Calculation and Optimization of Volumetric Efficiency of Aviation External Gear Pump

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Abstract. The power consumption of aviation gear pump has a direct impact on the performance of aero-engine. To improve the efficiency of aviation external gear pump, this paper analyses the leakage of gear pump, and calculates the leakage of axial, radial clearance, gear face meshing clearance, and the elastic loss of liquid compression. Mathematical models of volumetric efficiency are established including the effect of temperature and pressure. Comparing the theoretical value with experimental results, the effectiveness of the models is verified with the error less than 5%. On this basis, the design variables of gear pump are optimized with the maximum theoretical displacement as the optimization objective function. The simulation results show significant volumetric efficiency improvement of the external gear pump. And some suggestions on actual products design and analysis in the aviation fuel device are proposed accordingly.

External gear pump is widely used in aviation as the main fuel pump, and oil transfer pump, booster pump and return pump of lubricating oil system in aviation power plant, and as the main hydraulic pump in aircraft hydraulic system. With the development of aviation power plant, high-pressure, low pulsation, low noise and large displacement have become the development trend of gear pump [1]. In order to reduce the power consumption of aero-engine, the total efficiency of aero fuel pump is required to be optimized, and the volumetric efficiency of gear pump must be improved accordingly.

Various researches in this field mainly studied the improvement of volumetric efficiency and the solution to leakage problem respectively. The method of improving gear pump volumetric efficiency in engineering is studied in [2]. The leakage problem of axial clearance of gear pump is analysed and a new processing technology is proposed in [3]. The mathematical model of leakage in external gear pump is established in [4][5]. And the model of volumetric efficiency of gear pump by experiment is studied in [6][7]. Few studies are taking a holistic view of the problem. Therefore, by establishing and verifying the mathematical model of volume efficiency, the paper proposes a possible integrated method to optimize the volumetric efficiency with theoretical solution to the leakage problem and achieve energy saving consequently.
1 Calculation of external gear pump leakage

The main causes of leakage of external gear pump are clearance leakage, hydraulic impact and temperature rise, which reduce the oil viscosity. The leakage caused by temperature rise in the hydraulic system is mainly due to the decrease of oil viscosity, the increase of pressure and gap caused by heat shock, and the oil deterioration caused by heat.

The total leakage of external gear pump mainly includes axial, radial, tooth surface meshing clearance leakage and compression elastic loss. The first three can be taken as the two-dimensional flow theory of plane gap, calculated from N-S equation [8]. The last can be written by the formula for the modulus of elasticity of volume.

The axial leakage of gear pump is calculated as:

\[ Q_1 = \frac{\delta^3}{3\pi \mu} \sum_{l=1}^{\infty} \frac{1 - (R_1/R_2)^{2l}}{1 + (R_1/R_2)^{2l}} \left[ \sin(\pi - \theta_1) + \frac{\sin(\pi - \theta_0)}{\pi - \theta_0} \right] \frac{1 + \cos l\pi}{l^2} \]

where: \( \delta \) is gear pump end clearance; \( \mu \) is dynamic viscosity of oil; \( p_1, p_2 \) is pressure of high and low pressure cavity of gear pump; \( R_1, R_2 \) is circular radius of gear shaft and tooth tip; \( l \) is constant; \( \theta_1, \theta_2 \) is transition zone angle and pressure zone angle.

The radial leakage of gear pump is calculated as:

\[ Q_2 = \frac{(p_1 - p_2) b h^3}{6 \mu S Z_0} - b v_0 h \]

where: \( h \) is the height of the radial clearance between the tooth tip and the shell; \( b \) is the width of radial clearance between the tooth tip and the shell (can be tooth width); \( S \) is addendum thickness; \( Z_0 \) is the number of teeth in contact between the tooth tip and the shell; \( v_0 \) is the linear velocity of the addendum.

The tooth surface meshing clearance leakage of the gear can be:

\[ Q_3 = \int_0^c v_b dz = \int_0^c \left( v_0 - \frac{v_0 z}{c} \right) b dz = \frac{b v_0 c}{2} \]

where: \( v \) is flow rate, \( z \) is height from the tooth top, \( c \) is the meshing gap of the two gears.

The Elastic loss calculation of gear pump can be:

\[ \Delta Q_4 = 2(-\Delta V) n = 2 \frac{\Delta p V}{E} n \]

where: \( V \) is inter-tooth volume; \( \Delta V \) is the change value of liquid under pressure; \( \Delta p \) is differential pressure between high and low pressure chambers of gear pump, \( \Delta p = p_1 - p_2 \); \( E \) is elastic modulus of liquid volume for mineral oil, set \( E = 15.63 \times 10^3 (P_a) \), \( n \) is rotate speed.

