A Price Formula of Deposit Insurance Based on Merton Deposit Insurance Pricing Model

Meng-le GU¹, Tao ZHOU² and Yi-rong YING³

¹School of Economics, Shanghai University, Shanghai, 200444, China
²School of Economics, Shanghai University, Shanghai, 200444, China
³Xianda School of Economics and Humanities, Shanghai International Studies University, Shanghai, 200083, China

*Corresponding author

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Abstract. A reasonable deposit insurance price can effectively not only reduce the adverse selection problem, but plays an important role in reducing the systemic risk of Banks. By studying the heteroscedasticity of bank asset returns and the sequence of bank debt repayment, this paper deduces the closed form deposit insurance price formula, which further improves and expands Merton's deposit insurance pricing model.

Literature Review

Merton [1] proposed in 1977 that deposit insurance can be regarded as a put option whose subject matter is the market value of the assets of the insured institution. He used the option pricing formula for the first time in the study of deposit insurance price. Since then, many scholars have made some modifications and extensions on the basis of Merton’s classical model. Marcus and Shaked [2] found that after insurance, the value of bank assets changed, so they added the impact of insurance on the value of bank assets to Merton’s pricing model. Ronn and Verma [3] also modified the deposit insurance pricing model. They considered regulatory factors, thus improving the practical value of the model. Moreover, Duan [4] first proposed the method of maximum likelihood estimation to price deposit insurance model, and then created GARCH option pricing theory [5], which has been widely used in the process of deposit insurance pricing.

Liu et al. (2018) focused on deposit insurance pricing under a GARCH framework [6]. They derived a closed-form pricing formula, and presented an estimation method for the pricing model with market data. They also applied the pricing model on a sample of 40 U.S. exchange-listed banks and their results reaffirmed the importance of GARCH framework. Wong et al. (2018) have put forward a formula about the deposit insurance valuation problem as a zero-sum optimal stopping game using Israeli option with bankruptcy cost [7]. Specifically, closure of a bank is framed as a game between the insured bank and the deposit insurer, in which a bank with financial difficulties is choosing an optimal self-closure point to maximize its benefits from the deposit insurance scheme; and the deposit insurer is choosing an optimal regulatory closure point to minimize their cost of offering the insurance. In such setting, the deposit insurance itself could be regarded as an Israeli put option. Recently, Tone et al. (2018) proposed a dynamic two-stage network data envelopment analysis (DEA) model with and without carry-over variables to evaluate corporate performance [8]. They employed a multi-criteria decision analysis to compare all insurance companies in a common setting, including each ratio of liquidity, profitability, and leverage. The literature above provides great inspiration for our research.

The Pricing Formula

Merton (1977) assumed that the bank’s liabilities were all deposits and all insured. So once the bank was faced with payment difficulty, the compensation of deposit insurance institutions would be the
gap between bank assets and liabilities. However, with the continuous innovation of commercial Banks’ products and services, there are more and more types of bank debts and a growing proportion of liabilities other than deposits. In addition, different types of debts have different order of payment, and some types of debts may affect the compensation of deposit insurance institutions. For example, for the liabilities of mortgage, the creditor has the priority to claim the collateral, and the increase of such liabilities is bound to increase the risk and compensation amount of the deposit insurance institution. Therefore, when calculating the price of deposit insurance, it is necessary to consider the sequence of repayment of bank debts.

Among all the liabilities of the bank, there are two main types of debt with a higher priority than deposits: the debt that enjoys priority according to law (including the wages owed to employees, legal fees and state taxes) and the secured debt, while the debt with priority lower than deposits is mainly subprime debt. Assuming that the total liabilities of the bank are $K$. Without loss of generality, we divide the liabilities of banks into three categories: class A liabilities represent the debts with priority higher than deposits, and the amount is denoted as $K_1$; class B liabilities represent debts with the same priority as deposits, and the amount is recorded as $K_2$, where the deposit is $K_D$; class C liabilities are those with lower priority than deposits, and the amount is $K_3$. When a bank goes bankrupt, its assets first pay off class A liabilities, and the rest can pay off class B liabilities such as deposits. And then finally class C liabilities can be repaid. Therefore, it can be seen that the liquidation sequence of bank debt will inevitably have an impact on the payment of deposit insurance institutions, thus affecting the price of deposit insurance.

