fig. 4, the contact load distribution \(q(\theta)\) has a significant impact on the radial stress \(\sigma_r/(F/\pi R t)\) compared with hoop stress \(\sigma_\theta/(F/\pi R t)\). With the increased of \(r/R\), the influence to normalized stress \(\sigma/(F/\pi R t)\) is more and more under the condition that the contact angle \(2\alpha\) is same. And the contact angle \(2\alpha\) is smaller, the influence of contact load distribution \(q(\theta)\) to the normalized stress \(\sigma/(F/\pi R t)\) is more.

Figure 2. The normalized radial stress \(\sigma_r/(F/\pi R t)\) versus the normalized distance \(r/R\) for \(\theta=0\) in various case of the contact load distribution \(q(\theta)\) and the different contact angle \(2\alpha\). (a) \(\alpha=12^\circ\), (b) \(\alpha=15^\circ\), (c) \(\alpha=18^\circ\).

Figure 3. The normalized hoop stress \(\sigma_\theta/(F/\pi R t)\) versus the normalized distance \(r/R\) for \(\theta=0\) in various case of the contact load distribution \(q(\theta)\) and contact angle \(2\alpha\). (a)\(\alpha=12^\circ\), (b)\(\alpha=15^\circ\), (c)\(\alpha=18^\circ\).
The Tensile Strength $\sigma_t$ under The Seven Kind of Boundary Conditions

The precondition for the determine of tensile strength $\sigma_t$ is the crack initiated at the center of the Brazilian disc. For the Brazilian disc considered here, we can know $3\sigma_\theta+\sigma_r<0$ ($\sigma_1=\sigma_\theta$, $\sigma_3=\sigma_r$) in the range of contact angle considered from Fig. 2 and 3, thus $\sigma_G = -(\sigma_\theta-\sigma_r)^2/(8(\sigma_\theta+\sigma_r))$. Now we investigate the influence of contact angle $2\alpha$ on equivalent stress $\sigma_G$. Fig. 4 shows the curves of normalized equivalent stress $\sigma_G/(F/\pi R_t)$ for various distribution forms and contact angle $2\alpha$. As shown in Fig. 4, the place of the maximum value of equivalent stress $\sigma_G$ appears at the disc center ($r=0$, $\theta=0$) when $\alpha \geq 15^\circ$, this means that the crack initiated at the center of the Brazilian disc. Hence, we obtain the following expression of tensile strength:

$$\sigma_t = \frac{(\sigma_\theta - \sigma_r)^2}{8(\sigma_\theta + \sigma_r)} \bigg|_{r=0,\theta=0} = \frac{q_{\max} C_\theta^2}{C_0}, \quad q_{\max} = \max q(\theta) \text{ if } \alpha \geq 15^\circ. \quad (8)$$

![Figure 4. The normalized equivalent stress $\sigma_G/(F/\pi R_t)$ versus the normalized distance $r/R$ for $\theta=0$ in various case of the contact load distribution $q(\theta)$ and contact angle $2\alpha$. (a) $\alpha=12^\circ$, (b) $\alpha=15^\circ$, (c) $\alpha=18^\circ$.](image)

**Experiment**

Disc specimens are made of Long Ma Xi shale rock that is taken from a depth of about 800m. Disc specimen parallelism of two parallel surfaces is less than 0.05 mm, the surface flatness within 0.02 mm. The geometric parameters of disc specimens are listed by Table 1. Making a pair of concave aluminum alloy clamp holders, the radius of the clamp holder is 40mm. The British disc test was carried by electronic universal testing machine, and Fig. 5 is the test picture. The outline of the test is briefly described here. The test procedure was recommended by ISRM[6]. Static load was acted on disc specimen though concave arc surface of the clamp holder. At the same time, the test brought in thin cloth with stamp-pad ink to measure the contact angle $2\alpha$ between clamp holders and specimen. Observation shows that the crack initiates at disc specimen center and extends along the vertical direction, resulting in a run through the vertical fracture surfaces. The load-displacement curve was shown in Fig. 6. In the initial stages, the loading curve is nonlinear for the shale specimen is in a compacting state. Further loading, the curve shows a straight line segment, which indicates that shale specimen in elastic deformation stage. Finally the load reached the maximum value corresponded with crack initiate and the crack propagates unsteadily.
A series of the Brazilian disc test was carried out using specimens made from shale. The test was displacement controlled at rate of 0.02mm/min. Load versus diameter deformations were plotted on an xy-plotter (Fig.6).

Based on the recorded complete load-displacement curve and Eq. 8. Shale tensile strength can be obtained. Though the form of boundary condition was not exactly known, we discuss the influences of seven different contact load distributions on tensile strength $\sigma_t$ which is represented by Table 2.

**Figure 5.** The Brazilian disc test.

- **Figure 6.** A typical test record of force-displacement curve by using the Brazilian disc test specimen.

<table>
<thead>
<tr>
<th>Test sample</th>
<th>$t$ (mm)</th>
<th>$\alpha$</th>
<th>$P_{\text{max}}$ (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>28</td>
<td>17°</td>
<td>50.3</td>
</tr>
<tr>
<td>Sample 2</td>
<td>27</td>
<td>16°</td>
<td>50.4</td>
</tr>
<tr>
<td>Sample 3</td>
<td>27</td>
<td>18°</td>
<td>51.0</td>
</tr>
</tbody>
</table>

**Table 1.** The Brazilian disc test parameter.

<table>
<thead>
<tr>
<th>Test sample</th>
<th>Uniform $\sigma_t$ (Mpa)</th>
<th>Semiellipse $\sigma_t$ (Mpa)</th>
<th>Biquadratic parabola $\sigma_t$ (Mpa)</th>
<th>Catenary $\sigma_t$ (Mpa)</th>
<th>Quadratic parabola $\sigma_t$ (Mpa)</th>
<th>Hyperbola $\sigma_t$ (Mpa)</th>
<th>Cosine $\sigma_t$ (Mpa)</th>
<th>Average $\sigma_t$ (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>14.7</td>
<td>15.1</td>
<td>15.2</td>
<td>15.3</td>
<td>15.4</td>
<td>15.3</td>
<td>15.4</td>
<td>15.2</td>
</tr>
<tr>
<td>Sample 2</td>
<td>15.5</td>
<td>15.9</td>
<td>15.9</td>
<td>16.0</td>
<td>16.1</td>
<td>16.0</td>
<td>16.1</td>
<td>15.9</td>
</tr>
<tr>
<td>Sample 3</td>
<td>15.3</td>
<td>15.7</td>
<td>15.8</td>
<td>16.0</td>
<td>16.0</td>
<td>15.9</td>
<td>16.1</td>
<td>15.8</td>
</tr>
</tbody>
</table>
Conclusion

(1) Due to the shale rock brittleness, softer aluminum clamp holder was made instead of hard rigid in order to avoid disc specimen of shale crushing failure in the loading location.

(2) The British disc test is an indirect testing method to test the tensile strength of specimen. Its theoretical basis is the solution of stress for pressed disc specimen. However, the distribution of contact load between clamp and specimen is not exactly known. Based on the complex function theory for elastic plane problem, the complex solution of stress is obtained under the general distribution of contact load. Discuss the influence of seven kinds of contact load distributions and different contact angles on stress-field distribution, and obtain the theoretical equation of shale tensile strength.

(3) British disc test was carried out, and by applying the formulas and test results, the value of tensile strength of shale was got at 15.6MPa. Analysis shows that the influence of contact load distribution forms to the measurement of the tensile strength is small.

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References


