A Random Switching Frequency De-Re-Coupling Current MPC Method for Single-Phase MC

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Abstract. Model predictive control (MPC) is a highlighted control method for power electronic realm. For Matrix Converter (MC) with bi-directional switches and difficulty model, MPC has a complex structure and control process. A random switching frequency de-re-coupling current MPC method is proposed in this paper for a single-phase MC. A voltage function is obtained by the simplified and decoupled discrete-time model of MC, and a random switching frequency generating strategy is inserted to modulate function and to improve harmonic of MPC variable switching frequency. The principle effectiveness and advantages of the proposed method such as THD, settling times and static errors are verified by simulation results comparing with the conventional MPC and de-re-coupling fixed switching frequency current MPC methods.

1 Introduction

MC is a frequently used topology in industry because of its structure and application flexibility, such as combining with a high frequency link (HFL) or wireless power transmission (WPT) system to converting alternative currents with arbitrary frequency, amplitude and phase because of the flexible bi-directional switches on arms [1-4]. Unipolar or bipolar phase-shift control strategies are the basic methods for MC topologies to realize soft switches and to utilize parasitic parameters [5, 6].

MPC is an effective strategy in power electronic realm and highlighted by researchers because of advantages including high bandwitch, good dynamics, easy handing and algorithm close to model [7]. Based on the Venturini method, a MPC strategy with variable switching frequency for MC is proposed to realize converging frequency [8, 9], and some improving limitations of controlled instantaneous reactive power and switching frequency are inserted to the cost function [10].

Inevitably, because of voltage vector table selecting process, variable switching frequency of MPC generates larger ripples of the controlled variable. In order to limit switching frequency and to decrease ripples, some limitations can be seen as control objectives and inserted to the cost function [8, 11, 12]. Another thought of decreasing switching frequency is that a MPC with fixed switching frequency is proposed according to the cost function and discrete-time model calculating process [13, 14].
function is obtained by predictive variables and a modulation strategy is inserted to the structure to generate driving pulses [15]. The high frequency harmonics are concentrated on the switching frequency and its integral times frequencies because of PWM block [16].

De-re-coupling is an effective method for MC topologies to divide a bi-directional switch into two si-directional switches and to decouple the MC into two inverters with separate control for positive and negative half-cycle of input voltage [3, 17]. A random switching frequency de-re-coupling current MPC method for single-phase MC is proposed in this paper. A voltage function is obtained by the fixed switching frequency MPC and combined with a de-re-coupling modulation to decouple and to simply single-phase MC model. A random switching frequency with pseudo-random numbers is applied to replace fixed switching frequency carrier wave and to extend to a selected harmonic range. Performance advantages including THD and MPC of the proposed method are verified by simulation results and compared with the de-re-coupling fixed switching frequency current MPC and conventional current MPC methods.

2 Proposed method

2.1 Decoupled single-phase MC structure

The topology of single-phase MC includes 4 bi-directional switches, a high-frequency input sinusoidal voltage $u_{in}$ and an output filter induction $L$. A bi-directional switch could be divided as two si-directional switches to control separately and to name as $S_{pah}$, $S_{nah}$, $S_{pal}$, $S_{nal}$, $S_{pbl}$, $S_{nbl}$, $S_{phl}$ and $S_{nbL}$ respectively according to the operating state in steady process in Tab. I. The si-directional switches are divided as a positive group and a negative group and regrouped as two single phase bridge inverters which are operated by the sign of input voltage. According to the operating state in Tab. I, the decoupled single phase MC is shown in Fig. 1. When the input voltage is operating at positive half-cycle, the switches $S_{pah}$ and $S_{pbl}$ are closed while the positive half-cycle output voltage is requiring, and the switches $S_{pal}$ and $S_{phl}$ are closed while the output voltage $u_o$ is requiring a negative half-cycle to realize converting frequency. Similarly, the operating states in the input voltage negative half-cycle are closed by the requiring sign of the output voltage $u_o$.