Deposit insurance institutions usually insure only part of a bank’s deposits. The main reasons are as follows: on the one hand, deposit insurance system often apply underwriting limit. For example, China’s deposit insurance regulations stipulate that the maximum repayment of deposit insurance is 500,000 yuan, and the same depositor mustn’t participate in the insurance when the principal and interest of all deposits exceed the limit in the same bank. On the other hand, in order to prevent moral hazard, inter-bank deposits of financial institutions and deposits of bank executives could not be insured either. In order to reflect this, we assumed that the insured proportion of bank deposits is $\rho$ ($\rho \in [0,1]$). At the end of each accounting period, according to the bank’s asset value, which is $V_T$, there are four situations for the compensation of deposit insurance institutions:

(i) While $V_T \geq K$, the value of the bank’s assets worth more than its liabilities. There is no loss on any of its debts.

(ii) While $K_1 + K_2 \leq V_T < K$, the bank goes into liquidation, but the assets of the bank are enough to pay the liabilities of class A and class B. At this time, there is no loss on the deposits, and the compensation amount of deposit insurance is zero.

(iii) While $K_1 \leq V_T < K_1 + K_2$, The assets of the bank are only enough to cover class A liabilities, and only part of class B liabilities are compensated. The amount of loss is $K_1 + K_2 - V_T$. As the priority is the same, insured deposits will bear losses in proportion. At this time, the compensation amount of deposit insurance is $\frac{\rho K_D}{K_2} (K_1 + K_2 - V_T)$.

(iv) While $V_T < K_1$, The assets of the bank are insufficient to pay the liabilities of class A, and the liabilities of class B include the deposit suffer a complete loss. At this time, the compensation amount of the deposit insurance is $\rho K_D$.

Based on the above analysis, the payoff function of deposit insurance institutions can be written as follows:

$$G(V_T) = \begin{cases} 
0 & V_T \geq K_1 + K_2 \\
\frac{\rho K_D}{K_2} (K_1 + K_2 - V_T) & K_1 \leq V_T < K_1 + K_2 \\
\rho K_D & V_T < K_1 
\end{cases}$$

(1)

It can be seen from equation (1) that the deposit insurance price is related to class A liabilities and class B liabilities, and is unrelated to class C liabilities. The larger the total amount ($K_1 + K_2$) of class
A liabilities and class B liabilities is, the higher the risk of deposit loss \((Prob(V_T < K_1 + K_2))\) is, and the higher the price of deposit insurance is. The more class A liabilities \((K_1)\), the higher the risk of total loss of deposits \((Prob(V_T < K_1))\), and the higher the deposit insurance price. When there is no class A liability and class C liability \((K_1 = K_3 = 0)\) in the bank debt, the problem of deposit insurance pricing will transform into a situation that does not consider the sequence of debt repayment.

Based on the structure of bank assets and the payment function of deposit insurance institutions, the closed-form deposit insurance price formula can be obtained, as shown in the following proposition.

Assume that the value of the bank’s assets fits the equation as follows:

\[
\log(V_t) = \log(V_{t-1}) + r + \left(\frac{1}{2} \right) h_t + \sqrt{h_t} \varepsilon_t \tag{2}
\]

\[
h_t = \alpha + \alpha \varepsilon_{t-1} - \gamma \sqrt{h_{t-1}}^2 + \beta h_{t-1} \tag{3}
\]

From where, \(V_T\) represents the asset value of the bank, \(\gamma\) is defined as the risk-free rate of return; \(h_t\) represents the volatility of the return on assets; \(\varepsilon_t\) is defined as a normal perturbation term; Parameter \(\gamma\) reflects the asymmetry of conditional volatility in impact response.

**Proposition**: Suppose variable \(V_T\) satisfies with equations (2) and (3), and there are differences in the sequence of repayment of bank debts. Then the deposit insurance premium that the bank pays is as follows:

\[
P = \frac{\rho K_0}{\pi} \exp \left[ -r(T-t) \right] \left[ \int_{0}^{\infty} \text{Re} \left[ H(\varphi)(f(i\varphi)(K_1+K_2) - f(i\varphi+1)) \right] d\varphi + \pi K_2 G(K_1) \right] \tag{4}
\]

Where

\[
H(\varphi) = \frac{1}{i\varphi} \left( e^{-\text{i} \ln(K_1)} - e^{-\text{i} \ln(K_1+K_2)} \right)
\]

\[
G(K_1) = \frac{1}{2} - \frac{1}{\pi} \int_{0}^{\infty} \text{Re} \left[ \frac{e^{\text{i} \ln K_1}}{i\varphi} f(i\varphi) \right] d\varphi
\]

The proposition means that, the price of deposit insurance should be equal to the present value that deposit insurance institutions expect to pay.

