Parameter Identification for Lithium Ion Supercapacitor Based on a Modified RLS Method

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Abstract
As a kind of renewable energy source, lithium ion supercapacitors have bright prospects in national grids, new energy technologies and tram applications. It has the good features of high energy/power density, extremely low resistance, no demand for maintenance and long cycle life. In this paper, we build up the 2nd-order RC equivalent circuit model, which is based on the Thevenin equivalent circuit model. In order to improve the accuracy and stability, and achieve accurate identification of model parameters, such as OCV, R<sub>0</sub>, R<sub>p1</sub>, R<sub>p2</sub>, a variable forgetting factor method is introduced into the traditional fixed forgetting factor recursive least squares method. This provides a basic method and reference for the subsequent state estimation and life prediction of lithium ion supercapacitors. The accuracy of the model parameter identification method is verified by the experimental result.

Keywords: lithium ion supercapacitor, parameter identification, variable forgetting factor recursive least squares

1. Introduction
In recent years, global warming and the energy crisis have caused human to show the most urgent need of the development of new energy vehicles. It is highly expected that new energy vehicles will be able to replace traditional fuel vehicles to reduce the emissions of carbon dioxide and gradually get rid of dependence on oil and coal. At present, due to its high energy density, high power density, extremely low resistance, over 30,000 times cycle, extremely small self-discharge current, support for fast charging and other advantages, lithium ion supercapacitor has been widely used in high-speed trains, trams and national grids. However, there are still some concerns and worries about the safety of lithium ion supercapacitors. It must be ensured that the voltage and current of lithium ion supercapacitors can be within the proper range when it’s working. Therefore, parameter identification and state estimation of lithium ion supercapacitors are of great use.

State estimation and life prediction are very important functions in the battery management system, which are closely related to battery model parameters. Among them, OCV corresponds to SOC, R<sub>0</sub> is related to the aging of lithium ion supercapacitor, and R<sub>p1</sub> and R<sub>p2</sub> are closely related to the dynamic characteristics of lithium ion supercapacitor. There are two main methods for parameter identification: offline parameter identification and online parameter identification. The off-line parameter identification produces the reference values of the battery model parameters under various SOC values by artificially designed battery experiment. However, the amount of calculation in the offline parameter identification process is large and the calculation time is long. And when the battery characteristics change with time and the temperature, the data obtained by the offline parameter identification will generate large errors and instability in the online application condition. For all of the above reasons, we use online parameter identification for lithium ion supercapacitors in this paper. Online parameter identification is based on real-time voltage and current data to calculate real-time model parameters of the battery. The existing online parameter identification methods include least squares, recursive least squares, recursive least squares with fixed forgetting factor (FFF-RLS), recursive least squares method with variable forgetting factor (VFF-RLS) and so on. Among them, LS, RLS and FFF-RLS are more suitable for the parameter identification of the first-order RC equivalent circuit model of batteries and capacitors, while VFF-RLS is more suitable for the parameter identification of the second-order RC equivalent circuit model of lithium ion supercapacitors [1].

In this paper, we use the recursive least squares method of variable forgetting factor to identify the...
model parameters of lithium ion supercapacitors and then the process of capturing the dynamic characteristics of the battery will be more sensitive and accurate, which provides a referenced way to improve the function of state estimation and life prediction for the existing battery management system.

The rest of the structure of this article is shown below: Section 2 describes the structural characteristics of lithium ion supercapacitors. Section 3 introduces the establishment of the equivalent circuit model for lithium ion supercapacitors. Section 4 uses the recursive least squares method with variable forgetting factors to identify the parameters of lithium ion supercapacitors. Section 5 gives the conclusion based on the full article.

2. Structure and characteristics

The lithium ion supercapacitor is mainly composed of the following four parts: 1. The positive electrode of the battery: LFP (lithium iron phosphate electrode). 2. The negative electrode of the battery: activated carbon. 3. Electrolyte: The main component is Li2SO4-LiPF6, which is filled in the space between the positive electrodes and negative electrodes and transports lithium ions. 4. Separator: The main function is to separate the positive and negative electrodes of the battery to prevent short circuit between positive electrodes and negative electrodes and allow ions in the electrolyte to get through.