2 Gear pump volumetric efficiency calculation

According to the definition of volumetric efficiency, equations (1)–(4) are substituted into the calculation mathematical model of volumetric efficiency as follows:

\[ \eta_v = \frac{Q_T - \Delta Q}{Q_T} = 1 - \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_T} = 1 - \frac{K_1 \frac{\Delta p V}{E} + \frac{\Delta p h^3}{6 \mu S Z_0} b v_0 h + b v_0 c + 2 \frac{\Delta p V}{E}}{2\pi K Z m^2 b n} \]

where: \( Q_T \) is theoretical flow rate, \( \Delta Q \) is leakage flow rate; \( K \) is compensation coefficient, normally \( K = 1.02 \sim 1.12 \), \( Z \) is number of teeth; \( m \) is gear module.

MATLAB is used to calculate CB-X aviation gear pump. The parameters of the pump are: \( Z = 8, m = 3.5, b = 17mm, \alpha = 20^\circ, d_a = 39.5mm, n = 4850r/min, \delta = 0.10mm, Z_0 = 5, h = 0.01mm, c = 0.02mm \).

The volumetric efficiency of the gear pump under different rotational speed and pressure difference is shown in Fig. 1. The maximum error between the theoretical calculation of the volume efficiency and the measured value is less than 5%. Theoretical
results: Axial leakage accounts for 70%~85% of the total leakage, radial leakage accounts for 13%~20% of the total leakage, other meshing leakage and elastic leakage accounts for 2%~10% of the total leakage, which is consistent with the experimental results. The theoretical calculation model is verified.

3 The relationship between gear pump volumetric efficiency and temperature and pressure

Considering that the dynamic viscosity of hydraulic oil varies with temperature and pressure, the viscosity is:

$$\mu = \mu_0 e^{\frac{p-p_0}{p_0} - \beta(t-t_0)}$$  \hspace{1cm} (6)

where: $\mu_0$ is basic dynamic viscosity at $p_0 = 1\text{atm}$, $t_0 = 50^\circ\text{C}$; $\beta$ - temperature viscosity coefficient; $\gamma$ - pressure viscosity coefficient, aviation fuel $\gamma = 1/432, \beta = 0.0144$.

The derivative of equation (5) for the dynamic viscosity can be obtained:

$$\frac{d\eta_v}{d\mu} = -\frac{k_1 \Delta p b h_3}{2\pi K Z_0 b n} \left( \frac{b_v}{2} + 2 \frac{\Delta p V_n}{K} \right) < 0$$  \hspace{1cm} (7)

It can be seen that the volumetric efficiency of gear pump decreases with the increase of temperature and the increase of pressure differential, which is shown in Fig. 2:

Figure 1. The calculated and measured value of volume efficiency under different conditions.

Figure 2. The curve of volume efficiency with respect to temperature and pressure.
4 Optimization of gear pump volumetric efficiency

4.1 Optimize parameters and objective functions

The independent parameters of gear pump include six variables $z, x, m, h, a, B, \alpha$. In order to obtain the maximum flow rate, tooth width $B$ selected, the basic design variable should be:

$$X = [x_1, x_2, x_3, x_4, x_5]^T = [z, x, m, h, B, \alpha]^T$$ (8)

Under the condition that the gear center distance is certain, the gear parameters are optimized by taking the gear pump flow rate as the objective function:

$$f(x) = -q_t = -2\pi nB \left\{x_2 \left(x_4 - \frac{x_1}{2}\right) - \frac{a^2}{4} - \left(\frac{\pi x_3 \cos x_5}{12}\right)^2\right\}$$ (9)

4.2 Constraints

According to the gear meshing theory, the limits of each design variable can be defined as:

$$g \leq x_1 \leq 20, 0 \leq x_2 \leq 1, 10 \leq x_3 \leq 30, 1.0 \leq x_4 \leq 1.2, 15 \leq x_5 \leq 30$$

The corresponding constraint function is:

$$g_1(X) = 8 - x_1 \leq 0, g_2(X) = x_1 - 20 \leq 0, g_3(X) = -x_2 \leq 0, g_4(X) = x_2 - 1 \leq 0, g_5(X) = 10 - x_3 \leq 0, g_6(X) = x_3 - 30 \leq 0, g_7(X) = 1 - x_4 \leq 0, g_8(X) = x_4 - 1.2 \leq 0, g_9(X) = 15 - x_5 \leq 0, g_{10}(X) = x_5 - 30 \leq 0.$$ (10)