<table>
<thead>
<tr>
<th>Power frequency output sinusoidal voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive half-cycle</td>
</tr>
<tr>
<td>High frequency input sinusoidal voltage</td>
</tr>
<tr>
<td>Positive half-cycle</td>
</tr>
<tr>
<td>Negative half-cycle</td>
</tr>
</tbody>
</table>

2.2 Discrete-time model and control strategy

The structure of the decoupled single-phase MC in Fig. 1 has a resistive-inductive load is decoupled and simplified as two single-phase inverters, and each of them with load can be expressed as:

$$L \frac{di_o(t)}{dt} = u_o(t) - Ri_o(t) \quad (1)$$

The backward Euler interpolation method has been applied to discrete and to predict the derivative of output current $i_o$ at time $k+1$ with sampling period $T_s$:  

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\[
\begin{align*}
\frac{di_o}{dt} &= \frac{i_o(k+1) - i_o(k)}{T_s} \\

\text{including error:} \\

i_{o,k+1} - i_o(k+1) &\approx \frac{T_s^2}{2} i_{o,k}^* \\

\text{where } i_{o,k+1} \text{ is the real value of output current } i_o \text{ at time } k+1.
\end{align*}
\]

Substituting (2) into (1), the predictive variable \(i_o(k+1)\) could be expressed as:

\[
\begin{align*}
i_o(k+1) &= \left(1 - \frac{RT_s}{L}\right)i_o(k) + \frac{T}{L} u_o(k)
\end{align*}
\]

The main objective for current controller is that the predictive output current \(i_o(k+1)\) tracks reference signal \(i_o^*\), and some additional objectives such as current ripple limitation and reactive power limitation also can be inserted to the cost function. Because the weighting factors in cost function with multiple objectives are eliminated during voltage function calculating processes of the fixed switching frequency MPC, only main objective is selected to combine as the cost function which can be expressed as:

\[
g = \left(i_o - i_o(k+1)\right)^2
\]

Making:

\[
\frac{dg}{du_o(k)} = 0
\]

and substituting (4) into (6), the voltage function with fixed switching frequency can be expressed as:

\[
\begin{align*}
u_o(k+1) &= \frac{L}{T_s} \left(i_o^* - \left(1 - \frac{RT_s}{L}\right)i_o(k)\right)
\end{align*}
\]

**2.3 Random pulse width**

A random switching frequency generation method based on the pseudo-random number method can be expressed as:

\[
f_{n+1} = f_s + s \times R
\]
where $s$ is a number from range $[-1, 1]$, $R$ is the random gain of the method, $f_s$ is the rated switching frequency and $f_{n+1}$ is $n+1$th generated switching frequency of the system. The random switching function $g(t)$ of the random PWM is expressed as:

$$g(t) = \lim_{N \to \infty} \sum_{k=1}^{N} g_k (t-t_k)$$

(9)

where $\varepsilon_k$ is delay coefficient of the pulse rising edge, $\alpha_k$ is pulse coefficient and $T_k$ is switching period. A limitation of three parameters can be expressed as:

$$0 < \varepsilon_k T_s \leq t-t_k \leq (\varepsilon_k + \alpha_k)/T_k < T_k$$

(10)

Three parameters in (3) are independent and the range of $\varepsilon_k + \alpha_k$ is $[0, 1]$. The switching period $T_k$, pulse rising edge $\varepsilon_k$ or both of variables can be selected to realize random PMW. When $T_k$ is changing by a selected random rule, the random degree $R_T$ can be expressed as:

$$R_T = \frac{T_{\text{max}} - T_{\text{min}}}{T_s}$$

(11)

where $T_{\text{max}}$ is the maximal switching period, $T_{\text{min}}$ is the minimal switching period, and $T_s$ is the average switching period which equals the inverse of $f_s$ ideally.

When $\varepsilon_k$ is selected as the random variable, the random degree $R_{\varepsilon}$ can be expressed as:

$$R_{\varepsilon} = \frac{\varepsilon_2 T_s - \varepsilon_1 T_s}{T_s} = \varepsilon_2 - \varepsilon_1$$

(12)

where $\varepsilon_1 T_s$ and $\varepsilon_2 T_s$ are the maximal and minimal pulse locations at each switching period.

### 3 Simulation results

A simulation environment has been built in the MATLAB/ Simulink software according to the structure in Fig. 2. A sinusoidal wave with amplitude 500V, frequency 50kHz and zero initial phase is applied to the input voltage $u_{in}$, and another sinusoidal wave with amplitude 311V, frequency 50Hz and zero initial phase is applied to the reference signal $u_{o\ast}$. An uniform random number block with average value 20kHz and random degree $R_T$ 2kHz is applied to generate pseudo-random numbers. The input voltage and random switching frequency with pseudo-random numbers are shown in Fig. 3 and Fig. 4 respectively.