In lithium ion supercapacitors, Faradic and non-Faradic processes occur at the same time. When two electrodes are connected by a load, the ions, which contains the dissolved lithium salt, move through the electrolyte during the non-Faradic process. It’s partially responsible for electrical conduction. When lithium ions react with the negative electrode in the Faradic reaction, another partial conduction occurs [2].

The structural mechanism comparison diagrams of supercapacitor, lithium ion battery and lithium ion supercapacitor are as follows.

In order to select the appropriate equivalent circuit model and model parameter identification method, the HPPC (Hybrid Pulse Power Characterization) experiment is performed on lithium ion supercapacitor. Subsequently, the model parameter such as OCV and ohmic internal resistance R can be extracted from its experimental data. And the further analysis of its characteristics will be carried out in more detail.

Figure 2 OCV curve at different SOC

In Fig. 2, the variation of the OCV of the single-cell lithium ion supercapacitor with the depth of discharge (DOD) is shown in the figure below. As seen from the figure, at the beginning, when the SOC drops by about 10%, the open circuit voltage OCV drops by about 0.3V. After that, the OCV curve entered another trend, when the OCV dropped again by 0.3V, and the SOC dropped by about 40%. At the same time, with the increase of DOD, the declining speed of OCV is gradually getting faster and faster. According to our previous experimental data and papers, when the SOC of a lithium ion supercapacitor drops from 90% to 0, its variation trend of OCV are very similar to the OCV curve of supercapacitor, suggesting that lithium ion supercapacitors have the part of the characteristics of the traditional supercapacitors. When the SOC of the lithium ion supercapacitor drops from 100% to 90%, the trend that OCV changes with SOC indicates that the lithium ion supercapacitor also has a few of the characteristics of a typical lithium ion battery. The above phenomenon is consistent with the characteristics of the lithium ion supercapacitors model we have established.

Figure 3 ohmic internal resistance curve of a single lithium ion supercapacitor

Fig. 3 shows the internal resistance of a lithium ion supercapacitor monomer as a function of SOC. As shown in the figure, the internal resistance curve of the monomer shows a linear change from the beginning. During the process of SOC decreasing from 100% to
80%, it just looks like that the internal resistance of the monomer increases linearly from 0.37 milliohms to 0.39 milliohms. When the SOC continues to decline from 80%, the subsequent trend of the curve is very similar to the quadratic function, and the rate that internal resistance changes with the SOC is increasing. When DOD increases from 20% to 40%, the internal resistance only increased by 0.006 milliohms; and when the DOD increased from 80% to 100%, the internal resistance increased by 0.05 milliohms relatively quickly. In general, at 25 ° C, when the SOC drops from 100% to 0, the internal resistance of the lithium ion supercapacitor monomer changes by 0.277 milliohms and the average internal resistance is 0.45 milliohms. Therefore, it can be concluded that the internal resistance of the ion supercapacitor is relatively stable during the discharging process and its value is also extremely small.

3. Lithium ion supercapacitor modeling

The significance of the choosing the proper battery model type for model-based state estimation and life prediction methods is self-evident. The accuracy of state estimation has a high correlation with the accuracy of the model. The more the established battery model conforms with the dynamic characteristics and actual conditions of the battery, the higher the accuracy and validity of state estimation are.

In this paper, in view of the polarization characteristics of lithium ion supercapacitors, a new 2nd order RC equivalent circuit model is formed by adding an RC branch to the existing Thevenin equivalent circuit model (1st order RC model).

The anode of a lithium ion supercapacitor consists of a lithium iron phosphate battery, which indicates that there should be a voltage source in the equivalent circuit model of the lithium ion supercapacitor. Lithium ion supercapacitors can be quickly charged and discharged with a large current, which is consistent with the characteristics of the capacitor, and the model should also contain a main capacitor. And its self-discharge phenomenon shows that this main capacitor also has a parallel branch of self-discharge resistor. On the basis of the analysis in the previous chapter, lithium ion supercapacitors have a certain internal resistance, so the resistance is also one of the components of the circuit model. According to the sudden drop of the OCV curve in its early stage, it can be inferred that the lithium ion supercapacitor has dynamic variation characteristics, so its model should include two RC parallel branches that are not identical. From the structural characteristics of lithium ion supercapacitors, we have initially obtained the mechanism model of lithium ion supercapacitors, which is shown below in the Fig. 4.