**Proof**: It is obviously, the price of deposit insurance can be written as follows:

\[
P = e^{-r(T-t)} E_t^0 \left[ G(V_t) \right] \tag{5}
\]

According to the function shown in formula (1), the expected payment of a deposit insurance institution under the risk neutral measure is as follows:

\[
E_t^0 \left[ G(V_t) \right] = \int_{0}^{K_1+K_2} \frac{\rho K_0}{K_1} (K_1 + K_2 - V_t) dF(V_t) + \int_{0}^{K_1} \rho K_0 dF(V_t)
\]

\[
= \int_{0}^{\ln(K_1+K_2)} \frac{\rho K_0}{K_1} (K_1 + K_2 - e^x) p(x) dx + \int_{-\infty}^{\ln(K_1)} \rho K_0 p(x) dx
\]

\[
= \frac{\rho K_0}{K_1} (K_1 + K_2) \int_{0}^{\ln(K_1+K_2)} p(x) dx - \frac{\rho K_0}{K_1} \int_{0}^{\ln(K_1)} e^x p(x) dx
\]

\[
+ \rho K_0 \int_{-\infty}^{\ln(K_1)} p(x) dx \tag{6}
\]

According to differential mean value theorem, there exists a parameter \(\theta: 0 < \theta < 1\), such that

\[
H(\varphi) = \frac{K_2}{i\varphi} \frac{e^{-\text{i} \ln(K_1)} - e^{-\text{i} \ln(K_1+K_2)}}{(K_1 + K_2) - K_1}
\]

\[
= K_2 \cdot \exp[-i \ln(K_1 + \theta(K_2 - K_1))] \tag{7}
\]
From where, \( F(V_T) \) represents the cumulative probability distribution function of bank asset \( V_T \), and we made a variable conversion for second equation. Let \( x = \ln(V_T) \), then \( p(x) \) represents the probability density function of \( x \).

Note that \( f(\varphi) \) is the moment generating function for \( p(x) \). Let \( q(x) = e^{x}p(x) \cdot (f(1))^{-1} \), obviously, \( q(x) \) is also a density function. Therefore, according to the properties of the moment generating function, we obtain: \( f(i\varphi) \) is the characteristic function for \( p(x) \); \( \frac{f(i\varphi)}{f(1)} \) is the characteristic function for \( q(x) \).

Then we have

\[
\int_{-\infty}^{\infty} p(x)dx = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \text{Re}\left[\frac{e^{-i\varphi x}}{i\varphi}\right]d\varphi
\]

(8)

\[
\int_{-\infty}^{\infty} q(x)dx = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \text{Re}\left[\frac{e^{-i\varphi(x+1)}}{i\varphi f(1)}\right]d\varphi
\]

(9)

Substitute (8) and (9) into equation (5) and combine them with equation (6), then we have

\[
\int_{\ln(K_{1+K_2})}^{\infty} p(x)dx = \int_{\ln(K_{1})}^{\infty} p(x)dx - \int_{\ln(K_{1+K_2})}^{\infty} p(x)dx
\]

(10)

\[
\int_{\ln(K_{1})}^{\ln(K_{1+K_2})} e^{x}p(x)dx = f(1)\int_{\ln(K_{1})}^{\ln(K_{1+K_2})} q(x)dx
\]

(11)

Substitute equations (10) and (11) into equation (5), we obtain

\[
E_0^q\left[G(V_T)\right] = \frac{\rho K_0(K_{1+K_2})}{\pi K_2} \int_{0}^{\infty} \text{Re}\left[\frac{e^{-ip\ln(K_1)} - e^{-ip\ln(K_{1+K_2})}}{i\varphi} \times \frac{f(i\varphi)}{f(1)}\right]d\varphi
\]

(12)

This means above proposition is true.

**Conclusion**

At present, there are a huge number of researches on the pricing model of deposit insurance, and at the same time, great achievements have been achieved. However, existing research methods rarely take into account the influence of the above factors on deposit insurance price comprehensively, and most literatures only consider one research object. Based on the reference of existing literature, we considered the variance of bank asset income and the sequence of bank debt repayment comprehensively, deduced a new deposit insurance pricing formula. And we also analyzed the differences in ignore the sequence of debt repayment, which have a positive effect for the deposit insurance pricing in our country.
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References


