Heterogeneous Characteristics of Power Flow and Its Mathematical Representation Based on Gini Coefficient

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Abstract. The heterogeneous characteristics of the components of complex power grids in terms of operating conditions, grid structure, status, etc., will play a negative role in fuel grid cascading failures, thus posing a serious threat to grid security. This paper analyzes the heterogeneity of complex power grids in terms of tidal current operation, mainly as the trend distribution curve exhibits exponential and power-law characteristics in semi-logarithmic and double-log curves. Based on the Lorentz curve and the Gini coefficient concept in economics, it is proposed that the Gini coefficient can be used as a characterization index for the heterogeneity of tidal current distribution.

Introduction

In recent years, large power outages have occurred worldwide. After the massive blackouts in Venezuela in March 2019, the national blackouts occurred in Argentina and Uruguay in June 2019, and large-scale reoccurrence in Manhattan, New York on July 13, 2019. The power outage has affected more than 70,000 people. The frequent occurrence of blackouts has caused people to think about the structure and operational safety of the power grid. The results of some studies like literature [1] and [2], have shown that the heterogeneous characteristics of complex power grids will have a negative effect on grid cascading failures, which poses a serious threat to grid security. Due to the differences in geographic location and resource conditions between the transmission, transmission, and substation nodes or transmission lines in the power grid, some of the power components are significantly higher in power circulation stability and information smoothness than other nodes. A non-uniform feature like the distribution of network structures. For example, the power delivered by a high-voltage line is much higher than the power of a low-voltage line, which is determined by the physical parameters and operating parameters of the line itself, which makes the high-voltage line have a higher role in power transmission. For low voltage grade lines. The actual operation of the grid data shows that there is always a small part of the transmission line in the power grid to undertake most of the power transmission tasks of the grid, and most of the lines bear a small load.

The heterogeneous characteristics of the distribution of the components of the grid in terms of operating conditions, grid structure, status, etc. are called the heterogeneity of the grid [3]. Any complex network, from a philosophical point of view, is homogeneous. Non-homogeneous is absolute. The heterogeneous nature of this complex network has a special impact on the transmission performance, safety and stability, and fault defense of the power grid, which increases the complexity of grid fault analysis. On the other hand, it can be better studied. A new approach is provided for the impact of complex grid characteristics and cascading failure propagation.
The Heterogeneity Phenomenon of Power Flow

Distribution Curve of Power Flow

In the stable operation of the actual power system, although the power flowing in each transmission line, transformer branch or other power transmission components is within its normal operating range the power system power flow is not evenly distributed among the various sections. On the line, the power flowing through different lines is very different. Some transmission lines transmit a large amount of power, operating near their capacity limits, while some lines flow very little, far below their rated operating capacity limits, thus showing the load rate of line operation in the grid. Not the same difference is significant. The magnitude of the load current in the transmission line in the power grid can reflect its direct role in the whole power transmission process. Therefore, the importance of the magnitude of the load flow to the power grid is self-evident. The heterogeneity of the power flow distribution of the transmission line will be the transmission performance of the power grid has a significant impact on the safety and stability of energy transmission. Taking China's regional power grid, A and B as examples, the paper analyzes the tidal power heterogeneity in the two actual power grids.

The active power flow distribution curves of the 330kV transmission line of the A grid and the 220kV transmission line of the B grid in a certain operation mode are shown in Figure I.

![Figure 1. 330kV of A power grid and 220kV of B power grid transmission line power flow distribution.](image)

It can be seen from Figure I that the active power flow distribution of the A power grid and the B power grid transmission line exhibits a significant imbalance. Taking the A power grid as an example, among the 207 330kV transmission lines in the province, the active power of 174 transmission lines is less than 400MW, accounting for nearly 85%, and only 15% of the transmission lines have a working load exceeding 400MW. The distribution of currents is very uneven. The situation of the B power grid is more prominent. Among the 519 high-voltage transmission lines in the province, the current trend of 473 transmission lines is less than 500MW, accounting for more than 91%, while only 46 transmission lines are higher than 500MW. In the above two provincial power grids, the distribution of the whole network power flow in the transmission line is very uneven and has significant heterogeneity characteristics.