![Figure 4](image)

**Figure 4** Mechanism model of lithium ion supercapacitor

The trend of OCV with SOC indicates that the charging-discharging behavior of capacitor C₀ has little effect on the voltage source V, which is almost negligible. Therefore, we can simplify the combination of the voltage source V and capacitor C₀ in a new voltage source OCV. According to the self-discharging experiment of lithium ion supercapacitor monomer that we have carried out, the self-discharging speed is very slow: after the lithium ion supercapacitor monomer is full charged, its voltage only drops by about 0.05V in 30 days. It can be known that the self-discharging resistance of lithium ion supercapacitors is extremely small, so we can ignore the self-discharge resistance Rₘ on in the model. The internal resistance Rᵢ of the lithium iron phosphate battery and the resistance R₀ of the internal connecting wire of the lithium ion supercapacitor can be combined into an equivalent internal resistance R₀.

After reasonable and appropriate simplification, a new model is obtained like this:

![Figure 5](image)

**Figure 5** 2nd order RC equivalent circuit model

As shown in Fig. 5, OCV represents the open circuit voltage, R₀ represents the ohmic internal resistance, Rₚ₁ and Rₚ₂ are the polarization resistances of the two RC parallel branches, and Cₚ₁ and Cₚ₂ are the polarization capacitances of the two RC parallel branches [3]. In order to distinguish two RC parallel branches, we artificially set their time constants in this paper: Let τᵢ₁=5 and τᵢ₂=20.

4. RLS and parameter identification

4.1 State space equation

By discretization of the full response of the first-order circuit, the dynamic equation of the first-order RC parallel circuit is deduced as follows:

\[ u_{n,k+1} = a_n u_{n,k} + b_n I_{i_n}, \quad n = 1, 2 \]  \hspace{1cm} (1)

where,

\[ \begin{align*}
  a_n &= e^{-\tau_i / r_i}, \\
  b_n &= R_i (1 - a_n)
\end{align*} \]  \hspace{1cm} (2)
The equation (1) is Z-transformed to get
\[ u_{n,k} = \frac{b_n}{z} - \frac{a_n}{z} i_k \]  \hspace{1cm} (3)

After the model is built, the parameters of the model need to be identified. Firstly, a state equation based on the established model is constructed. According to the Thevenin equivalent theorem, the following equation can be obtained and discretized.
\[ u = u_0 + u_1 + R_i I + OCV \]  \hspace{1cm} (4)

where \( u \) is the terminal voltage, \( u_0 \) is the voltage of the ohmic resistor, \( u_1 \) is the voltage of the RC parallel circuit with the time constant \( \text{Tau}_1 = 5 \), \( u_2 \) is the voltage of the RC parallel circuit with the time constant \( \text{Tau}_2 = 20 \), \( I \) represents the total current of the lithium ion supercapacitor, and \( R_0 \) represents the ohmic internal resistance of the lithium ion supercapacitor.

\[ u_k = u_{1,k} + u_{2,k} + R_i I_k + OCV_k \]  \hspace{1cm} (5)

where,
\[ \begin{cases} 
\theta_{a,1} = a_1 + a_2 \\
\theta_{a,0} = -a_1 a_2 \\
\theta_{b,2} = R_0 \\
\theta_{b,1} = b_1 + b_2 - R_i (a_1 + a_2) \\
\theta_{b,0} = R_i a_1 a_2 - a_1 b_2 - a_2 b_1 
\end{cases} \]  \hspace{1cm} (6)

Expand the above formula,
\[ u_{k+2} - \theta_{d,1} u_{k+1} - \theta_{d,0} u_k = \theta_{n,2} i_{k+2} + \theta_{n,1} i_{k+1} + \theta_{n,0} i_k + (z^2 - \theta_{d,1} z - \theta_{d,0}) OCV_k \]  \hspace{1cm} (7)

Here, the derivation equation of OCV is given, which is in order to eliminate the \( Z \) variable contained in the coefficient in front of the OCV.
\[ OCV_{i+1} = OCV_i - \frac{i_{T+1}}{Q} \]  \hspace{1cm} (8)