The output voltage and 10 times enlarged output current waveforms at start operating process are shown in Fig. 5. As shown in the figure, output voltage tracks the reference signal successfully, and operates into the steady state with unit power factor during about 0.5 cycles. As shown in Fig. 6, a step changing load torque is uploaded onto the shaft at 0.1s, the system returns to steady state and tracks reference with unit power factor after about 0.5 cycles.

A FFT analysis of output voltage for the proposed method is shown in Fig. 7. The THD of output voltage is 1.08% without any static error. The harmonic contents are decreased mainly when the order increase, and the high-frequency harmonic contents concentrate nearing switching frequency and its integral times.

A comparison among the proposed method, de-re-coupling fixed switching frequency current MPC and conventional current MPC is made with same parameters. The start operating and step changing load processes of output voltage and enlarged output current waveforms with the de-re-coupling fixed switching frequency current MPC are shown in Fig. 8 and Fig. 9, and the output voltage FFT is shown in Fig. 10 respectively. The output voltage can also track the reference and resist disturbance successfully. The THD is 1.14%.
and the high frequency harmonics are around with the switching frequency 20kHz and its integral times.

![Figure 2](image2.png)

**Figure 2.** Structure of single-phase MC with random pulse width de-re-coupling current MPC method.

![Figure 3](image3.png)

**Figure 3.** Input voltage waveform.

![Figure 4](image4.png)

**Figure 4.** Random switching frequency waveform.

![Figure 5](image5.png)

**Figure 5.** Voltage and current waveform at start state.
Figure 6. Voltage and current waveform at resisting state.

Figure 7. FFT of the output voltage.

To analyze the advantage of the proposed method, the contents nearby switching frequency has been extracted and fitted in Fig. 11. According to the analysis above-mentioned, the harmonic nearing switching frequency should be distributed in the range [18kHz, 22kHz] equably. Although the accuracy is finite, the harmonic range is extended and the contents near the switching frequency are decreased obviously because of random switching frequency.

Combining with the data of the conventional current MPC and the de-re-coupling with PI controller in [17], some control performances including settling times, static error and THD based on same parameters are summarized in Tab. II. The proposed method takes advantages including the smallest THD, shortest settling times and less static error over other methods.

Figure 8. Voltage and current waveform at start state of fixed switching frequency current MPC.
Figure 9. Voltage and current waveform at resisting state of fixed switching frequency current MPC.

Figure 10. FFT of the output voltage of fixed switching frequency current MPC.

Figure 11. Fitted harmonic content waveforms of two methods.

<table>
<thead>
<tr>
<th>Current Controller</th>
<th>Settling time at start</th>
<th>Settling time at changing load</th>
<th>Static error</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>De-re-coupling with PI</td>
<td>1 cycle</td>
<td>1.5 cycles</td>
<td>0</td>
<td>2.65%</td>
</tr>
<tr>
<td>Conventional current MPC</td>
<td>0.5 cycles</td>
<td>0.5 cycles</td>
<td>0.514%</td>
<td>4.52%</td>
</tr>
<tr>
<td>De-re-coupling fixed switching frequency current MPC</td>
<td>0.5 cycles</td>
<td>0.5 cycles</td>
<td>0</td>
<td>1.14%</td>
</tr>
<tr>
<td>Random pulse width de-re-coupling current MPC</td>
<td>0.5 cycles</td>
<td>0.5 cycles</td>
<td>0</td>
<td>1.08%</td>
</tr>
</tbody>
</table>

### 3 Conclusions

A random switching frequency de-re-coupling current MPC is proposed in this paper, and applied into a single-phase MC system. The decoupled MC generates two single-phase bridge inverters which can be controlled separately. The performances of the proposed method are compared with the de-re-coupling fixed switching frequency current MPC and the conventional current MPC methods. Test results and comparisons verify that the
random switching frequency makes harmonic extended to a selected range with lower contents for each single order and lower THD.

Summarily, comparing with other three methods, the proposed method decreased THDs of the output voltage near 5.26%, 76.11% and 59.25% respectively with shortest settling times of MPC and without any static error. The advantages including fast dynamics, good steady states and less aberration rate of the proposed method are shown obviously. Moreover, better control performances might be obtained by a modulation with a more reliable and balanced pseudo-random numbers generating strategy such as 2-order or high-order Markov stochastic chains.

References