In order to avoid all kinds of interference and damage which may occur in gear cutting and meshing, it is necessary to make some restrictions on the related parameters. In order to reduce flow fluctuation and improve oil trap, the coincidence coefficient should be limited:

1. Addendum thickness constraint: the number of pump gears is small, and the tooth tip tends to become sharp with the adoption of positive displacement gear. General requirements:
   $$g_{11}(X) = 0.15m - s_a \leq 0, s_a \text{ is addendum thickness.}$$

2. Gear mesh coincidence coefficient:
   $$g_{12}(X) = 1.05 - \varepsilon \leq 0, g_{13}(X) = \varepsilon - 1.08 \leq 0.$$ (11)

3. Avoid the interference of transition curve when meshing:
   $$g_{14}(x) = \tan x_5 - 2\tan a' + \tan a_a - \frac{4(x_4 - x_2)}{x_1 \sin 2x_5} \leq 0,$$
   $$a_a = \arccos \frac{T_b}{r_a} = \arccos \frac{x_1 x_3 \cos x_5}{x_3 (2x_4 - 2x_2 - x_1) + 2a}, a' = \arccos \frac{mz \cos \alpha}{\alpha} = \arccos \frac{x_1 x_3 \cos x_5}{\alpha}.$$ (12)

   In order to ensure that all the arccosines are localized during optimization, there should be:

   $$0 \leq \frac{x_1 x_3 \cos x_5}{x_3 (2x_4 - 2x_2 - x_1) + 2a} \leq 1, 0 \leq \frac{x_1 x_3 \cos x_5}{x_3} \leq 1,$$
   $$g_{16}(X) = \frac{x_1 x_3 \cos x_5}{x_3 (2x_4 - 2x_2 - x_1) + 2a}, g_{17}(X) = \frac{x_1 x_3 \cos x_5}{x_3}, g_{18}(X) = \frac{x_1 x_3 \cos x_5}{x_3} - 1.$$ (13)

4.3 Optimization algorithm

Since the objective function and most constraint functions are nonlinear, the optimization design of gear parameters is a nonlinear programming problem. So an indirect solution --- improved constrained variable scale method is chosen:

$$\min f(x) \quad x \in R^n \quad s. t. g_i(x) \leq 0 (i = 1, 2, ..., m), h_j(x) = 0 (j = 1, 2, ..., p < n)$$ (14)

Where: $g_i(x)$ is inequality constraint equation; $h_j(x)$ is equality constraint equation, and $p < n$. The constraint variable scale method is shown in Fig.3.
4.4 Optimization results

CB-X aviation oil gear pump is still used for calculation, $X_0 = [8,0,12,1.0,20]$, and center distance is determined. The optimal solution $X = [z, x, m, h_\alpha, \alpha] = [13,0.5,12,1.04,25]$ is obtained by MATLAB optimization toolbox. The gear pump characteristic parameter pre and post optimization are shown in table 1.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Actual flow($m^3/h$)</th>
<th>Volumetric efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=1000 r/min</td>
<td>Pre-opt. 617.91</td>
<td>87.43%</td>
</tr>
<tr>
<td></td>
<td>Post-opt. 705.05</td>
<td>97.74%</td>
</tr>
<tr>
<td>n=2000 r/min</td>
<td>Pre-opt. 1408.59</td>
<td>92.43%</td>
</tr>
<tr>
<td></td>
<td>Post-opt. 1235.81</td>
<td>97.63%</td>
</tr>
<tr>
<td>n=3000 r/min</td>
<td>Pre-opt. 1853.72</td>
<td>90.43%</td>
</tr>
<tr>
<td></td>
<td>Post-opt. 2111.16</td>
<td>97.55%</td>
</tr>
</tbody>
</table>

It can be seen that the actual flow rate of the optimized gear pump significantly increases, and the volume efficiency increases compared with the original parameters of the gear pump.

5 Conclusions

(1) The gear pump leakage increases with the rise of temperature and differential pressure, irrelevant to oil pump speed. In practice, the inlet and outlet oil pressure of gear pump remains basically unchanged, which reduces the influence factors, and makes the gear pump flow change only with the change of speed, thus ensuring the accurate realization of complex control in the aviation fuel device.

(2) The volumetric efficiency is improved from promoting the gear pump flow, and the optimization of constraint variable scale method improves its overall performance.

(3) The established mathematical model of leakage and volumetric efficiency of gear pump is applicable in analysing actual products and of certain significance in improving the volumetric efficiency of gear pump.
References