Among them, the constant
\[ M = \frac{T_i U_i}{Q_s} = \frac{1 + 3.65}{20.57 \times 3600} \]  \hspace{1cm} (9)

The equation (8) is substituted into the equation (7) and the following equation is obtained:
\[ u_{k+2} - \theta_{d,1} u_{k+1} - \theta_{d,0} u_k = \theta_{n,2} i_{k+2} + (\theta_{n,1} + M \theta_{n,0}) i_{k+1} + (1 - \theta_{d,1} z - \theta_{d,0}) OCV_k \]  \hspace{1cm} (10)

let
\[ y_k = u_{k+2} - \theta_{d,1} u_{k+1} - \theta_{d,0} u_k \]  \hspace{1cm} (11)

After separating variables between measured values and estimated values, we can get:
\[ y_k = [i_{k+2}^T \quad i_{k+1}^T \quad i_k^T \quad 1 - \theta_{d,1} - \theta_{d,0}]^T \quad \theta_{d,2}^T \theta_{d,1} - M \theta_{b,0} + M \theta_{b,1} - 1 \quad OCV_k] \]  \hspace{1cm} (12)

\[ \phi_k^T = \left[ \begin{array}{c} i_{k+2} \\
\theta_{d,2}^T \theta_{d,1} - M \\
\theta_{b,0} + M \theta_{b,1} - 1 \end{array} \right] \quad \text{OCV}_k \]  \hspace{1cm} (13)

\[ \theta_k = \left[ \begin{array}{c} \theta_{d,2}^T \theta_{d,1} - M \\
\theta_{b,0} + M \theta_{b,1} - 1 \end{array} \right] \quad \text{OCV}_k \]  \hspace{1cm} (14)

Among them, \( y_k \) and \( \phi_k \) are known variables, which can be obtained from measured voltage data and current data. \( \theta_k \) is the target vector to be solved [4].

4.2 Recursive least squares method

After building up the model of the lithium ion supercapacitor, it is necessary to use the appropriate parameter identification method for the model to obtain the parameters to be identified, like the OCV, the ohmic internal resistance \( R_0 \), and so on.

The core idea of the least square method is to minimize the sum of the squares of the deviation between the output of the identification model and the output of the real system. Under this premise, the model parameters are estimated:
\[ J(\hat{\theta}) = \sum_{i=1}^{N} \varepsilon^2(k) \]  \hspace{1cm} (15)

Among them, \( \varepsilon(k) \) represents the deviation between the model output and the real system output.

The least squares method obtains model parameters by minimizing the sum of the squares of the errors of the identification model output and the real system output. The accuracy of using the least squares method to identify parameters is related to the amount of data. The larger the amount of data, the higher the accuracy. But the large amount of data is not suitable for online parameter identification. Moreover, when the dispersion of the measured values is large or there are many abnormal values in measured values, a large error will occur when using the least squares method. Hence, the recursive least squares method is considered to be a feasible method: after each new voltage/current data is acquired, the estimation result is corrected based on the previous estimation result by use of the newly introduced voltage/current data and the recursive algorithm. Afterwards, a new parameter estimation value is obtained recursively [5].

4.3 The modified RLS method design

The equation of state for a single-input single-output system (SISO system) is as follows:
\[ y_k = \phi_k^T \theta_k \]  \hspace{1cm} (16)

The effect of introducing the forgetting factor is to eliminate data saturation for the purpose of strengthening the impact of current data and weakening the impact of historical data. The recursive least squares method with the fixed forgetting factor is as follows:
\[ K_k = P_{k-1} \phi_k^T (\lambda + \phi_k^T P_{k-1} \phi_k) \]
\[ \hat{\theta}_k = \hat{\theta}_{k-1} + K_k (Z_k - \hat{\phi}_k \phi_k) \]
\[ P_k = (I - K_k \phi_k \phi_k^T) P_{k-1} / \lambda \]  \hspace{1cm} (17)
Where $K_k$ represents the gain matrix, $P_k$ represents the estimated error covariance matrix, Lambda is the forgetting factor, $^\wedge$ shows the estimated value. According to the above formula, the target vector $\theta_k$ at time $k$ can be obtained. After the above two steps are repeated, the value of the target vector at the next moment can be figured out [6].