Function Characterization of Power Flow Distribution Curve

The heterogeneity of this trend distribution is very similar to the “80/20 rule” proposed by the French economist Pareto in the study of personal income statistics, that is, 20% of the population accounts for 80% of social wealth, and personal income X does not. There is a simple inverse relationship between the probability of being less than a certain value X and the constant power of X (Pareto's law). When studying the frequency of English words, Zipf, a linguistics expert at Harvard University, also found that if the frequency of occurrence of words is arranged in descending order, the frequency of occurrence of each word is simply the same as the constant power of its ranking. The inverse relationship (Zipf's law). Both Pareto's law and Zipf's law
conform to a simple power function relationship and can be represented by a power law function. Therefore, in order to find the inherent regularity of the power flow distribution of the power grid, the power flow distribution curves of the A power grid and the B power grid transmission line are plotted on the double logarithmic coordinates, as shown in Figure 2-3 and Figure 2-4. The horizontal axis represents the active power flow distribution (MW) of all transmission lines of the power grid, and the vertical axis represents the cumulative probability distribution of not less than the power flow value.

![Figure 2. 330kV of A power grid and 220kV of B power grid transmission line power flow distribution in log-log coordinate.](image)

It can be seen from Figure II that under the double logarithmic coordinates, the line tidal current distribution of A grid and B grid tends to be a straight line, and obvious power law distribution characteristics appear. The power function fitting of the two curves is obtained by the power function fitting. The power law fitting function of the current distribution of the 330 kV transmission line of the A grid is:

\[ y = 1.077x – 153.44 \]  

The power law fitting function of the power flow distribution of the 220 kV transmission line of the B grid is:

\[ y = 2.246x – 16853 \]  

The above statistical analysis of the power flow of the A power grid and the B power grid shows that the power flow distribution in the power grid has significant heterogeneity characteristics, that is, the power flow of most power transmission lines is small, and only a few power transmission lines have abnormally increased power flows. The transmission line bears the main power flow transmission task of the power grid; the power flow distribution of the power transmission line conforms to the power law characteristic, and the relationship between the active power flow of the line and its cumulative probability can be expressed by a power function.

**Mathematical Characterization of Grid Power Flow Heterogeneity**

Drawing on the Lorentz curve and the application of the Gini coefficient in economics [4], similarly, the Gini coefficient which characterizes the heterogeneity of the power flow distribution of the grid can also be obtained. Suppose a complex power grid contains N transmission lines. The tidal current load rate of each line is sorted from small to large according to different interval steps. The tidal current load rate of each line is recorded as k1, k2,..., kN (k1≤k2≤...≤kN). The construction method is: the ratio of the number of lines not limited to a certain load current rate to the total number of lines N (ie, the ratio of the number of accumulated lines to the total number of lines) is the abscissa, which is equivalent to the proportion of the population in the income distribution; The ratio pro (i) of the cumulative line tidal current load rate to the total tidal current load rate is the ordinate, which is equivalent to the proportion of income in the income distribution.
Since the rated active power transmission power of each transmission line is different, when the pro \( i \) is determined, the load current rate cannot be directly added and summed, and the line active power flow and the rated active power should be accumulated separately before the accumulation is obtained. The load current rate, the pro \( i \) expression is:

\[
pro(i) = \frac{\sum_{q=1}^{I} P_{0,q} / \sum_{q=1}^{I} P_{\max,q}}{\sum_{q=1}^{N} P_{0,q} / \sum_{q=1}^{N} P_{\max,q}}
\]

(3)

Where: \( P_0 \) is line active power, \( P_{\max} \) is the rated active power of the line. The Lorenz curve for the complex power flow distribution is shown in Figure III.

![Figure 3. Lorenz curve of complex power grid flow distribution.](image)

The formula for calculating the Gini coefficient of the complex power flow distribution is:

\[
G = 1 - \frac{1}{N} \sum_{i=1}^{N} [pro(i) + pro(i - 1)]
\]

(4)

It can be seen from the above analysis that the Lorenz curve and the Gini coefficient can effectively characterize the heterogeneity of the complex network in the network topology and power flow distribution, the size of the Gini coefficient and the node degree value or the line load current rate in the grid network structure. The difference is related, the greater the difference, the larger the Gini coefficient. Compared with the non-uniform case with exponential or power-law distribution, the Gini coefficient is simpler to calculate, and the Gini coefficient of different networks can be compared qualitatively on the Lorenz curve.

**Conclusion**

This paper proposes the heterogeneity phenomenon in complex power grids, and proposes metrics to characterize the heterogeneity characteristics of complex power grids. Complex power grids have significant heterogeneity in terms of tidal current operation, mainly as the trend distribution curve exhibits exponential and power-law characteristics in semi-logarithmic and double-log curves. Based on the Lorentz curve and the Gini coefficient concept in economics, it is proposed that the Gini coefficient can be used as a characterization index for the heterogeneity of tidal current distribution.
References