4.3.1 The debugging process of forgetting factor

When the model parameter is identified by the recursive least squares method with fixed forgetting factor, we have set forgetting factors as three different sizes: $\lambda_1=0.995$, $\lambda_2=0.99$, and $\lambda_3=0.98$. The identification result of OCV parameter under different forgetting factors are shown as follows.

![Figure 6 OCV comparison of online parameter identification and offline parameter identification under different forgetting factors](image)

As shown in the Fig. 6, when $\lambda=0.955$, in the first half, the online identification result of OCV has a large error with the offline identification result; in the second half, the online identification result of OCV are in good agreement with the offline identification result. When the forgetting factor $\lambda$ is gradually reduced, especially when $\lambda=0.98$, it can be found that the online identification result of OCV is much better in the early stage. But the deviation is more serious in the later stage when the online identification result curve is compared with the offline identification result curve.

In order to adjust the dynamic balance of the relationship between historical data and noise sensitivity, the range of forgetting factors is generally set to $[0.9, 1]$.

Therefore, it’s found that if the forgetting factor is properly reduced in the early stage, the convergence speed of the OCV online identification result curve get increased, which can make itself be in good agreement with the offline identification result curve. And in the later stage, if the forgetting factor is properly increased, which reduces the sensitivity of the online identification result curve to noise, the error between itself and the offline identification result will become smaller.

So, in this paper, a variable forgetting factor method is introduced to the recursive least squares method with forgetting factor, improving the accuracy and validity of the parameter identification results of the model.

4.3.2 Setting of variable forgetting factor

On the basis of the analysis in the previous section, the intervals and the forgetting factors are set as follows:

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Forgetting factor $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100% SOC~70% SOC</td>
<td>0.98</td>
</tr>
<tr>
<td>70% SOC~50% SOC</td>
<td>0.99</td>
</tr>
<tr>
<td>50% SOC~ 0% SOC</td>
<td>0.995</td>
</tr>
</tbody>
</table>

5. Experimental Verification

5.1 Introduction to the experiment

The experimental equipment includes a battery test system produced by Neware Company of Shenzhen, China, and a temperature chamber produced by Sunwood Company of Dongguan, China.

The Neware battery test system has a current range of -100A to 100A and a voltage range of 0V to 30V. The temperature chamber has a temperature range of -40 °C to 150 °C with an error of ± 1.5 °C. During the battery charging process, the Neware battery test system provides power to the battery; while the battery is discharged, it provides a discharging path for the battery. The temperature chamber can simulate the working environment temperature of the battery. The experimental mechanism diagram of the lithium ion supercapacitor is shown in the figure below.

![Figure 7 Flow chart of lithium ion supercapacitor experimental platform](image)
voltage charging experiment are conducted in the temperature chamber, and then the lithium ion supercapacitor is put to be static. After ensuring the lithium ion supercapacitor is under the temperature of 25°C for at least 3 hours, the FUDS (Federal Urban Driving Schedule) test experiment is conducted on the lithium ion supercapacitor at the temperature of 25 °C until the whole battery experiment ends.

5.2 Verification of the method

By using the VFFRLS method, the recognition results are as follows:

![Figure 8 The identification result figure of model parameter OCV, R₀, R₁, R₂.](image)

It can be seen that when using the recursive least square method with fixed forgetting factor, only in the partial interval, the online identification result curve are in good agreement with the offline identification result curve. And after the recursive least square method with the variable forgetting factor is used, the online identification result curve is in good agreement with the offline identification result curve throughout the whole process and its improvement effect is obvious.

6. Conclusion

In this paper, a method of variable forgetting factor in the recursive least squares method is proposed for the online identification of model parameters for the second-order RC equivalent circuit model of the lithium ion supercapacitor. After the forgetting factor is adjusted with the errors between the previous online parameter identification result and the offline parameter identification result, the result of model parameter identification will relatively get better on the whole.

The results show that VFFRLS can adjust the forgetting factor in time with the dynamic change of battery parameter characteristics to improve the accuracy and validity of parameter identification result. After the battery experiment at 25°C, the estimation accuracy was verified. It means VFFRLS (Variable Forgetting Factor RLS) method can provide much help in the parameter identification of the lithium ion supercapacitor, which provide some reference information for the subsequent work of state estimation and lifetime prediction.

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Reference